



Always moving forward

Принципы построения самосогласованных гидро-геомеханических моделей многофазных физических процессов в пласте и скважине



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Goal of the lecture



- ❑ In the short course I will try to present main basic principles of construction selfconsistent hydro-geomechanical models for complicated reservoir physics description
- ❑ A special attention I put on clear formulation of continuum mechanics models for one and two phase media, suitable for one-point-view description of reservoir and well flow+deformation processes. I intentionally do not get too deep in rigorous formulations to let the auditory feel that the procedure is simple (as, in fact, it is, if not to catch sight of mathematical tricks).
- ❑ I hope that a few practical examples I brought up would help people to believe that they could construct the models themselves if needed, or, at least to understand if someone constructed such models for them in a wrong way.
- ❑ I describe the experimental rock mechanic in detail sufficient enough to start believe in its constructive power and underline the value of in-situ field measurements.
- ❑ I emphasize the pivotal role of the MEM construction process – the best achievement in reservoir engineering in last decades (in my view).
- ❑ I truly thank organizers for the invitation to read the course, wish my audience for patience and thank it in advance for attention





Plan of the lecture



Part 1. Introduction. Examples of geomechanical effects.

Part 2. Basic concepts of continuum mechanics.

Part 3. Experimental Rock mechanics.

Part 4. Building and calibration of MEM.



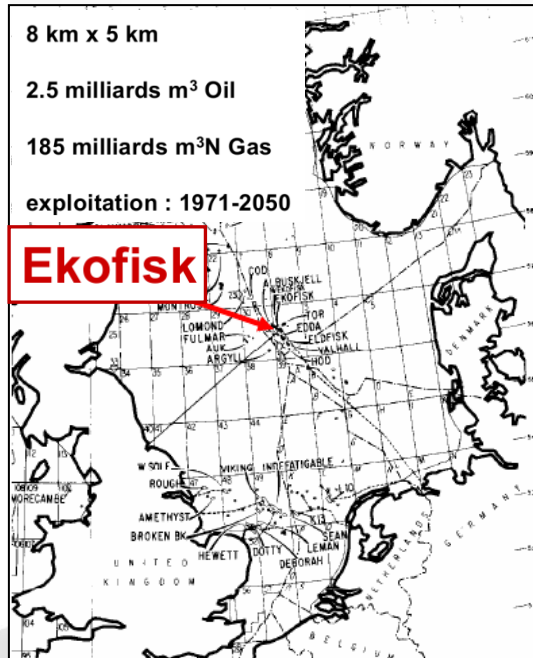


Part 1. Introduction. Examples of geomechanical effects.

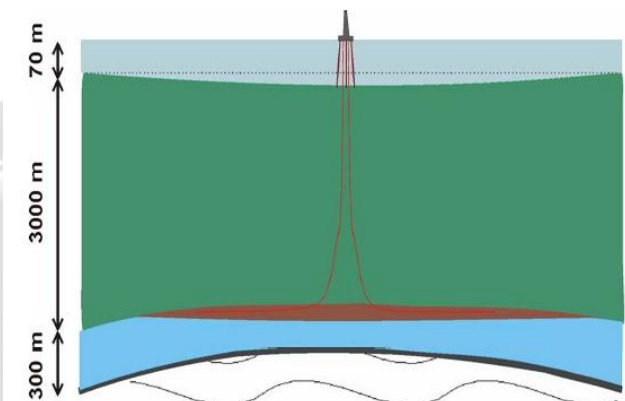


Introduction. Examples of geomechanical effects.

1. Compaction and subsidence. Ekofisk – a godfather of GM

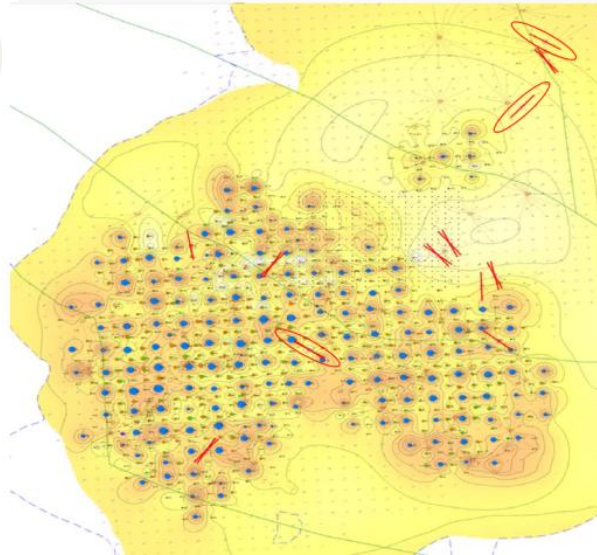


- ❑ Andersen M.A. (1995) reported the history of Ekofisk field development
- ❑ 1970: Start of production caused pore pressure depletion with followed compaction
- ❑ 1975-1977: Reinjection of gas
- ❑ 1985: Subsidence (3 meters)
- ❑ 1986: Injection of sea water
- ❑ 1987: Conoco-Philips had to rise up the rigs on 6 meters. Several rigs were completely reconstructed
- ❑ Permanently problems with well stability were observed
- ❑ Since 1982 oil production companies started collaboration with Norwegian government for geomechanical research of chinks in North Sea
- ❑ 2000: subsidence up to 8 meters
- ❑ Now: subsidence rate is 50cm/year, evolution of pore pressure from 49 MPa to 24MPa



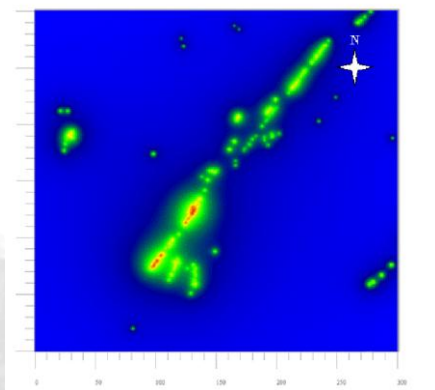
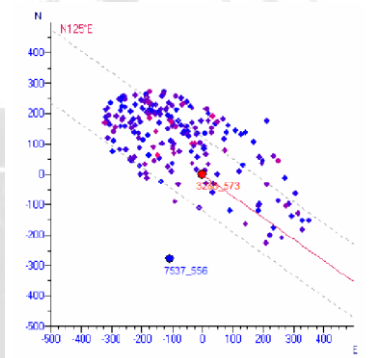
Introduction. Examples of geomechanical effects.

2. EOR. Hydraulic fracturing design (1/2): field data

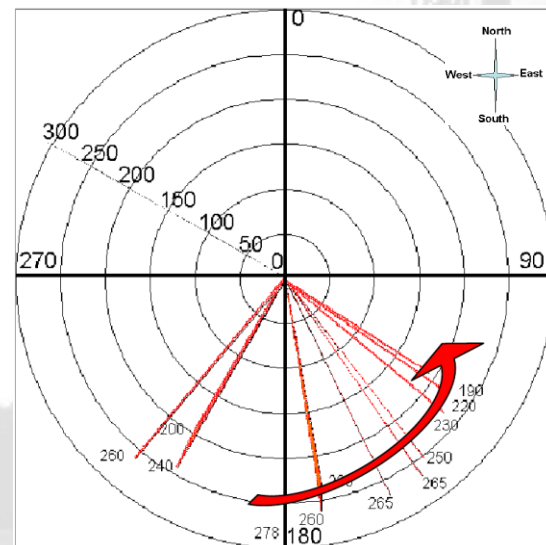


Malo-Balykskoe oilfield and recorded fracture azimuths. Ovals correspond to the micro seismic measurements

- ❑ *Kuzmina et al. (SPE 12074)* reported on depletion influence on HF orientation in Malo-Balykskoe oilfield of Western Siberia
- ❑ Production from the oilfield started in 1987, but until 2008, drilling was carried out only in about 40% of the field
- ❑ Anisotropy in horizontal stress is small in tectonically relaxed Western Siberian region (2-3%), so small depletion in pressure could cause significant rotation of the preferred fracture plane (155 degree from North)



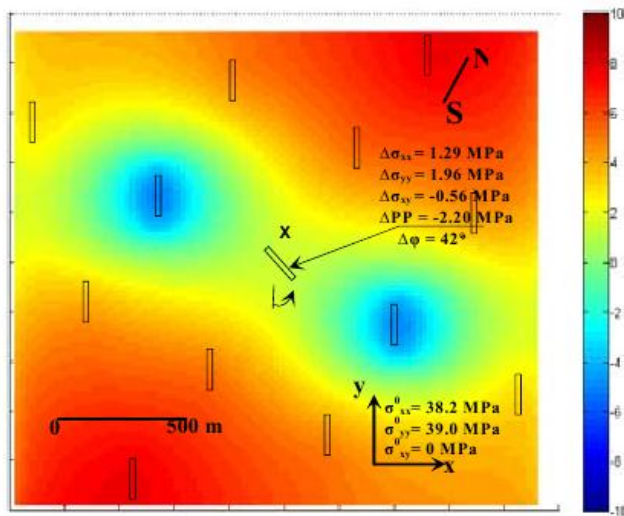
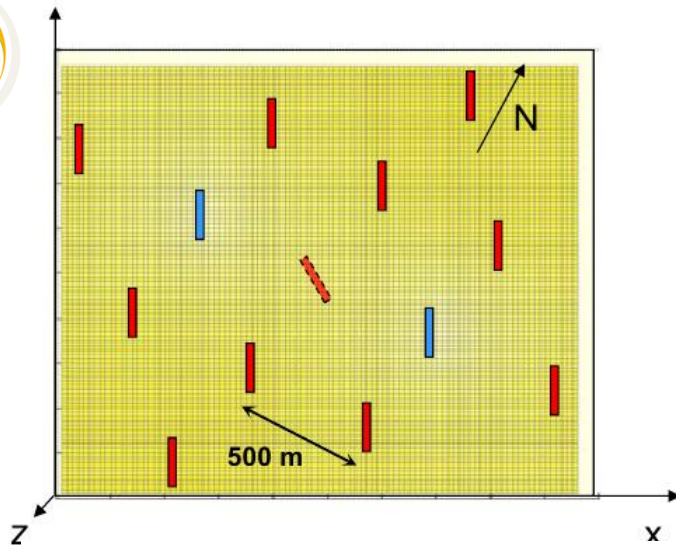
Micro seismic recording on well 3288-573 and 5538-605



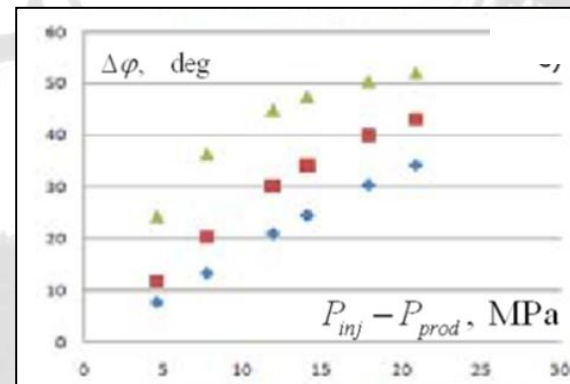
Fracture orientation tends to turn towards a SE direction

Introduction: examples of geomechanical effects:

3. EOR. Hydraulic fracturing design (2/2): modeling

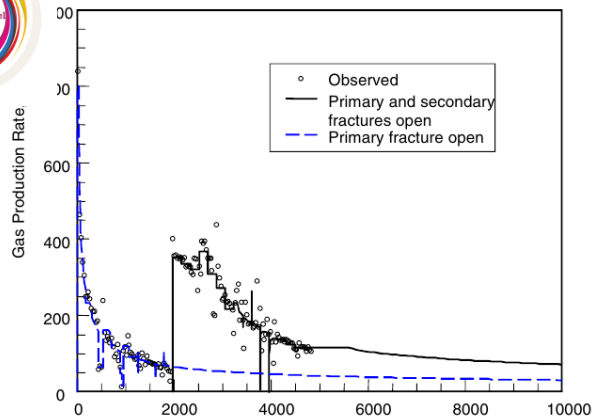


- ❑ *Chertov et al. (SPE 135679)* studied numerically the depletion influence on HF orientation reported by *Kuzmina et al. (SPE 12074)*
- ❑ There is no well in the center of the pattern, where stress changes are analyzed to evaluate how the fracture orientation may change in a newly drilled well at that location
- ❑ The parametric study shows that thicker pay zones result in higher rotations and that the swing angle is very sensitive to the initial stress anisotropy. Initial stress anisotropy less 20% allows measureable fracture rotations (1,2 and 3%) are shown at the figure

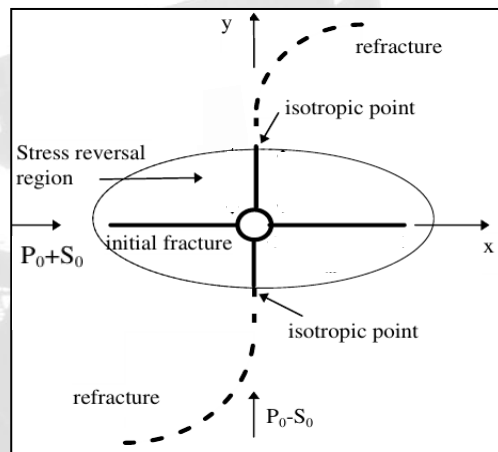


Introduction: examples of geomechanical effects:

4. EOR. Refracturing optimization

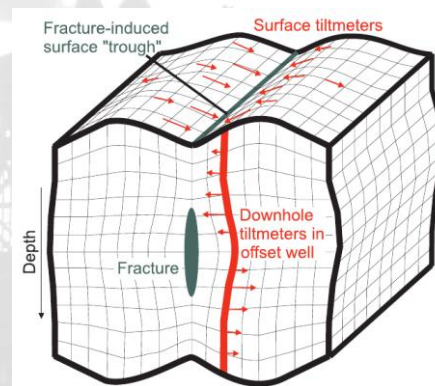


Production history of Well C before and after successful refracturing

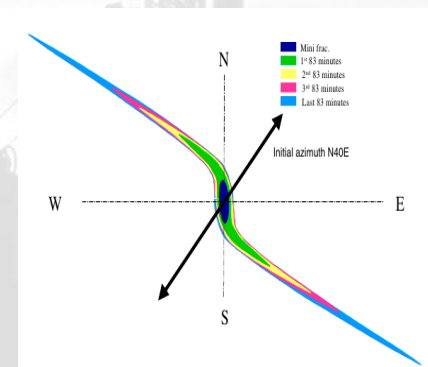


Refracture reorientation concept

- ❑ *Siebrits et al. (SPE 63030)* provided evidence of increased production due to refracturing two tight gas wells in the Barnett Shale, north of Fort Worth having deeply penetrating initial fractures
- ❑ If the induced stress changes are large enough to overcome the effect of the initial horizontal deviatoric stress, then the direction of the minimum horizontal stress becomes the maximum within a reversal region and a refracture will propagate at 90 degrees to the initial fracture azimuth.
- ❑ Fracture length can not be determined from the surface tilt data, but azimuth is detailed quite accurately



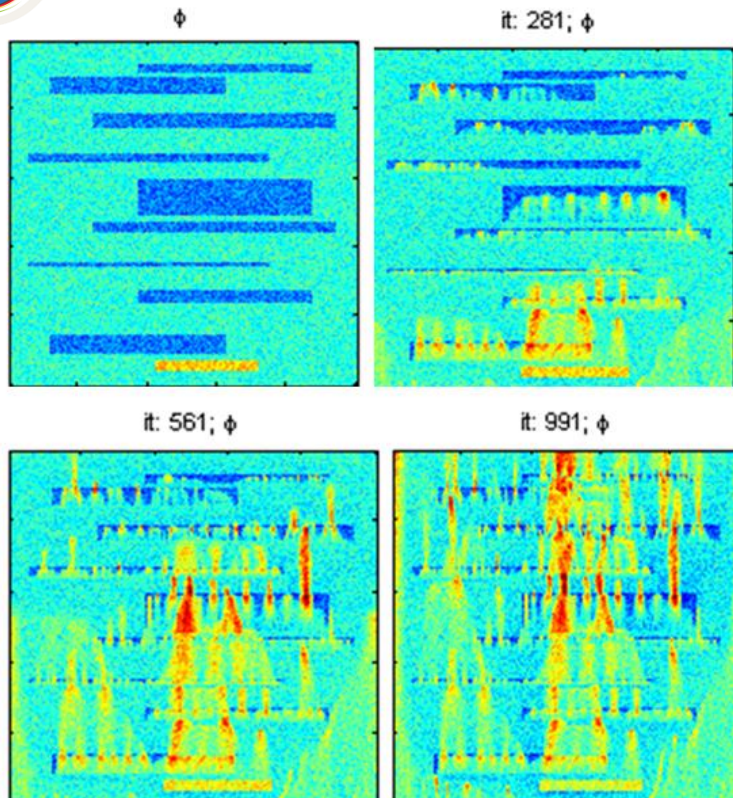
Induced deformation can be measured with a surface and downhole arrays of tiltmeters



Refracture azimuth as a function of time, from the tiltmeter data.

Introduction. Examples of geomechanical effects.

5. Cap rock integrity test (CO₂ injection at Sleipner)

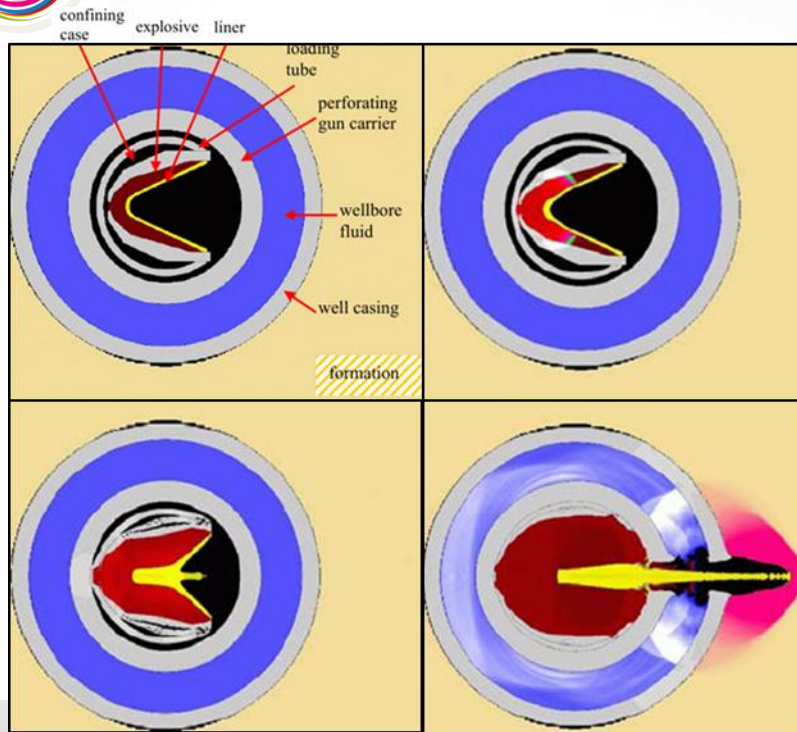


Evolution of porosity for four different times, showing the development of high porosity pathway flow (chimneys) and breaching of the shale layers

- ❑ *N.S.C. Simon and Y.Y. Podladchikov (2013)* performed a cap rock integrity test regarding CO₂ storage in North Sea
- ❑ Since 1996, one million ton of CO₂ per year has been injected into the Utsira formation at Sleipner, North Sea.
- ❑ This operation is usually regarded as a show-case for safe CO₂ underground storage.
- ❑ A recent puzzling observation is the occurrence of so- called chimneys visible in the time laps seismic images which act as pathways for fast CO₂ transport upwards from the injection well to the reservoir-cap rock interface.
- ❑ Developed couple hydro-geomechanical model makes possible to observe porosity evolution in time and examine various senario to avoid chimney formation

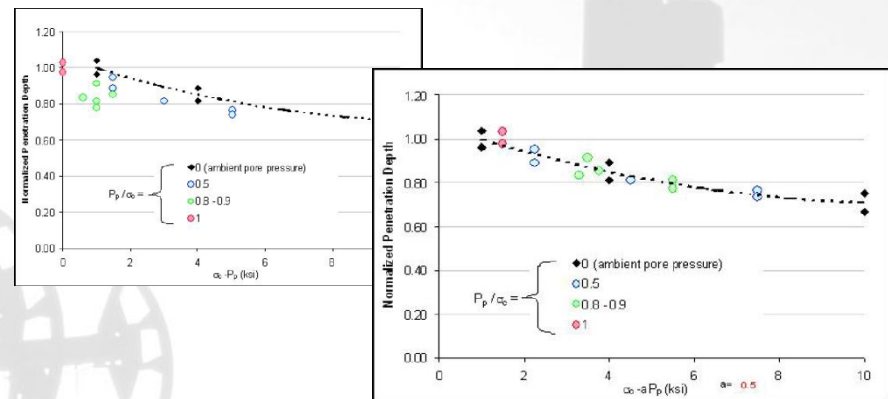
Introduction. Examples of geomechanical effects.

6. Completion. Cumulative perforation



Computer simulation of perforating shaped charge function sequence: 0,2,6,25 msc. 2 msc - liner has begun to collapse; 6 msc - explosive detonation is complete and jet tip has been formed; 25 msc - perforation tunnel creation is underway, jet tail continues to form

- Thompson (1962), Weeks (1974), Behrmann et al (1988) conducted a comprehensive experimental efforts to evaluate that perforation depth decreases as formation strength or effective stress increases
- Grove et al (2008) suggested a new definition of effective stress which better fits experimental data

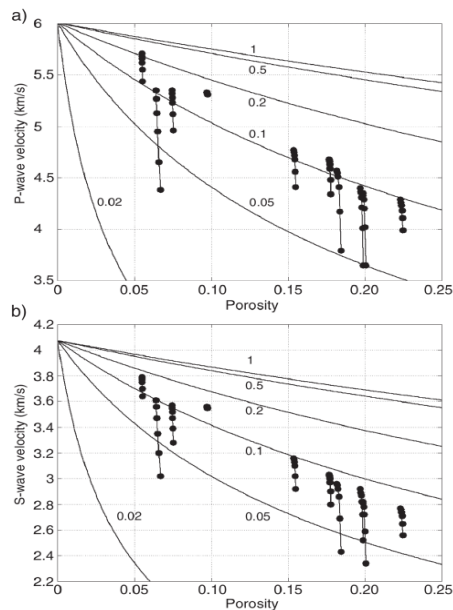


Shaped charge penetration depth vs. rock effective stress – classical and effective definitions

- Myasnikov & Zazovsky (2009) suggested to improve perforation design optimization by splitting the process by penetrating and channel shaping parts

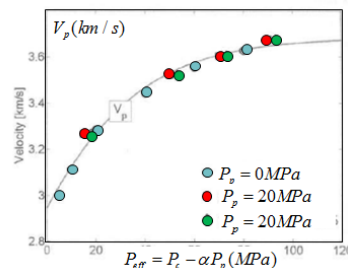
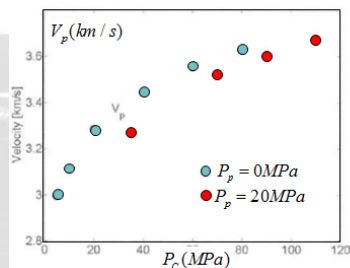
Introduction. Examples of geomechanical effects.

7. Seismic interpretation

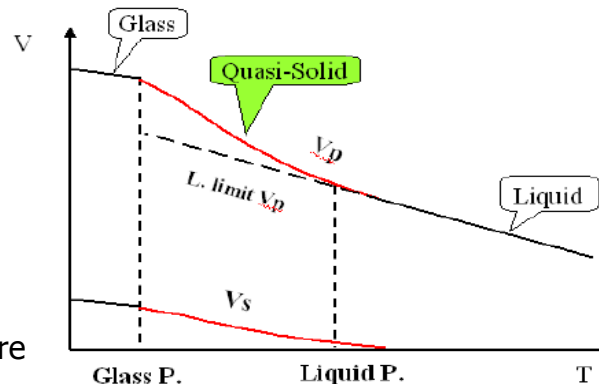


Velocity measurements for 24 room-dry sandstones, among which were 10 clean sands stated to have 0% clay

- ❑ *Han (1986)* measured compressional and shear-wave velocities for 24 sandstones and established significant dependence on confining stresses, porosities and clay content.
- ❑ *Hornby (1995, 1998)* measured ultrasonic P-wave velocity as a function of confining stress on samples with zero and non-zero pore pressure and found it increase with effective pressure increase
- ❑ *Han et al (2006)* measured ultrasonic velocities on 10 heavy oil samples at different phases as a function of temperature. With temperature decreases below the liquid point, heavy oil transfers from liquid phase to a quasi-solid phase with drastic increase of viscosity, S-wave velocity appears measurable and P-wave velocity deviated up from the light oil trend.
- ❑ *Myasnikov et al (2007)* suggested to estimate rheological content of rocks by classification of peculiarities of measured attenuation (rheological monitoring concept)

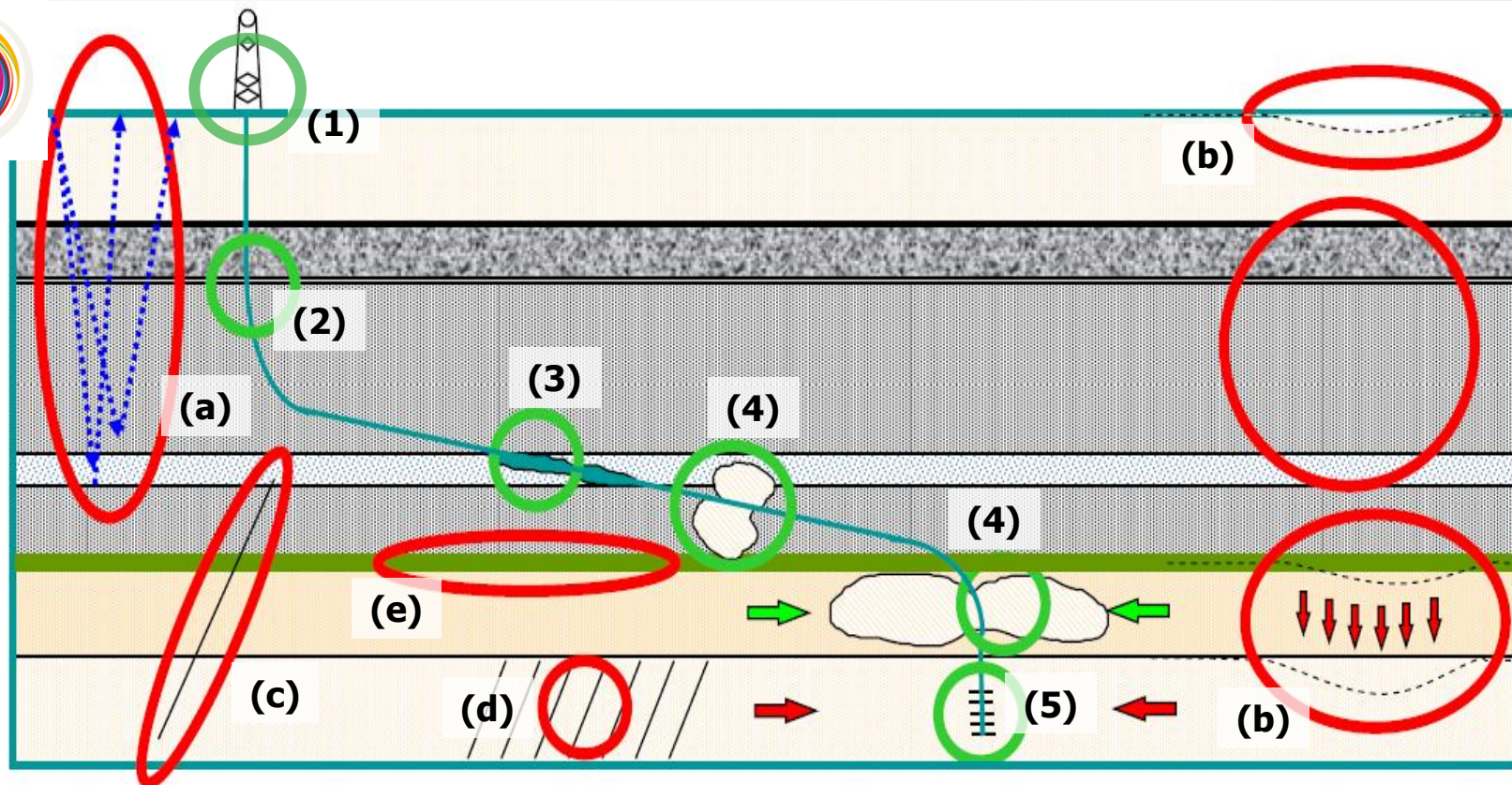


P-wave velocity depends on both confining and pore pressure through their combination – effective pressure



Velocity of heavy oil relates to temperature in three phases

Introduction. Examples of geomechanical effects. Recapitulating.



At Near Wellbore scale:

- 1) Well Placement & Trajectory
- 2) Drilling bit and fluid selection
- 3) Wellbore Stability
- 4) Reservoir Stimulation
- 5) Sand Production Control

At Reservoir scale:

- a) 3D, 4D seismic, microseismic
- b) Compaction and subsidence
- c) Fault reactivation
- d) Fractured reservoirs
- e) Slips along soft layers and bedding places



Part 2. Basic concepts of continuum mechanics.

Basic concepts of continuum mechanics.

Forces in continuum mechanics

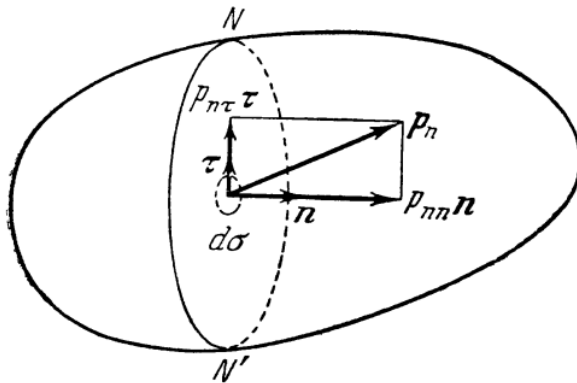


- ❑ Forces - is a fundamental concept in continuum mechanics They cause a motion and, in turn appear as a result of bodies interaction
- ❑ Forces are classified as concentrated and distributed (formally, since all forces in continuum mechanics are distributed)
- ❑ Forces are classified as external and internal.
 - External forces are caused by external bodies
 - Internal forces are caused by internal processes in a body
- ❑ Forces are classified as surface and volume (or mass)
 - Surface forces result interaction between bodies happen through surface of their contact
 - Volume (mass) surface is the result of penetration of bodies each other

$$\mathbf{F}_V = \rho \mathbf{F}_M, \quad \rho = \lim \frac{\Delta m}{\Delta V}$$

Basic concepts of continuum mechanics.

Internal stress force. Stress tensor definition



- The forces $\mathbf{p}_n$ distributed over a unit of surface are called the internal stress forces

$$d\mathbf{P}_n = \boxed{\mathbf{p}_n} d\sigma$$

- Properties of surface forces of internal interaction:

- certain general characteristics defined by particular form of the continuum
- certain universal properties which apply to any body, followed from general dynamic equations for the change of linear and angular momentum for any material media in arbitrary motion

$$\frac{d}{dt} \int_V \rho \mathbf{v} d\tau = \int_V \rho \mathbf{F} d\tau + \int_{\Sigma} \mathbf{p}_n d\sigma,$$

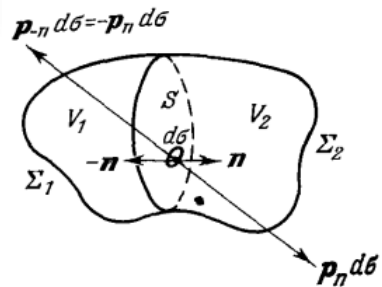
$$\frac{d}{dt} \int_V [\mathbf{r} \times \mathbf{v}] \rho d\tau = \int_V [\mathbf{r} \times \mathbf{F}] \rho d\tau + \int_{\Sigma} [\mathbf{r} \times \mathbf{p}_n] d\sigma$$

Basic concepts of continuum mechanics.

Stress tensor properties



- If acceleration is finite, $\mathbf{p}_n = -\mathbf{p}_{-n}$ for any direction of $\mathbf{n}$



$$\frac{d}{dt} \int_V \rho \mathbf{v} d\tau = \int_V \rho \mathbf{F} d\tau + \int_{\Sigma} \mathbf{p}_n d\sigma$$

$$\frac{d}{dt} \int_{V_1} \rho \mathbf{v} d\tau = \int_{V_1} \rho \mathbf{F} d\tau + \int_{\Sigma_1} \mathbf{p}_n d\sigma + \int_S \mathbf{p}_n d\sigma$$

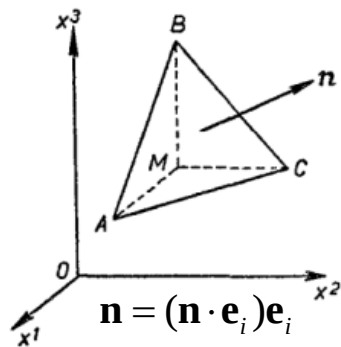
$$\frac{d}{dt} \int_{V_2} \rho \mathbf{v} d\tau = \int_{V_2} \rho \mathbf{F} d\tau + \int_{\Sigma_2} \mathbf{p}_n d\sigma + \int_S \mathbf{p}_{-n} d\sigma$$

- The stress on any element of area defined by $\mathbf{n}$ can be considered as a linear vector function $\mathbf{p}_n = \mathbf{P} \cdot \mathbf{n}$ defined by

$$\mathbf{p}_n = \mathbf{p}^i (\mathbf{n} \cdot \mathbf{e}_i) = P^{ki} \mathbf{e}_k n_i = P^{ki} \mathbf{e}_k (\mathbf{e}_i \cdot \mathbf{n})$$

$$\mathbf{P} = P^{\alpha\beta} \mathbf{e}_\alpha \mathbf{e}_\beta$$

- From classical angular momentum equation one can derive $P^{\alpha\beta} = P^{\beta\alpha}$



$$\int_{\Sigma} [\mathbf{r} \times \mathbf{p}_n] d\sigma = \int_{\Sigma} [\mathbf{r} \times \mathbf{p}^i n_i] d\sigma = \int_V \nabla_i [\mathbf{r} \times \mathbf{p}^i] d\tau = \int_V [\mathbf{r} \times \nabla_i \mathbf{p}^i] d\tau + \int_V [\mathbf{e}_i \times \mathbf{e}_j] P^{ij} d\tau$$

$$\frac{d}{dt} \int_V [\mathbf{r} \times \mathbf{v}] \rho d\tau = \int_V [\mathbf{r} \times \mathbf{F}] \rho d\tau + \int_{\Sigma} [\mathbf{r} \times \mathbf{p}_n] d\sigma$$

$$\int_V \mathbf{r} \times \left[\frac{d}{dt} \mathbf{v} - \mathbf{F} - \frac{1}{\rho} \nabla_i \mathbf{p}^i \right] \rho d\tau = \int_V [\mathbf{e}_i \times \mathbf{e}_j] P^{ij} d\tau$$

$$[\mathbf{e}_i \times \mathbf{e}_j]_{i < j} (P^{ij} - P^{ji}) = 0$$

Basic concepts of continuum mechanics. Principal axis of Stress tensor. More circles



- Principal axis of stress tensor are determined by conditions

$$\mathbf{p}_n \parallel k\mathbf{n} \Rightarrow \mathbf{P} \cdot \mathbf{n} - k\mathbf{n} = 0 \Rightarrow \det(\mathbf{P} - k\mathbf{U}) = 0$$

$$k^3 - I_1 k^2 + I_2 k - I_3 = 0$$

$$P_{(1)}, P_{(2)}, P_{(3)} - \text{principal components}$$

$$\begin{cases} I_1 = P_{11} + P_{22} + P_{33} \\ I_2 = P_{11}P_{22} + P_{22}P_{33} + P_{33}P_{11} \\ I_3 = P_{11}P_{22}P_{33} \end{cases} \quad \left(\mathbf{P} - P_{(k)}\mathbf{U} \right) \cdot \mathbf{n}^{(k)} = 0$$

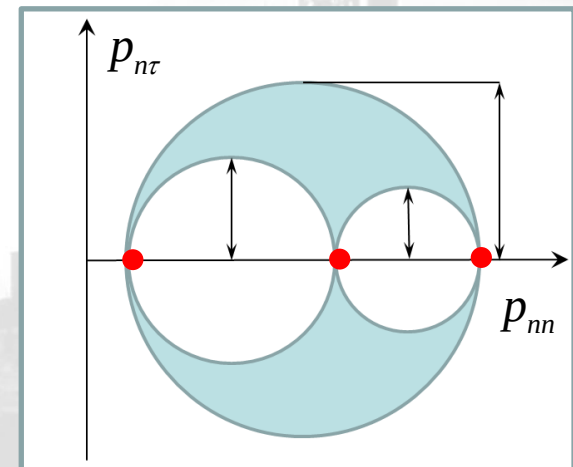
(no summation)

- Since principal axis coincide to principal axis of Cauchy stress surface $P^{ij}x_i x_j = P_N r^2 = \text{const}$, maximal and minimal normal stresses at the point belong to the set of principal components

$$\mathbf{p}_n = p_{nn}\mathbf{n} + p_{n\tau}\boldsymbol{\tau}, \quad p_{nn} = P^{ij}n_i n_j, \quad p_{n\tau}^2 = \mathbf{p}_n \cdot \mathbf{p}_n - p_{nn}^2$$

in principal axis $P_{(1)} > P_{(2)} > P_{(3)}$

$$\begin{cases} p_{n\tau}^2 + p_{nn}^2 = P_{(k)}^2 n_k^2 \\ p_{nn} = P_{(k)} n_k^2 \\ \sum_k n_k^2 = 1 \end{cases} \quad n_k^2 = \frac{\prod_{i \neq k} (p_{nn} - P_{(i)}) + p_{n\tau}^2}{\prod_{i \neq k} (P_{(k)} - P_{(i)})}$$



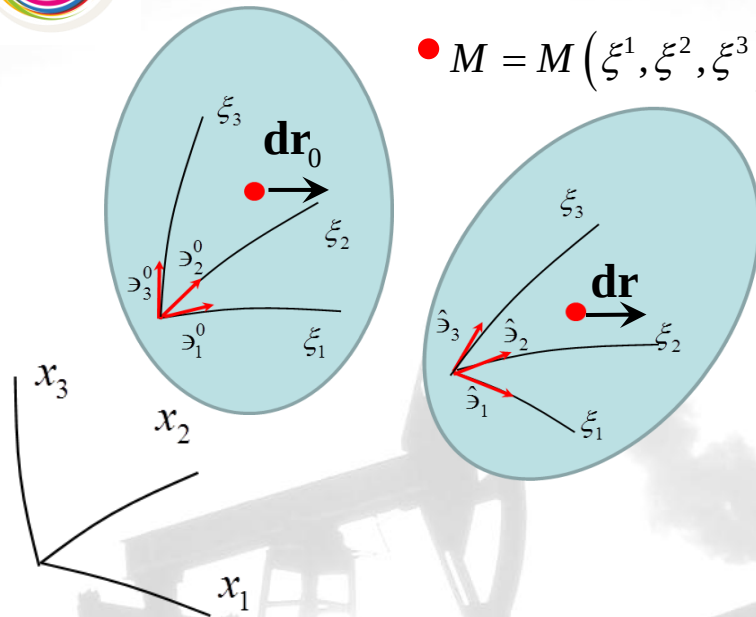
Basic concepts of continuum mechanics.

Deformation. Strain tensor.



- Eulerian and Lagrangian approaches for deformation description:

- $M = M(\xi^1, \xi^2, \xi^3)$
 - Concomitant coordinates $\xi^i = \xi^i(x^1, x^2, x^3, t)$
 - Reference coordinates $v^i = v^i(x^1, x^2, x^3, t)$



$$d\mathbf{r} = \frac{\partial \mathbf{r}}{\partial \xi^\alpha} d\xi^\alpha \equiv d\xi^\alpha \mathfrak{e}_\alpha$$

$$|d\mathbf{r}|^2 = \mathfrak{e}_\alpha \cdot \mathfrak{e}_\beta d\xi^\alpha d\xi^\beta \equiv \hat{g}_{\alpha\beta} d\xi^\alpha d\xi^\beta$$

$$|d\mathbf{r}_0|^2 = \mathfrak{e}_\alpha^0 \cdot \mathfrak{e}_\beta^0 d\xi^\alpha d\xi^\beta \equiv g_{\alpha\beta}^0 d\xi^\alpha d\xi^\beta$$

$$|d\mathbf{r}|^2 - |d\mathbf{r}_0|^2 = 2\varepsilon_{\alpha\beta} d\xi^\alpha d\xi^\beta, \quad \varepsilon_{\alpha\beta} = \frac{1}{2}(\hat{g}_{\alpha\beta} - g_{\alpha\beta}^0)$$

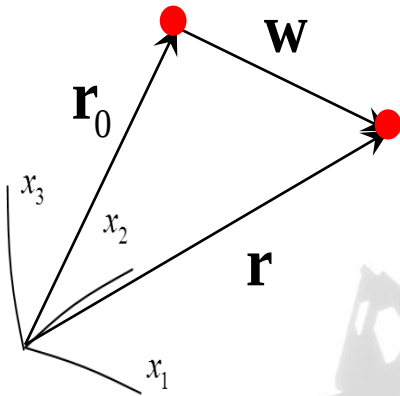
$$\varepsilon_{\alpha\beta} \rightarrow \begin{array}{ll} \boldsymbol{\varepsilon}^0 = \varepsilon_{\alpha\beta} \mathfrak{e}^{0\alpha} \mathfrak{e}^{0\beta}, & \hat{\boldsymbol{\varepsilon}} = \varepsilon_{\alpha\beta} \hat{\mathfrak{e}}^\alpha \hat{\mathfrak{e}}^\beta \\ \text{Green tensor} & \text{Almansi tensor} \end{array} \quad \mathfrak{e}^\alpha = g^{\alpha\beta} \mathfrak{e}_\beta, \quad \|g^{\alpha\beta}\| = \|g_{\alpha\beta}\|^{-1}$$

Basic concepts of continuum mechanics.

Displacement vector. Strain and displacement.



If displacement vector $\mathbf{w}$ exists, $\mathbf{r} = \mathbf{r}_0 + \mathbf{w}$



$$\mathfrak{r}_i = \mathfrak{r}_i^0 + \frac{\partial \mathbf{w}}{\partial \xi^i}$$

$$\mathbf{w} = w^{0\alpha} \mathfrak{r}_\alpha^0 = w^\alpha \mathfrak{r}_\alpha$$

$$\frac{\partial \mathbf{w}}{\partial \xi^i} = \frac{\partial w^{0\alpha}}{\partial \xi^i} \mathfrak{r}_\alpha^0 + w^{0\alpha} \frac{\partial \mathfrak{r}_\alpha^0}{\partial \xi^i} = (\nabla_i^0 w^{0s}) \mathfrak{r}_s^0$$

$$\nabla_i^0 w^{0s} = \frac{\partial w^{0s}}{\partial \xi^i} + w^{0\alpha} \Gamma_{\alpha i}^{0s}$$

$$2\varepsilon_{\alpha\beta} = \widehat{\mathfrak{r}}_\alpha \cdot \widehat{\mathfrak{r}}_\beta - \mathfrak{r}_\alpha^0 \cdot \mathfrak{r}_\beta^0 = (\nabla_\beta^0 w^{0s}) g_{\alpha s}^0 + (\nabla_\alpha^0 w^{0s}) g_{\beta s}^0 + (\nabla_\alpha^0 w^{0s}) (\nabla_\beta^0 w^{0l}) g_{ls}^0$$

$$2\varepsilon_{\alpha\beta} = \nabla_\beta^0 w_\alpha^0 + \nabla_\alpha^0 w_\beta^0 + \nabla_\alpha^0 w^{0s} \nabla_\beta w_s^0 \quad (\nabla_\alpha^0 g_{\beta s}^0 = 0)$$

$$\varepsilon_{\alpha\beta} = \frac{1}{2} (\nabla_\beta w_\alpha + \nabla_\alpha w_\beta - \nabla_\alpha w^s \nabla_\beta w_s)$$

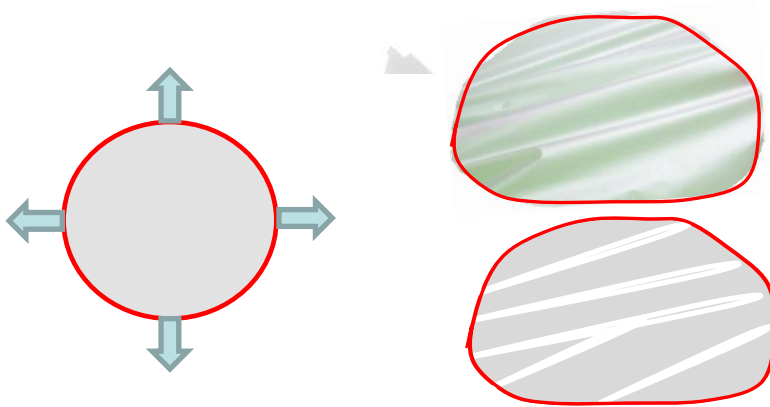
$$\boxed{\boldsymbol{\varepsilon} = (\nabla \mathbf{w})^S} \quad \text{for small deformations}$$

Basic concepts of continuum mechanics.

Compatibility relations



- ❑ Six independent strain tensor components depend on derivatives of the independent functions.
- ❑ Thus, additional constraint conditions must exist.
- ❑ In Riemann space, locally, one can find a new coordinate system $\eta^i = \eta^i(\xi^1, \xi^2, \xi^3)$, in which



$$\Gamma_{ij}^{0\gamma} = \left(\Gamma_{\alpha\beta}^{\omega} \frac{\partial \xi^{\alpha}}{\partial \eta^i} \frac{\partial \xi^{\beta}}{\partial \eta^j} + \frac{\partial^2 \partial \xi^{\omega}}{\partial \eta^i \partial \eta^j} \right) \frac{\partial \eta^{\gamma}}{\partial \xi^{\omega}} = 0$$

- ❑ In Euclidian space, such coordinate system can be found totally. The condition for that is the integrability condition for the equations

$$\Gamma_{ij}^{0\gamma} = 0$$

that is, for small deformations

$$R_{ij\mu\nu} = \frac{1}{2} \left[\frac{\partial^2 \varepsilon_{vi}}{\partial x^j \partial x^{\mu}} + \frac{\partial^2 \varepsilon_{\mu j}}{\partial x^i \partial x^{\nu}} - \frac{\partial^2 \varepsilon_{\mu i}}{\partial x^j \partial x^{\nu}} - \frac{\partial^2 \varepsilon_{\nu j}}{\partial x^i \partial x^{\mu}} \right] = 0$$

Basic concepts of continuum mechanics.

Strain rate tensor. Geometrical meaning of the components



- For a fixed material point $M = M(\xi^1, \xi^2, \xi^3)$ we define a change rate in deformation between two states at $t = t_0$ and $t = t_0 + \Delta t$ as

$$e_{\alpha\beta} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \varepsilon_{\alpha\beta}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{1}{2} \frac{\hat{g}'_{\alpha\beta} - \hat{g}_{\alpha\beta}}{\Delta t} = \frac{1}{2} \frac{g'_{\alpha\beta} - g_{\alpha\beta}^0}{\Delta t} = \frac{1}{2} \frac{d\hat{g}_{\alpha\beta}}{dt} - \frac{1}{2} \frac{dg_{\alpha\beta}^0}{dt}$$

$$\frac{dg^0}{dt} = 0 \Rightarrow \mathbf{e} = \hat{\mathbf{e}} = \frac{d\hat{\boldsymbol{\varepsilon}}}{dt} = (\nabla \mathbf{v})^s$$

in the reference coordinate (*Cotter-Rivlin rate*)

$$\frac{D\boldsymbol{\varepsilon}}{dt} \equiv \frac{d\boldsymbol{\varepsilon}}{dt} + \boldsymbol{\varepsilon} \cdot \nabla \mathbf{v} + (\nabla \mathbf{v})^T \cdot \boldsymbol{\varepsilon} = (\nabla \mathbf{v})^s$$

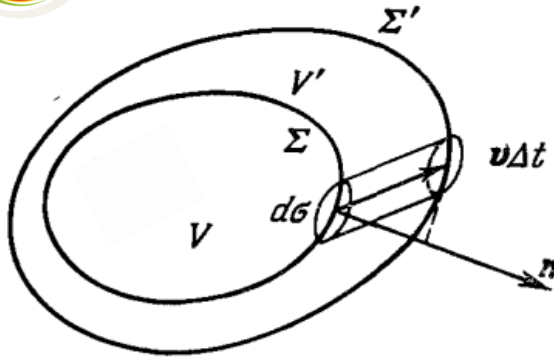
- Note that deformation rate tensor is defined with respect to the current moment of time
- Geometrical meaning of the components:
 - diagonal terms $e_{\alpha\alpha}(\varepsilon_{\alpha\alpha})$ equal to rate of linear dilation (linear dilation) in direction ξ_α
 - non-diagonal terms $e_{\alpha\beta}(\varepsilon_{\alpha\beta})$ equal to distortion related to directions ξ_α, ξ_β
 - principal component $e_{\alpha\alpha} = \frac{d\varepsilon_{\alpha\alpha}}{dt} = \frac{d}{dt} \frac{dV - dV_0}{dV_0} = \text{div} \mathbf{v}$

Basic concepts of continuum mechanics.

Differentiation of a integral over a moving volume



- Differentiation of a integral over a moving volume with respect to time:



$$\frac{d}{dt} \int_{V(t)} f(\mathbf{r}, t) d\tau = \int_V \left(\frac{\partial f}{\partial t}(\mathbf{r}, t) + \nabla \cdot f\mathbf{v} \right) d\tau$$

- Indeed, since

$$d\tau = v_n d\sigma \Delta t, \quad \Sigma' \rightarrow \Sigma, \quad f(\mathbf{r}, t + \Delta t) \rightarrow f(\mathbf{r}, t)$$

Thus,

$$\begin{aligned} \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \left[\int_{V'} f(\mathbf{r}, t + \Delta t) d\tau - \int_V f(\mathbf{r}, t) d\tau \right] &= \\ \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \left[\int_V [f(t + \Delta t) - f(t)] d\tau - \int_{V' - V} f(t + \Delta t) d\tau \right] &= \\ = \int_V \frac{\partial f}{\partial t} d\tau + \int_{\Sigma} f v_n d\sigma \end{aligned}$$

Basic concepts of continuum mechanics.

Conservation of mass, momentum and total energy



□ Three immediate consequences:

1 mass conservation: $f = \rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V}, \quad m = \int_V \rho d\tau, \quad \frac{dm}{dt} = 0 \Rightarrow$

- Mass does not change

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

2 momentum conservation: $f = \rho \mathbf{v}, \quad m \mathbf{v}^* = \int_V \rho \mathbf{v} d\tau, \quad \frac{dm \mathbf{v}^*}{dt} = \int_V \rho \mathbf{F} d\tau + \int_{\Sigma} \mathbf{p}_n d\tau \Rightarrow$

- Momentum changes due to volume and surface forces applied to the system

$$\rho \frac{d\mathbf{v}}{dt} = \nabla \cdot \mathbf{P} + \rho \mathbf{F}$$

3 energy conservation: $f = \rho e, \quad E = \int_V \rho e d\tau, \quad \frac{dE}{dt} = \int_V \nabla \cdot (\mathbf{P} \cdot \mathbf{v} + \mathbf{J}_q) d\tau \Rightarrow$

- Energy changes due to energy flux generated by mechanical work over the system plus thermal flux

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot \mathbf{J}_e = 0, \quad \mathbf{J}_e = \rho e \mathbf{v} + \mathbf{P} \cdot \mathbf{v} + \mathbf{J}_q$$

Basic concepts of continuum mechanics.

Internal energy. First law of thermodynamics



- The kinetic energy theorem and work of internal forces:

Multiply both part of momentum equation $\rho \frac{d\mathbf{v}}{dt} = \nabla \cdot \mathbf{P} + \rho \mathbf{F}$ by $d\mathbf{r} = \mathbf{v} dt$:

$$\frac{dv^2}{2} = \boxed{\mathbf{F} \cdot d\mathbf{r}} + \boxed{\frac{1}{\rho} \nabla (\mathbf{P} \cdot \mathbf{v}) dt} - \boxed{\frac{1}{\rho} \mathbf{P} : \nabla \mathbf{v} dt}, \quad dK = \boxed{dA_M^{ext}} + \boxed{dA_{sur}^{ext}} + \boxed{dA_{sur}^{int}}$$

- Let $\mathbf{F} = -\nabla \Psi$, $\frac{\partial \Psi}{\partial t} = 0$. From continuity equation one then obtains:

$$\frac{\partial \Psi \rho}{\partial t} = -\nabla \cdot \Psi \rho \mathbf{v} - \rho \mathbf{v} \cdot \mathbf{F}$$

- If internal energy density u is defined as

$$e = u + \frac{1}{2} v^2 + \Psi$$

with conservation condition for the total energy density e as defined above

$$\rho \frac{du}{dt} = -\nabla \cdot \mathbf{J}_q + \mathbf{P} : \nabla \mathbf{v}$$

- Recall, the first law of thermodynamics states that internal energy is a one-valued function of the system state which can change only by external influence on its external or internal parameters

Basic concepts of continuum mechanics.

Entropy. Second law of Thermodynamics. Gibbs identity



- The second law of thermodynamics states, that for any macroscopic system a state function “entropy” exists which has the following properties:
 - The entropy change dS can be spitted as $dS = d_e S + d_i S$, where $d_i S > 0$ for irreversible processes and $d_i S = 0$ for reversible.
 - For closed systems $d_e S = dQ / T$, while for open systems it can has arbitrary value.
- Turning again to the “moving volume integration rule”:

[4] entropy conservation: $f = \rho s, \quad S = \int_V \rho s d\tau, \quad \frac{dS}{dt} = \int_V (-\nabla \cdot \mathbf{J}_s + \sigma) d\tau \Rightarrow$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot \mathbf{J}_{s,tot} = \sigma, \quad \mathbf{J}_{s,tot} = \rho s \mathbf{v} + \mathbf{J}_s, \quad \sigma \geq 0$$

- Combining the first and second laws of thermodynamics, $u = u(s, a_1, \dots, a_n, \mu_1, \dots, \mu_k)$ is a total differential of entropy, external a_n and internal μ_k parameters, that is

$$du = Tds + \sum \frac{\partial u}{\partial a_i} da_i + \sum \frac{\partial u}{\partial \mu_j} d\mu_j \quad \text{Gibbs identity}$$

- For an open systems it supposed to be correct along the trajectory of a particle, which is set then to be in local equilibrium conditions

Basic concepts of continuum mechanics.

One phase models: Recapitulating the laws



□ Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

□ Linear momentum equations:

$$\rho \frac{d\mathbf{v}}{dt} = \rho \mathbf{F} + \nabla \cdot \mathbf{P}$$

□ Angular momentum equations:

$$\mathbf{P} = \mathbf{P}^T$$

□ Energy equation:

$$\rho \frac{du}{dt} = -\nabla \cdot \mathbf{J}_q + \mathbf{P} : \nabla \mathbf{V}$$

□ Thermodynamical restrictions:

$$\frac{\partial \rho s}{\partial t} + \nabla \cdot \mathbf{J}_{s,tot} = \sigma, \quad \mathbf{J}_{s,tot} = \rho s \mathbf{v} + \mathbf{J}_s, \quad \sigma \geq 0$$

$$du(s, a_1, \dots, a_n, \mu_1, \dots, \mu_k) = Tds + \sum \frac{\partial u}{\partial a_i} da_i + \sum \frac{\partial u}{\partial \mu_j} d\mu_j$$

Basic concepts of continuum mechanics.

One phase models: Linear and nonlinear liquids



- We start our construction of continuum media models with the simplest case: linear and nonlinear fluids. Under “fluid” is understood a media, where internal energy depends only on volumetric deformation, but not on deviatoric:

$$u = u(1/\rho, s)$$

- After substitution of time derivatives of energy and density (from conservation laws) into Gibbs identity

$$T \frac{ds}{dt} = \frac{du}{dt} + p \frac{d(1/\rho)}{dt}$$

and splitting the total liquid stress into “non-dissipative” and “dissipative” parts

$$\mathbf{P} = -p\mathbf{U} + \mathbf{\Pi} = -p\mathbf{U} + \Pi\mathbf{U} + \mathbf{\Pi}^0, \quad \Pi = \frac{1}{3} \text{tr} \mathbf{\Pi}$$

- The entropy flux and production have a form

$$\rho \frac{ds}{dt} = -\nabla \cdot \mathbf{J}_s + \sigma, \quad \mathbf{J}_s = \frac{\mathbf{J}_q}{T}$$

$$\sigma = -\frac{1}{T^2} \mathbf{J}_q \cdot \nabla T + \frac{1}{T} \Pi \text{div} \mathbf{v} + \frac{1}{T} \mathbf{\Pi}^0 : (\nabla \mathbf{v})^{0s}$$

Basic concepts of continuum mechanics.

One phase models: Onsager theory.



- The entropy production can be presented in the form flux and production have a form of sum of products between generalized forces X_k^R and fluxes I_k^R

$$\sigma = \sum_{R=0,2} \sum_{k=1,K} I_k^R X_k^R = \sum_{R=0,2} \sigma^R$$

where, in our case forces and fluxes only with the same tensor rank R are coupled together. With small deviation from equilibrium, where all forces and fluxes are zero,

$$I_i^R = \sum_k L_{ik}^R X_k^R, \quad \sigma^R = \sum_{k,i} L_{ik}^R X_i^R X_k^R$$

- The necessary and sufficient condition for $\sigma^R > 0$ are:

$$\det L_{ik}^R > 0, \quad L_{ii}^R > 0, \quad \begin{vmatrix} L_{ii}^R & L_{ii+1}^R \\ L_{i+1i}^R & L_{i+1i+1}^R \end{vmatrix} > 0, \quad \begin{vmatrix} L_{ii}^R & L_{ii+1}^R & L_{ii+2}^R \\ L_{i+1i}^R & L_{i+1i+1}^R & L_{i+1i+2}^R \\ L_{i+2i}^R & L_{i+2i+1}^R & L_{i+2i+2}^R \end{vmatrix} > 0, \text{ etc}$$

- Further simplification of phenomenological coefficients is related with the Onsager's theoretical observation that

$$L_{ik}^R = L_{ki}^R$$

which is true except the cases with magnetic effects, which causes $L_{ik}^R(\mathbf{B}) = L_{ik}^R(-\mathbf{B})$

Basic concepts of continuum mechanics.

One phase models: Navier-Stokes equations



- In the case of linear liquid each flux is coupled with an only one force, so each of production is positive

$$\sigma^0 = \frac{1}{T} \Pi \nabla \cdot \mathbf{v} > 0, \quad \sigma^1 = -\frac{1}{T^2} \mathbf{J}_q \cdot \nabla T > 0, \quad \sigma^2 = \frac{1}{T} \Pi^0 : (\nabla \mathbf{v})^{0S} > 0$$

if

$$\Pi = \xi \nabla \cdot \mathbf{v}, \quad \mathbf{J}_q = -\lambda \nabla T, \quad \Pi^0 = 2\mu (\nabla \mathbf{v})^{0S}$$

- The total production is then

$$\sigma = \frac{\lambda (\nabla T)^2}{T^2} \mathbf{J}_q \cdot + \frac{\xi}{T} (\nabla \cdot \mathbf{v}) + \frac{2\mu}{T} (\nabla \mathbf{v})^{0S} : (\nabla \mathbf{v})^{0S} > 0$$

- And Navier-Stokes equations are obtained in a standard form

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \xi \Delta \mathbf{v} + \left(\frac{1}{3} \xi + \mu \right) \nabla (\nabla \cdot \mathbf{v}) + \rho \mathbf{F}$$

$$\rho \frac{du}{dt} = \lambda \Delta \mathbf{v} - p \nabla \cdot \mathbf{v} + \mu (\nabla \cdot \mathbf{v})^2 + 2\mu (\nabla \mathbf{v})^{0S} : (\nabla \mathbf{v})^{0S}$$

Basic concepts of continuum mechanics.

One phase models: Dissipative potential



- For the non-linear extension of the Onsager theory, the “dissipative potential” can be constructed (*Mosolov, Myasnikov V, 1978*). It can be proved, that for

$$\sigma^R(\mathbf{X}^R) = \sum_{k=1,K} I_k^R X_k^R = \sum_{k=1,K} I_k^R (X_k^R) X_k^R > 0$$

a function $\Phi(\mathbf{X}^R) = \Phi(X_k^R)$ exist so that

$$\Phi(\mathbf{X}^R) = \int_0^1 \sigma(\lambda \mathbf{X}^R) \frac{d\lambda}{\lambda}, \quad \frac{\partial \Phi}{\partial \mathbf{X}^R} = \mathbf{I}^R$$

- Indeed, for a non-rigorous prove, let

$$\Phi(\mu \mathbf{X}^R) = \int_0^1 \sigma(\mu \lambda \mathbf{X}^R) \frac{d\mu \lambda}{\mu \lambda} = \int_0^\mu \sigma(\tau \mathbf{X}^R) \frac{d\tau}{\tau}$$

then

$$\left. \frac{d\Phi(\mu \mathbf{X}^R)}{d\mu} \right|_{\mu=1} = \left. \frac{\sigma(\mu \mathbf{X}^R)}{\mu} \right|_{\mu=1} = \sigma(\mathbf{X}^R) = \sum_{k=1,K} I_k^R (X_k^R) X_k^R$$

on the other hand,

$$\left. \frac{d\Phi(\mu \mathbf{X}^R)}{d\mu} \right|_{\mu=1} = \frac{\partial \Phi}{\partial \mu \mathbf{X}^R} \cdot \left. \frac{d\mu \mathbf{X}^R}{d\mu} \right|_{\mu=1} = \sum_{k=1,K} \frac{\partial \Phi}{\partial X_k^R} X_k^R$$

Basic concepts of continuum mechanics.

One phase models: Dissipative potential (2)



- Let now dissipative potential be a non-negative convex function of its arguments:

$$\Phi(X_k^R) = \Phi(\mathbf{X}^R) \geq \Phi(0) = 0$$

$$\left. \frac{\partial \Phi}{\partial \mathbf{X}^R} \right|_{\mathbf{X}^R} \cdot (\mathbf{X}_*^R - \mathbf{X}^R) \leq \Phi(\mathbf{X}_*^R) - \Phi(\mathbf{X}^R) \Leftrightarrow X_i^R \frac{\partial^2 \Phi}{\partial X_i^R \partial X_j^R} X_j^R \geq 0$$

Recall: a scalar function $f(x)$ is convex if the tangent to the curve $y = f(x)$ at any point M is always below the secant connecting M to another point M_* :

$$\left. \frac{\partial f}{\partial x} \right|_x \cdot (x_* - x) \leq f(x_*) - f(x) \Leftrightarrow \frac{\partial^2 f}{\partial x^2} \geq 0$$

Since

$$\frac{\partial \Phi}{\partial X_k^R} = I_k^R, \quad \frac{\partial \Phi}{\partial \mathbf{X}^R} = \mathbf{I}^R \Rightarrow \sigma^R = \frac{\partial \Phi}{\partial \mathbf{X}^R} \cdot \mathbf{X}^R$$

Applying the convexity condition for $\mathbf{X}_*^R = 0$

$$\frac{\partial \Phi}{\partial \mathbf{X}^R} \cdot \mathbf{X}^R \geq \Phi(\mathbf{X}^R) \geq 0$$



Basic concepts of continuum mechanics.

One phase models: Linear and nonlinear liquids



- Therefore, non-negativity of entropy production for each $R = 0, 1, 2$

$$\sigma^0 = \frac{1}{T} \Pi \nabla \cdot \mathbf{v} \geq 0, \quad \sigma^1 = -\frac{1}{T^2} \mathbf{J}_q \cdot \nabla T \geq 0, \quad \sigma^2 = \frac{1}{T} \mathbf{\Pi}^0 : (\nabla \mathbf{v})^{0S} \geq 0$$

generates three dissipative potentials

$$\Pi = \frac{\partial \Phi^0(\nabla \cdot \mathbf{v})}{\partial \nabla \cdot \mathbf{v}}, \quad \mathbf{J}_q = \frac{\partial \Phi^1(\nabla T)}{\partial \nabla T}, \quad \mathbf{\Pi}^0 = \frac{\partial \Phi^2((\nabla \mathbf{v})^{0S})}{\partial (\nabla \mathbf{v})^{0S}}$$

with a class nonlinear liquid models:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \nabla \frac{\partial \Phi^0}{\partial \nabla \cdot \mathbf{v}} + \nabla \cdot \left(\frac{\partial \Phi^2}{\partial (\nabla \mathbf{v})^{0S}} \right) + \rho \mathbf{F}$$

$$\rho \frac{du}{dt} = \lambda \Delta T + \left(-p + \frac{\partial \Phi^0}{\partial \nabla \cdot \mathbf{v}} \right) \text{div} \mathbf{v} + \frac{\partial \Phi^2}{\partial (\nabla \mathbf{v})^{0S}} : (\nabla \mathbf{v})^{0S}$$

- Some special well studied rheologies:

$$\Phi(\lambda s) = \begin{cases} \lambda^2 \Phi(s) & \text{Newtonian liquid} \\ \lambda \Phi(s) & \text{Hard plastic liquid} \\ \lambda^2 \Phi(s) + \lambda \Phi(s) & \text{Viscoplastic liquid} \end{cases}$$

Basic concepts of continuum mechanics.

One phase models: Linear and nonlinear solids. Entropy production



- We turn now to “solid” media which we define as those, where internal energy depends not only on volumetric deformation, but not on deviatoric as well. Recall, that the first invariant of strain tensor is (for small deformations) $I_1 = dV / V$, while from the “moving volume integration rule” one obtains:

$$f = \frac{1}{V}, \quad \int_V \frac{d\tau}{V} = 1 \quad \Rightarrow \quad \nabla \cdot \mathbf{v} = \frac{1}{V} \frac{dV}{dt} = -\frac{1}{\rho} \frac{d\rho}{dt}$$

Thus, there is two similar, but non-equivalent forms of description solid materials:

$$u = u(\boldsymbol{\varepsilon}, s) \quad \text{or} \quad u = u(1/\rho, \boldsymbol{\varepsilon}^0, s)$$

- We start with the first one and consider the second one later. Substituting the conservation laws into the Gibbs identity

$$\frac{du}{dt} = T \frac{ds}{dt} + \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : \frac{d\boldsymbol{\varepsilon}}{dt}, \quad \boldsymbol{\varepsilon} = \frac{1}{2}(\mathbf{g} - \mathbf{g}^0)$$

we obtain entropy production in the form

$$\sigma = -\frac{1}{T^2} \mathbf{J}_q \cdot \nabla T + \frac{1}{T} \mathbf{P} : \nabla \mathbf{v} - \frac{\rho}{T} \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : \frac{d\boldsymbol{\varepsilon}}{dt} \geq 0$$

Basic concepts of continuum mechanics.

One phase models: Linear and nonlinear solids. Elasticity



- By definition, medium is called elastic if all isothermal process there are reversible:

$$\sigma|_{T=\text{const}} = -\frac{1}{T^2} \mathbf{J}_q \cdot \nabla T|_{T=\text{const}} + \frac{1}{T} \mathbf{P} : \nabla \mathbf{v} - \frac{\rho}{T} \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : \frac{d\boldsymbol{\varepsilon}}{dt} = 0$$

$$\left. \begin{aligned} \mathbf{P} : (\nabla \mathbf{v})^s - \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : \frac{d\boldsymbol{\varepsilon}}{dt} &= 0 \\ \frac{d\boldsymbol{\varepsilon}}{dt} = (\nabla \mathbf{v})^s \equiv \mathbf{e} \quad \text{if} \quad \frac{d\mathbf{g}^0}{dt} &= 0 \end{aligned} \right\} \Rightarrow \mathbf{P} = \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}}$$

- Note that relation above between stresses and strain in nonlinear, elasticity does not mean linearity, which is a special case for which internal energy is a positive quadratic form of the strain tensor, that is

$$u = \frac{1}{2\rho} \boldsymbol{\varepsilon} : \boldsymbol{\Lambda} : \boldsymbol{\varepsilon} \Rightarrow \mathbf{P} = \boldsymbol{\Lambda} : \boldsymbol{\varepsilon}$$

where $\boldsymbol{\Lambda}$ is a tensor of a forth order (elasticity matrix). In the isotropic case, for arbitrary coordinate system

$$\Lambda^{ij\alpha\beta} = \lambda g^{ij} g^{\alpha\beta} + \mu (g^{i\alpha} g^{j\beta} + g^{i\beta} g^{j\alpha})$$

λ, μ - Lamé coefficients.

Basic concepts of continuum mechanics.

One phase models: Non-elastic models



- If now irreversible deformations take into play

$$\frac{d\mathbf{g}^0}{dt} \equiv \mathbf{e}^p \neq 0, \quad \frac{d\boldsymbol{\varepsilon}}{dt} = \mathbf{e} - \mathbf{e}^p$$

entropy production reads:

$$T \sigma \Big|_{T=\text{const}} = \mathbf{P} : \nabla \mathbf{v} - \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : \frac{d\boldsymbol{\varepsilon}}{dt} = \mathbf{P} : \mathbf{e} - \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : (\mathbf{e} - \mathbf{e}^p) = \left(\mathbf{P} - \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}} \right) : \mathbf{e} + \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : \mathbf{e}^p \geq 0$$

- Since stresses do not work on reversible, elastic deformation:

$$\mathbf{P} = \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}} \Rightarrow \sigma = \mathbf{P} : \mathbf{e}^p \geq 0$$

- As before:

$$\sigma = \sigma^2(\mathbf{e}^p) = \mathbf{P} : \mathbf{e}^p = \mathbf{P}(\mathbf{e}^p) : \mathbf{e}^p > 0 \Rightarrow \exists \Phi(\mathbf{e}^p), \quad \mathbf{P} = \frac{\partial \Phi(\mathbf{e}^p)}{\partial \mathbf{e}^p}$$

- Thus, we have a system of ordinary differential equations resolved with respect to $\varepsilon_{\alpha\beta}$:

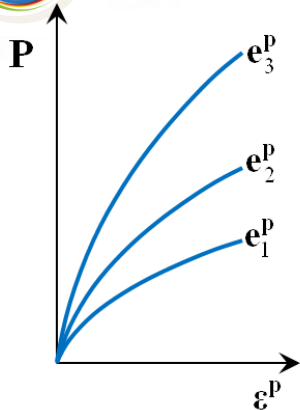
$$\frac{\partial \Phi[\mathbf{e} - d\boldsymbol{\varepsilon} / dt]}{\partial [\mathbf{e} - d\boldsymbol{\varepsilon} / dt]} = \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}}$$

Basic concepts of continuum mechanics.

One phase models: Non-elastic models(2)



- As examples of non-elastic models we schematically consider two limiting cases:



- $\mathbf{P}(\mathbf{e}^p) \neq \mathbf{P}(\lambda \mathbf{e}^p)$ - dissipative phenomena are dependent on time, thus viscosity effect come into play. The dissipative potential is homogeneous of order 2

$$\Phi(\lambda \mathbf{e}^p) = \lambda^2 \Phi(\mathbf{e}^p)$$

corresponds to Maxwell viscoelastic model, where the ODE system has a form:

$$\eta[\mathbf{e} - d\mathbf{e}/dt] = \rho \frac{\partial u}{\partial \mathbf{e}} \Rightarrow \frac{d\mathbf{e}}{dt} = (\nabla \mathbf{v})^s - \frac{1}{\eta} \mathbf{\Lambda} : \mathbf{e}$$

- $\mathbf{P}(\mathbf{e}^p) = \mathbf{P}(\lambda \mathbf{e}^p)$ - no viscosity effect, $\Phi(\mathbf{e}^p)$ is homogeneous of order 1:

$$\Phi(\lambda \mathbf{e}^p) = \lambda \Phi(\mathbf{e}^p)$$

- If one considers a conjugated potential $\Phi^*(\mathbf{P})$:

$$\Phi(\mathbf{e}^p), \mathbf{P} = \frac{\partial \Phi(\mathbf{e}^p)}{\partial \mathbf{e}^p} \rightarrow \Phi^*(\mathbf{P}), \mathbf{e}^p = \frac{\partial \Phi^*(\mathbf{P})}{\partial \mathbf{P}}, \quad \Phi(\mathbf{e}^p) + \Phi^*(\mathbf{P}) = \mathbf{e}^p : \mathbf{P}$$

from properties of conjugated potentials and the condition $\mathbf{P}(\mathbf{e}) = \mathbf{P}(\lambda \mathbf{e})$ one can define the convex with respect to elastic zone hypersurface $f(\mathbf{P}) = 0$ - "yield locus" with the properties:

$$\begin{cases} d\mathbf{e}^p = 0 & \text{if } f = 0, df < 0 \text{ or } f < 0 \\ d\mathbf{e}^p \neq 0 & \text{if } f = 0, df = 0 \end{cases} \quad \text{- postulated additionally, like } d\mathbf{e}^p = d\lambda \partial f / \partial \mathbf{P}$$

Basic concepts of continuum mechanics.

One phase models: A remark of a "solid definition"



- For some applications it is more convenient to explicitly split the dependence on spherical and deviator parts:

$$u = u(\boldsymbol{\varepsilon}, s) \quad \text{or} \quad u = u(1/\rho, \boldsymbol{\varepsilon}^0, s)$$

- After substitution of conservation laws into Gibbs identity

$$T \frac{ds}{dt} = \frac{du}{dt} - \frac{\partial u}{\partial(1/\rho)} \frac{d(1/\rho)}{dt} - \frac{\partial u}{\partial \boldsymbol{\varepsilon}^0} : \frac{d\boldsymbol{\varepsilon}^0}{dt}$$

one has for entropy production (for elastic models)

$$\sigma^0 + \sigma^2 = \frac{1}{T} \left(\frac{1}{3} \sum_k \mathbf{P}_{kk} - \frac{\partial u}{\partial(1/\rho)} \right) \nabla \cdot \mathbf{v} + \frac{1}{T} \left(\mathbf{P}^0 - \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}^0} \right) : \mathbf{e}^0, \quad \mathbf{P}^0 = \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}^0}, \quad p \equiv \frac{1}{3} \sum_k \mathbf{P}_{kk} = \frac{\partial u}{\partial(1/\rho)}$$

- Compare with

$$\sigma^0 + \sigma^2 = \frac{1}{3T} \left[\sum_k \mathbf{P}_{kk} - \rho \left(\frac{\partial u_s}{\partial \boldsymbol{\varepsilon}^0} \right)_{kk} \right] \nabla \cdot \mathbf{v} + \frac{1}{T} \left[\mathbf{P}^0 - \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}^0} \right] : \mathbf{e}^0, \quad \mathbf{P}^0 = \rho \frac{\partial u}{\partial \boldsymbol{\varepsilon}^0} \Leftrightarrow p \equiv \frac{1}{3} \sum_k \mathbf{P}_{kk} = \frac{1}{3} \rho \sum_k \left(\frac{\partial u_s}{\partial \boldsymbol{\varepsilon}^0} \right)_{kk}$$

which can be obtained by direct substitution of $\boldsymbol{\varepsilon} = \frac{1}{3} \sum_k \boldsymbol{\varepsilon}_{kk} + \boldsymbol{\varepsilon}^0$ into previously obtained

$$\sigma = -\frac{1}{T^2} \mathbf{J}_q \cdot \nabla T + \frac{1}{T} \mathbf{P} : \nabla \mathbf{v} - \frac{\rho}{T} \frac{\partial u}{\partial \boldsymbol{\varepsilon}} : \frac{d\boldsymbol{\varepsilon}}{dt}$$

Basic concepts of continuum mechanics.

One phase models: Discussion about elastic and plastic deformations



- Recall, that covariant components $\varepsilon_{\alpha\beta}$ generate two deformation tensors

$$\varepsilon_{\alpha\beta} \rightarrow \boldsymbol{\varepsilon}^0 = \varepsilon_{\alpha\beta} \mathfrak{D}^{0\alpha} \mathfrak{D}^{0\beta}, \quad \widehat{\boldsymbol{\varepsilon}} = \varepsilon_{\alpha\beta} \widehat{\mathfrak{D}}^\alpha \widehat{\mathfrak{D}}^\beta$$

Splitting deformations in elastic and plastic parts

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p$$

is valid only in the sense of splitting of Lagrangian components $\varepsilon_{\alpha\beta} = \varepsilon_{\alpha\beta}^e + \varepsilon_{\alpha\beta}^p$

- It is not possible for contravariant or mixed components or in Eulerian tensor of finite deformations. A lot of discussions can be found in literature with this relation and collected in (*Kovtanyuk, 2006*) with a conclusion that one of the most fruitful approach is (*Myasnikov V 1996*):

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p$$

$$u = u(\boldsymbol{\varepsilon}^e, \boldsymbol{\varepsilon}^p, s)$$

$$\frac{D\boldsymbol{\varepsilon}}{Dt} = \mathbf{e}, \quad \frac{D\boldsymbol{\varepsilon}^p}{Dt} = \rho \mathbf{I}_\Gamma - \nabla \cdot \boldsymbol{\Gamma}, \quad \frac{D\boldsymbol{\varepsilon}^e}{Dt} = \rho (\mathbf{e} - \mathbf{I}_\Gamma) + \nabla \cdot \boldsymbol{\Gamma}$$

$$\boldsymbol{\sigma} = - \left(\mathbf{J}_q + \boldsymbol{\Gamma} : \left(\frac{\partial u}{\partial \boldsymbol{\varepsilon}^e} - \frac{\partial u}{\partial \boldsymbol{\varepsilon}^p} \right) \right) \cdot \frac{\nabla T}{T^2} + \frac{\mathbf{P}^* : \mathbf{e}}{T} + \frac{(\boldsymbol{\Gamma} \cdot \nabla)}{T} : \left(\frac{\partial u}{\partial \boldsymbol{\varepsilon}^e} - \frac{\partial u}{\partial \boldsymbol{\varepsilon}^p} \right) - \frac{\rho}{T} \left(\frac{\partial u}{\partial \boldsymbol{\varepsilon}^e} - \frac{\partial u}{\partial \boldsymbol{\varepsilon}^p} \right) : \mathbf{I}_\Gamma$$

$$\Phi = \Phi(\mathbf{I}_\Gamma, \mathbf{e}, T)$$

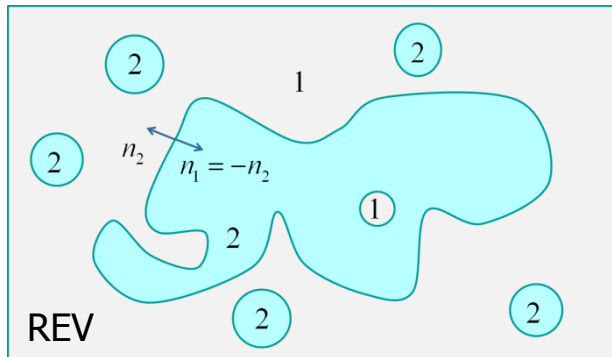
- Splitting is used here as phenomenological relation, which determine elastic and plastic deformations as internal parameters which can be produced and transferred in the media.

Basic concepts of continuum mechanics.

Two phase models: Conservation laws



- We consider a heterogeneous system consisting of several phases in which heat and matter exchange between the phases is possible. The balance equations for mass, momentum, and energy for the individual phase in the Euler representation are given by:



$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = \sum_{j \neq i} J_{ji}$$

$$\rho_i \frac{d_i \mathbf{v}_i}{dt} = \rho \mathbf{F}_i + \nabla \cdot \mathbf{P}_i + \sum_{j \neq i} (\mathbf{P}_{ji} - J_{ji} \mathbf{v}_i)$$

$$\rho_i \frac{d_i e_i}{dt} = -\nabla \cdot (\mathbf{P}_i \cdot \mathbf{v}_i - \mathbf{J}_{q,i}) + \rho_i \mathbf{F} \cdot \mathbf{v}_i + \sum_{j \neq i} (E_{ji} - J_{ji} e)$$

$$\rho_i = \alpha_i \rho_i^0, \quad e_i = u_i + \frac{1}{2} v_i^2, \quad \frac{d_i}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_i \nabla$$

- Basically, the equations are similar to consider for one phase models. The difference is that we have to deal with true densities ρ_i^0 and specify terms responsible for heat, momentum and mass exchange between phases

$$J_{ji} = -J_{ij}, \quad \mathbf{P}_{ji} = -\mathbf{P}_{ij}, \quad E_{ji} = -E_{ij}$$

in thermodynamically consistent way.

Basic concepts of continuum mechanics.

Two phase models: Material derivative



- To do this, we need to define material derivative for the mixture as the whole, which, as in the one phase case, define the change in time of any phase additive physical quantity:

$$\rho = \sum \rho_i, \quad \rho \mathbf{v} = \sum \rho_i \mathbf{v}_i, \quad \rho e = \sum \rho_i e_i, \quad \rho \Phi = \sum \rho_i \Phi_i$$

We define:

$$\forall \Phi, \quad \rho \Phi = \sum \rho_i \Phi_i, \quad \rho \frac{D\Phi}{Dt} \equiv \sum_i \left[\rho_i \frac{d_i \Phi_i}{dt} + \sum_j J_{ji} \Phi_i \right]$$

- Indeed, let's obtain the change rate of Φ for the mixture with the fixed mass $dM \equiv \sum dM_i$ (note that $dM / dV = \rho$, $dM_i / dV = \rho_i$).

Consider the mixture at t and t' :

$$\Phi dM = \sum \Phi_i dM_i, \quad \Phi' dM' = \sum \Phi'_i dM'_i$$

Since

$$dM' = dM, \quad dM'_i = dM_i + \sum_j J_{ji} dV dt$$

$$\Phi' dM' - \Phi dM = \sum (\Phi'_i dM'_i - \Phi_i dM_i)$$

We finally have

$$dM (\Phi' - \Phi) = \sum dM_i (\Phi'_i - \Phi_i) + \Phi'_i \sum_j J_{ji} dV dt$$

Basic concepts of continuum mechanics.

Two phase models: entropy production(1)



- Apply this formula to the mixture entropy for two phases, solid and fluid, s and f , and make use of the developed previously irreversible thermodynamics formalism:

$$\rho \frac{Ds}{Dt} \equiv \rho_s \frac{ds_s}{dt} + \rho_f \frac{ds_f}{dt} + J_{sf} (s_f - s_s)$$

(liquid is defined as in the one phase case) $u_f = u_f(1/\rho_f^0, s_f)$

$u_s = u_s(1/\rho_s^0, \epsilon_s^0, \epsilon_s^*, s_s)$ (we use the second way of the 'solid' definition here, and, $[*]$)

$$T_f \frac{ds_f}{dt} = \frac{du_f}{dt} - \frac{\partial u_f}{\partial(1/\rho_f^0)} \frac{d(1/\rho_f^0)}{dt}$$

$$T_s \frac{ds_s}{dt} = \frac{du_s}{dt} - \frac{\partial u_s}{\partial(1/\rho_s^0)} \frac{d(1/\rho_s^0)}{dt} - \frac{\partial u_s}{\partial \epsilon_s^0} \cdot \frac{d\epsilon_s^0}{dt} - \frac{\partial u_s}{\partial \epsilon_s^*} \cdot \frac{d\epsilon_s^*}{dt}$$

$$\frac{d_f \rho_f^0}{dt} = -\frac{1}{\alpha_f} \left(J_{fs} + \alpha_f \rho_f^0 \nabla \cdot \mathbf{v}_f + \rho_f^0 \frac{d_f \alpha_f}{dt} \right)$$

$$\frac{d_s \rho_s^0}{dt} = -\frac{1}{\alpha_s} \left(-J_{fs} + \alpha_s \rho_s^0 \nabla \cdot \mathbf{v}_s + \rho_s^0 \frac{d_s \alpha_s}{dt} \right)$$

$$\alpha_f \rho_f^0 \frac{d_f \mathbf{v}_f}{dt} = \alpha_f \rho_f^0 \mathbf{F}_f + \nabla \cdot \mathbf{P}_f - \mathbf{P}_{fs} + J_{fs} \mathbf{v}_f$$

$$\alpha_s \rho_s^0 \frac{d_s \mathbf{v}_s}{dt} = \alpha_s \rho_s^0 \mathbf{F}_i + \nabla \cdot \mathbf{P}_s + \mathbf{P}_{fs} - J_{fs} \mathbf{v}_s$$

$$\alpha_f \rho_f^0 \frac{d_f u_f}{dt} = -\nabla \cdot \mathbf{J}_{q,f} + \mathbf{P}_f : \nabla \mathbf{v}_f + \mathbf{v}_f \cdot \mathbf{P}_{fs} - E_{fs} + J_{fs} \left(u_f - \frac{\mathbf{v}_f^2}{2} \right)$$

$$\alpha_s \rho_s^0 \frac{d_s u_s}{dt} = -\nabla \cdot \mathbf{J}_{q,s} + \mathbf{P}_s : \nabla \mathbf{v}_s - \mathbf{v}_s \cdot \mathbf{P}_{fs} + E_{fs} - J_{fs} \left(u_s - \frac{\mathbf{v}_s^2}{2} \right)$$

[*] The pivotal point here is the term ϵ_s^* in the list of state variables for the function u_s . Additional strain appears, since, besides deformations generated by the external surface forces, applied to the boundary of REV, additional surface forces of interphase interaction exist, which should be considered as external forces for both phases. Since the true density for liquid is clearly defined, additional force appears only for the solid phase.

Basic concepts of continuum mechanics.

Two phase models: entropy production(2)



- After substitution of conservation laws into Gibbs identity for each phase, the total entropy change in material volume is

$$\rho \frac{Ds}{Dt} = -\nabla \cdot \mathbf{J}_{e,S} + \sigma_S + \sigma_f + \sigma_{fs}$$

with entropy flux

$$\mathbf{J}_{e,S} = \frac{1}{T_S} \mathbf{J}_{q,S} + \frac{1}{T_f} \mathbf{J}_{q,f}$$

internal phase entropy productions

$$\sigma_S = -\frac{1}{T_S^2} \mathbf{J}_{q,S} \cdot \nabla T_S + \frac{1}{T_S} \left(\frac{1}{3} (\mathbf{P}_S)_{kk} - \frac{\partial u_S}{\partial(1/\rho_S^0)} \alpha_S \right) \nabla \cdot \mathbf{v}_S + \frac{1}{T_S} \left(\mathbf{P}_S^0 - \rho_S^0 \frac{\partial u_S}{\partial \boldsymbol{\varepsilon}_S^0} \alpha_S \right) : \mathbf{e}_S^0$$

$$\sigma_f = -\frac{1}{T_f^2} \mathbf{J}_{q,f} \cdot \nabla T_f + \frac{1}{T_f} \left(-\alpha_f p_f + \frac{1}{3} (\mathbf{\Pi}_f)_{kk} - \frac{\partial u_f}{\partial(1/\rho_f^0)} \alpha_f \right) \nabla \cdot \mathbf{v}_f + \frac{1}{T_f} \mathbf{\Pi}_f^0 : \mathbf{e}_f^0$$

and entropy production of interphase interaction

$$\sigma_{fs} = \left(\frac{\Phi_f}{T_f} - \frac{\Phi_S}{T_S} \right) J_{fs} + \left(\frac{\mathbf{v}_f}{T_f} - \frac{\mathbf{v}_S}{T_S} \right) \cdot \mathbf{P}_{fs} - \left(\frac{1}{T_f} - \frac{1}{T_S} \right) E_{fs} - \frac{1}{T_f} \frac{\partial u_f}{\partial(1/\rho_f)} \frac{d_f \alpha_f}{dt} - \frac{1}{T_S} \frac{\partial u_S}{\partial(1/\rho_S)} \frac{d_S \alpha_S}{dt} - \frac{\partial u_S}{\partial \boldsymbol{\varepsilon}_S^*} \cdot \frac{d \boldsymbol{\varepsilon}_S^*}{dt}$$

where $\Phi_{f,S}$ is specific Gibbs free energy (not that only interphase exchange with volume deformation is considered)

$$\Phi_{f,S} = u_{f,S} - \frac{\mathbf{v}_{f,S}^2}{2} - \frac{1}{\rho_{f,S}^0} \frac{\partial u_{f,S}}{\partial(1/\rho_{f,S})}$$

Basic concepts of continuum mechanics.

Two phase models: entropy production (3)



- Thermodynamics implies that the total entropy production must be non negative

$$\sigma_s + \sigma_f + \sigma_{fs} \geq 0$$

- However, more strong condition is usually assumed

$$\sigma_s \geq 0, \quad \sigma_f \geq 0, \quad \sigma_{fs} \geq 0$$

which makes it possible to imply what was previously developed for one phase models

$$p_f = -\frac{\partial u_f}{\partial(1/\rho_f)}, \quad \mathbf{J}_{q,f} = \frac{\partial \Phi_1(\nabla T_f)}{\partial \nabla T_f}, \quad \Pi_f = \frac{\partial \Phi_2(\nabla \cdot \mathbf{v}_f)}{\partial \nabla \cdot \mathbf{v}_f}, \quad \Pi_f^0 = \frac{\partial \Phi_3(\mathbf{e}^0)}{\partial \mathbf{e}^0}$$

$$\mathbf{J}_{q,s} \cdot \nabla T_s, \quad \alpha_s p_s \equiv -\frac{1}{3}(\mathbf{P}_s)_{kk} = -\alpha_s \frac{\partial u_s}{\partial(1/\rho_s^0)}, \quad \alpha_s \sigma_s^0 \equiv \mathbf{P}_s^0 = \alpha_s \rho_s^0 \frac{\partial u_s}{\partial \boldsymbol{\varepsilon}_s^0}$$

- The condition

$$\sigma_{fs} = -\left[\frac{1}{T}\right] E_{fs} + \left[\frac{\Phi}{T}\right] J_{fs} + \left[\frac{\mathbf{v}}{T}\right] \cdot \mathbf{P}_{fs} + \frac{1}{T_f} p_f [\mathbf{v}] \cdot \nabla \alpha_f + \left[\frac{p}{T}\right] \frac{d_s \alpha_f}{dt} - \frac{1}{T_s} \frac{\partial u_s}{\partial \boldsymbol{\varepsilon}_s^*} : \frac{d \boldsymbol{\varepsilon}_s^*}{dt}$$

where $[A] = A_f - A_s$ is satisfied if, for example,

$$\left[\frac{1}{T}\right] = 0, \quad \left[\frac{\Phi}{T}\right] = 0, \quad \mathbf{P}_{fs} = -p_f \cdot \nabla \alpha_f + k[\mathbf{v}],$$

$$\left[\frac{p}{T}\right] \frac{d_s \alpha_f}{dt} = \frac{1}{T_s} \frac{\partial u_s}{\partial \boldsymbol{\varepsilon}_s^*} : \frac{d \boldsymbol{\varepsilon}_s^*}{dt}$$



$$\boldsymbol{\varepsilon}_s^* \equiv \alpha_f \mathbf{U},$$

$$[p] = \frac{\partial u_s}{\partial \boldsymbol{\varepsilon}_s^*} : \mathbf{U} = \frac{\partial u_s}{\partial \alpha_f}$$

Basic concepts of continuum mechanics. .

Two phase models: Darcy law and effective stress



- Lets now neglect mass and energy interphase exchange as well as viscous effects in liquid phase. With the specific choice $\mathbf{F}_f / \rho_f = \mathbf{F}_s / \rho_s = g\mathbf{e}_z$ the resulting equations read

$$\frac{d_f \phi \rho_f^0}{dt} + \phi \rho_f^0 \nabla \cdot \mathbf{v}_f = 0$$

$$\phi \rho_f^0 \frac{d_f \mathbf{v}_f}{dt} = -\phi \rho_f^0 g\mathbf{e}_z - \nabla \phi p_f + p_f \cdot \nabla \phi - k[\mathbf{v}]$$

$$\phi \rho_f^0 \frac{d_f u_f}{dt} = \nabla \cdot \lambda_T \nabla T - \phi p_f \nabla \cdot \mathbf{v}_f - p_f \mathbf{v}_f \cdot \nabla \phi + k \mathbf{v}_f \cdot [\mathbf{v}]$$

When inertia effects can be neglected, momentum equation is reduced to Darcy law:

$$[\mathbf{v}] = -\frac{\phi}{k} (\nabla p_f - \rho_f^0 g\mathbf{e}_z)$$

- After introducing total and effective stress, the equations for solid phase have a form

$$\Gamma = -\phi p_f \mathbf{U} - (1-\phi) p_s \mathbf{U} + (1-\phi) \boldsymbol{\sigma}_s^0$$

$$\boldsymbol{\sigma}_s^{eff} = \Gamma + p_f \mathbf{U} = (1-\phi) (p_f - p_s \mathbf{U} + \boldsymbol{\sigma}_s^0) = (1-\phi) (p_f + \boldsymbol{\sigma}_s)$$

$$\frac{d_s \phi \rho_s^0}{dt} + (1-\phi) \rho_s^0 \nabla \cdot \mathbf{v}_s = 0$$

$$(1-\phi) \rho_s^0 \frac{d_s \mathbf{v}_s}{dt} = -(1-\phi) \rho_s^0 g\mathbf{e}_z + \nabla \cdot \boldsymbol{\sigma}_s^{eff} - (1-\phi) \nabla p_f + k[\mathbf{v}]$$

$$(1-\phi) \rho_s^0 \frac{d_s u_s}{dt} = -\nabla \cdot \mathbf{J}_{q,s} + \boldsymbol{\sigma}_s^{eff} : \nabla \mathbf{v}_s + (1-\phi) \mathbf{v}_s \nabla p_f - \nabla p_f \mathbf{v}_s (1-\phi) - \mathbf{v}_s k[\mathbf{v}]$$

Basic concepts of continuum mechanics. .

Two phase models: Towards the Biot equations



- The dependence $u_s = u_s(1/\rho_s^0, \boldsymbol{\epsilon}_s^0, \boldsymbol{\epsilon}_s^*, s_s) = u_s(1/\rho_s^0, \boldsymbol{\epsilon}_s^0, \varphi, s_s)$ makes it possible to derive coupling interphase relations which in the linear case strives to classical Biot equations. For example, incremental form of the state equation for deviator stress in the solid phase has a form:

$$d\boldsymbol{\sigma}_s^0 = d\left(\rho_s^0 \frac{\partial u_s}{\partial \boldsymbol{\epsilon}_s^0}\right) = \frac{\partial u_s}{\partial \boldsymbol{\epsilon}_s} d\rho_s^0 + \rho_s^0 d\left(\frac{\partial u_s}{\partial \boldsymbol{\epsilon}_s}\right) =$$

$$= \frac{\partial u_s}{\partial \boldsymbol{\epsilon}_s} d\rho_s^0 + \rho_s^0 \left[\frac{\partial^2 u_s}{\partial \boldsymbol{\epsilon}_s^0 \partial (1/\rho_s^0)} d\left(\frac{1}{\rho_s^0}\right) + \frac{\partial^2 u_s}{\partial \boldsymbol{\epsilon}_s^0 \partial \boldsymbol{\epsilon}_s^0} d\boldsymbol{\epsilon}_s^0 + \frac{\partial^2 u_s}{\partial \boldsymbol{\epsilon}_s^0 \partial \varphi} d\varphi + \frac{\partial^2 u_s}{\partial \boldsymbol{\epsilon}_s^0 \partial s} ds \right]$$

So one can construct a matrix

$$\mathbf{A} = \mathbf{A}\left(\rho_s^0, \varphi, \frac{\partial u_s}{\partial \boldsymbol{\epsilon}_s^0}, \frac{\partial^2 u_s}{\partial \boldsymbol{\epsilon}_s^0 \partial \boldsymbol{\epsilon}_s^0}, \dots, \frac{\partial u_s}{\partial s_s}, \frac{\partial^2 u_s}{\partial s_s^2}\right)$$

with first and second derivatives on internal energy on state variables which relates their increments to the increment of conjugate variables

$$\left[dp_s, d\boldsymbol{\sigma}_s^0, dp_f, dT_s\right]^T = \mathbf{A} \left[d(1/\rho_s^0), d\boldsymbol{\epsilon}_s^0, d\varphi, ds_s\right]^T$$



Basic concepts of continuum mechanics. .

Two phase models: From core to pipe



- ❑ The observation made above that liquid and solid phase can be described in a similar manner makes it possible to apply the developed formalism to multiphase flow in pipes, wells, channels almost straightforwardly.
- ❑ *Schematically*, it can be done by replacing two separate phase internal energies

$$u_f = u_f(1/\rho_f^0, s_f), \quad u_s = u_s(1/\rho_s^0, \epsilon_s^0, \epsilon_s^*, s_s)$$

by the total internal energy of the mixture

$$u = u(u_f, u_s, \nabla \rho_f^0, \nabla \rho_s^0) = u(1/\rho_f^0, 1/\rho_s^0, \epsilon_s^0, s, \nabla \rho_f^0, \nabla \rho_s^0)$$

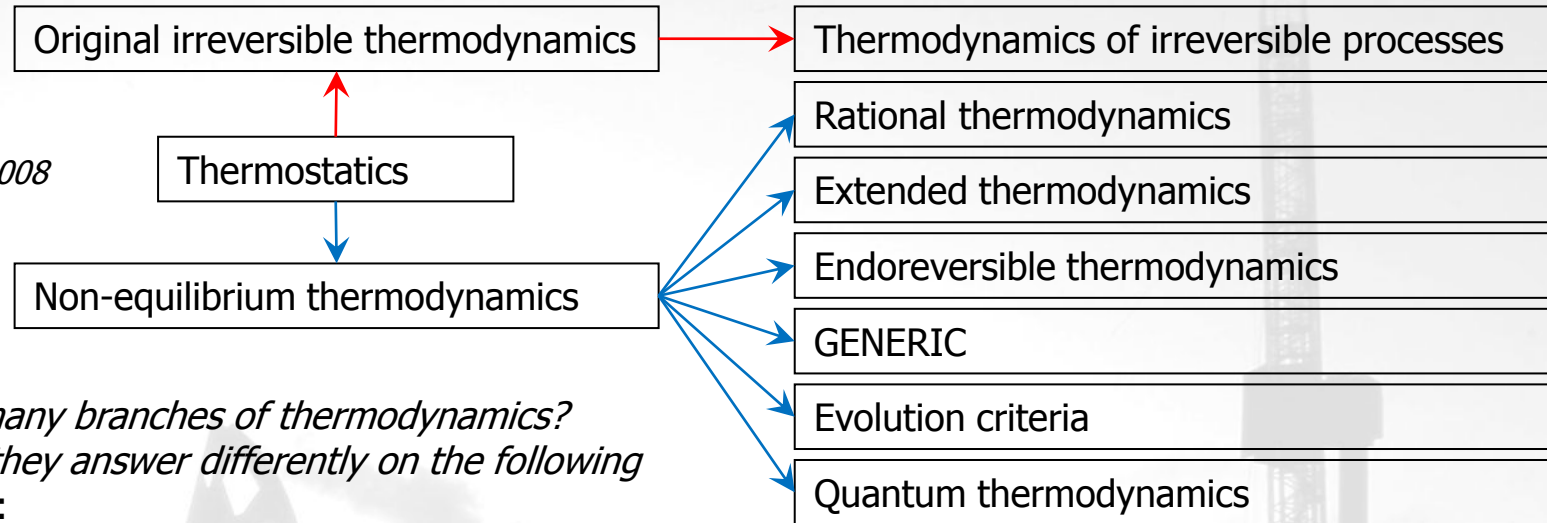
where entropy s now is the sum of fluid, liquid and *surface phase* entropy, and density gradients defines its production due to surface tension and, also, play a role of selector switch between phases rheologies: if density beyond the zone of high density gradient is "low", the media is fluid there, otherwise it is solid

- ❑ Note the absence of ϵ_s^* in newly defined internal energy. Its role is played now by some functional of phase densities
- ❑ The approach is called "Density functional theory" and is being developed by *Dinariev et al (2011)* in 15 years. The main problem is that thermodynamics makes too week constrain on the fluxes related to density gradient forces, so, coefficients related to selector switches should be tunes for another numerical grid resolution from scratch

Construction of general isotropic models. Some branches of modern thermodynamics



Muschik W., 2008



- *Why so many branches of thermodynamics?*
- *Because they answer differently on the following questions:*

- Is the considered system described as a discrete one or in field formulation?
- Are the temperature and entropy primitive concepts or derived quantities?
- Is the chosen state space small or large?
- Is the dissipation inequality global or local in time?
- Is the relation between heat flux density and entropy flux density universal or material dependent?
- What is the procedure for exploiting the dissipation inequality?
- Are complex material described by a mesoscopic theory or by introducing additional fields and their balances?
- Does the phenomenological description of a non-equilibrium system have a correct quantum mechanical background?



Part 3. Experimental Rock Mechanics.

Experimental Rock mechanics.

Classification of rocks for mechanical properties examination



- ❑ The mechanical properties of a sedimentary rock are determined by the entire process from erosion of rock fragments to transportation, deposition, compaction and lithification. Some knowledge of these processes is therefore has to be summarized

- ❑ Minerals are sorted with respected to their:

- ❑ Packing

- Grain size
- Grain shape: roundness (angularity of corners) and sphericity (proximity to a spherical shape)
- Grain sorting (grain size distribution)

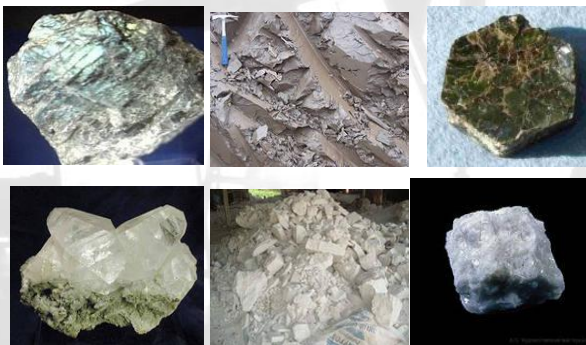
- ❑ Belonging to mineral group

- silicates, e.g. feldspar, clay, mica, tuff
- carbonates, e.g. calcite, dolomite, *marble*
- oxides, sulphides, sulphates, phosphates

- ❑ The way of formation

- clastic (originate from some sort of pre-existing rock)
- organic sediments (e.g. precipitation of salt crystals due to evaporation)
- chemical sediments (e.g. shells and skeletons from sea organisms forming a chalk or limestone)

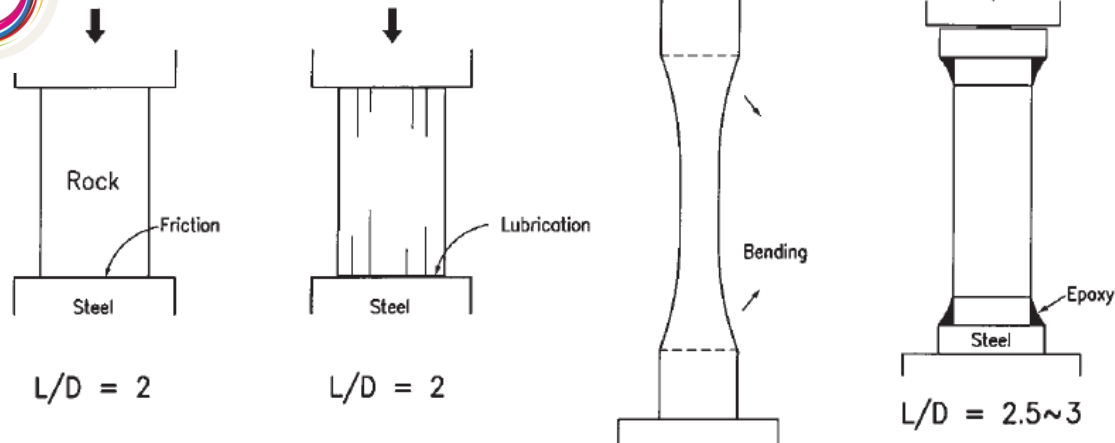
Grain diameter range [mm]	phi-scale	Term
>256	< -8	Boulder
64-256	-6 - -8	Cobble
4-64	-2 - -6	Pebble
2-4	-1 - -2	Granule
1/16-2	4 - 1	Sand
1/256-1/16	8-4	Silt
<1/256	>8	Clay



- ❑ What classification to chose for better identification of mechanical properties?
- ❑ One can give any answer, but the understanding that it was wrong often comes later on

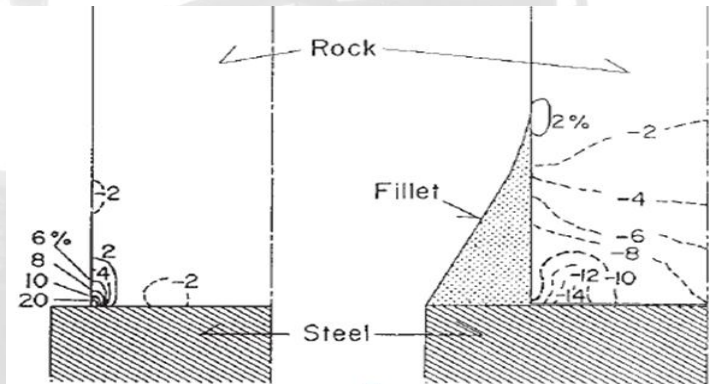
Experimental Rock mechanics.

Unconfined compressive stress.



- | | | | |
|------------------------|--|---------------------------|--------------------|
| ○ Easy fabrication | ○ Easy fabrication | ○ No stress concentration | ○ No end effect |
| ● Stress concentration | ○ No end clamping | ○ Low clamping effect | ○ No bending |
| ● End clamping effect | ● Vertical cracking at the end surface | ● Bending | ○ Easy fabrication |
| | | ● Difficult fabrication | |

- Deviation of axial stress from the mean applied stress in the middle of the specimen



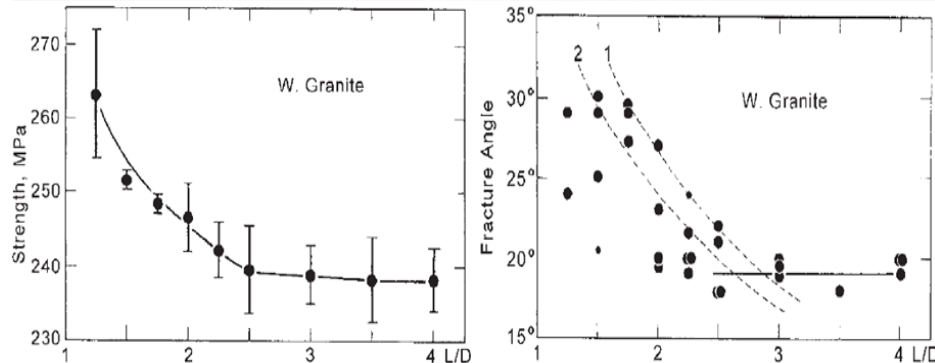
- *K. Mogi (2007) Experimental rock mechanics*

- Used in:
 - studies of both brittle and ductile behavior
 - in long term creep studies
 - in studies of elastic behavior

- Main problem:
 - radial shearing forces are generated at the interface when load is applied to the rock sample (clamping effect) .

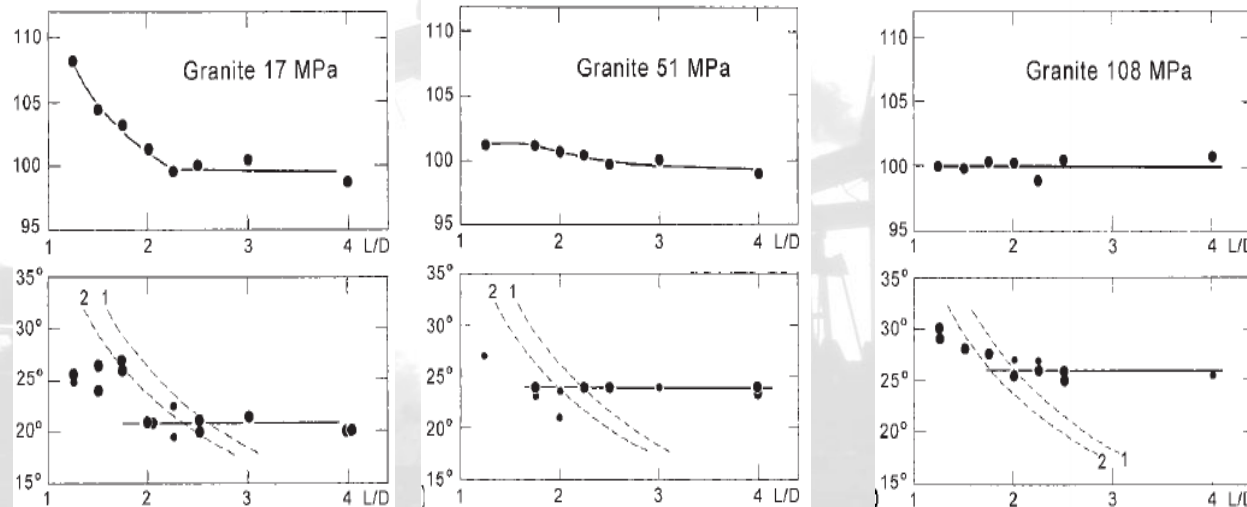
- Consequences:
 - stress concentration arises
 - if a fracture propagates into the region near the end of the sample, growth of the fracture may be impeded

Experimental Rock mechanics. Clamping effects and confining pressure



- Typical relations of uniaxial compressive strength and fracture angle to length/diameter ratio strongly indicates that for correct measurements of uniform compressive strength the ratio must be >2.5

- However, the end effect is drastically decreased by confining pressure:



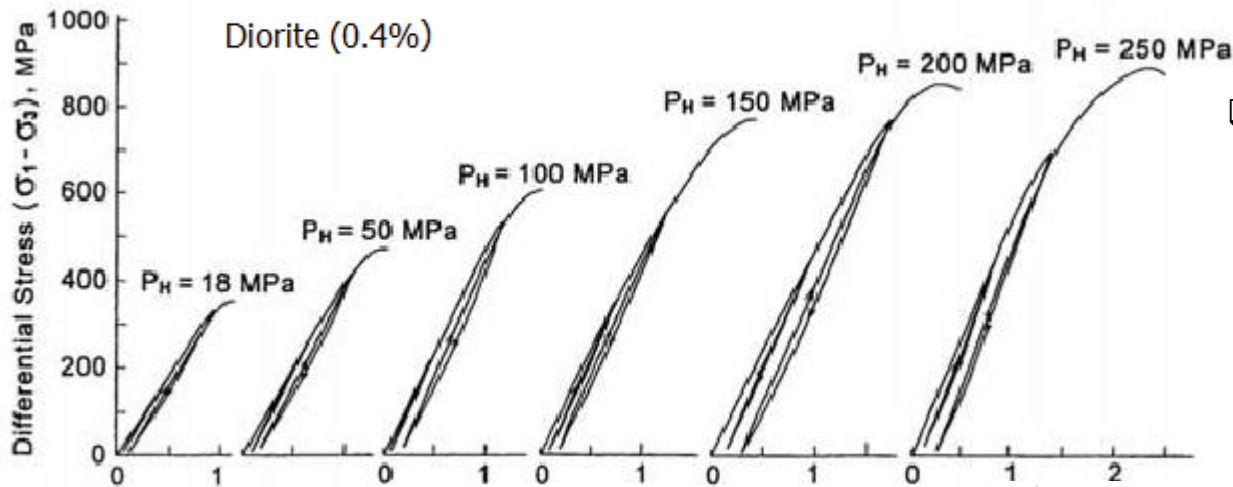
- the relative decrease of the effect of lateral restriction at the end part
- the increase of fracture angle under pressure
- the increase in the ductility of rocks

Experimental Rock mechanics.

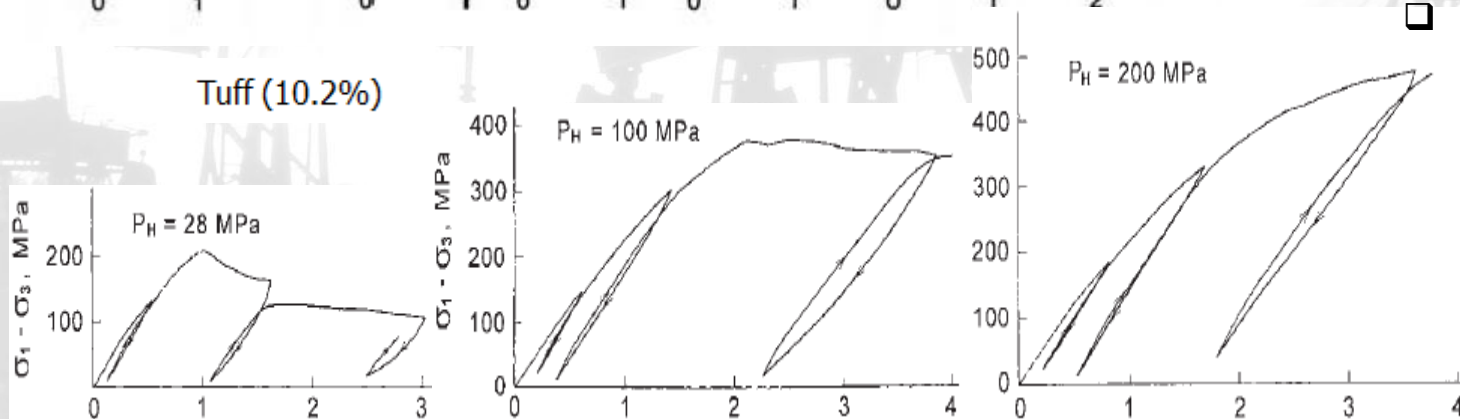
Brittle and ductile behavior of rocks



- The test specimens of peridotite (porosity 0.02%), marble (0,2%), diorite (0.4%), andesite (5,1%) and tuff (10%) were compressed to fracture or to 3–4 % strain for soft rocks, and at various stages of deformation the stress was reduced to zero or to very small values, then increased again.



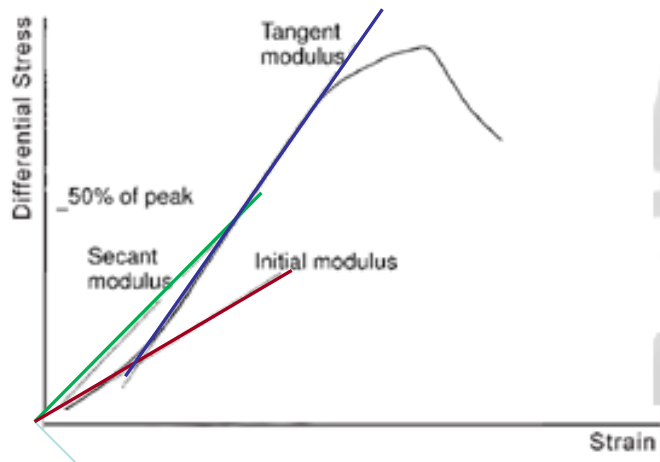
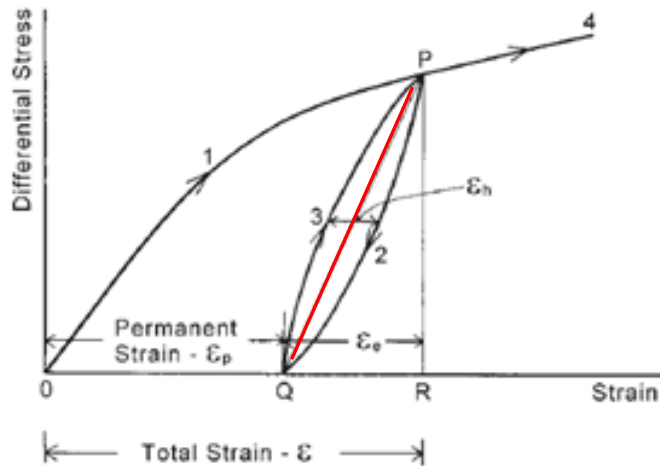
- Brittle behavior: a sudden change of slope in the stress-strain curve near the "yield point" followed by a complete loss of cohesion or an appreciable drop in differential stress.



- Ductile behavior: the deformation without any downward slope after the yield point

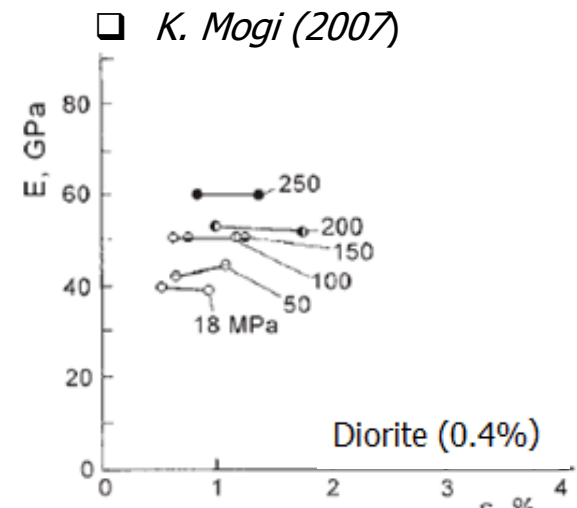
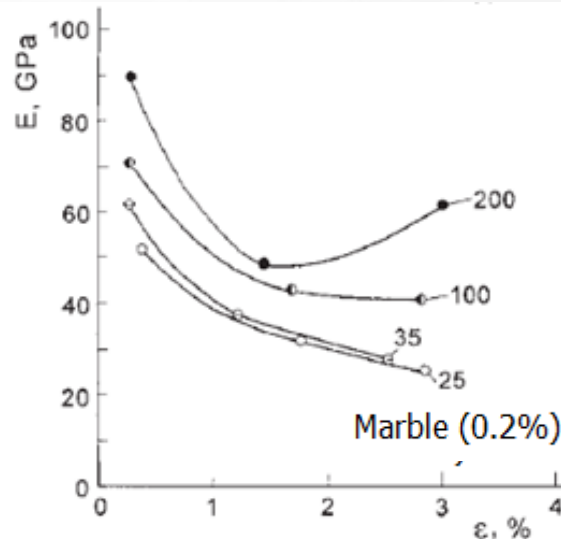
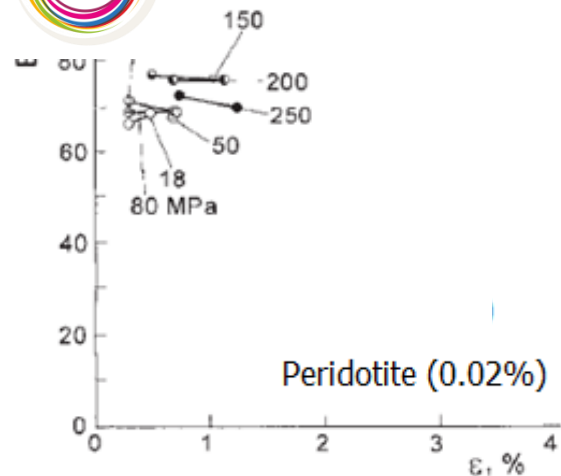
Experimental Rock mechanics.

Modulus of elasticity: definitions



- ❑ When the rock specimen is compressed to a point P and then unloaded and reloaded, the branches (P2Q, Q3P) are different from the virgin loading curve (O1P) in many cases.
- ❑ The unloading and reloading curves usually differ little from each other and form a narrow loop (P2Q3). The reloading curve (Q3P) passes very near the point P and continues to the virgin loading curve (O1P).
- ❑ This narrow loop circumscribed by the unloading and reloading branches may be approximately substituted by a straight line PQ. The slope of this line is taken as the mean Young's modulus.
- ❑ Other definitions:
 - Initial modulus, given as the initial slope of the stress-strain curve.
 - Secant modulus, measured up to a fixed percentage of the peak stress.
 - Tangent modulus, given at a specific percentage of the peak stress.

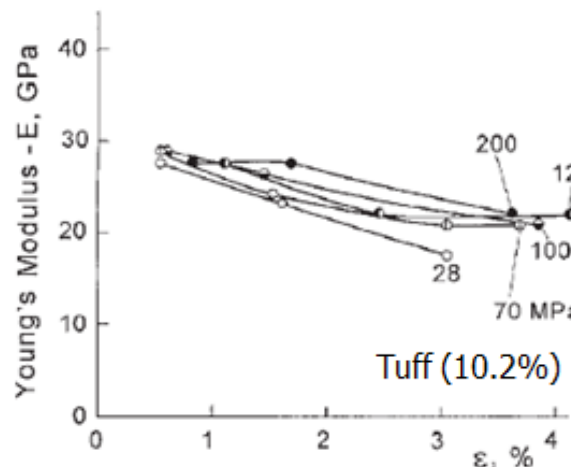
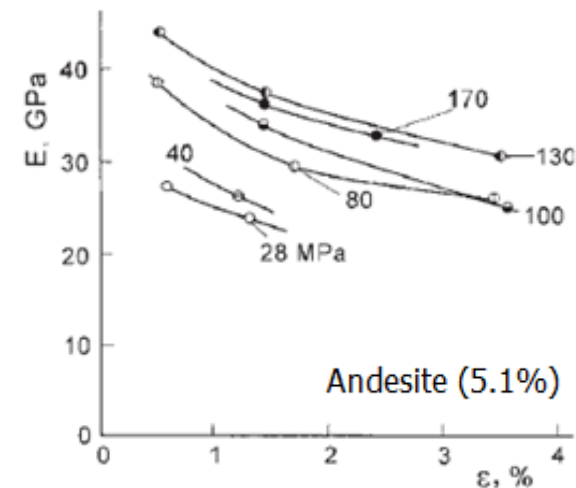
Experimental Rock mechanics. Modulus of elasticity vs strain



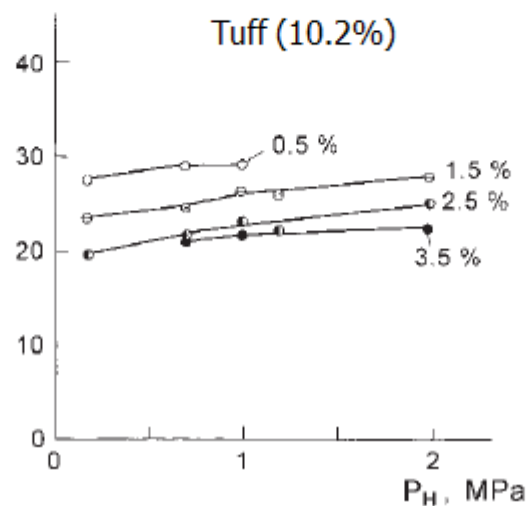
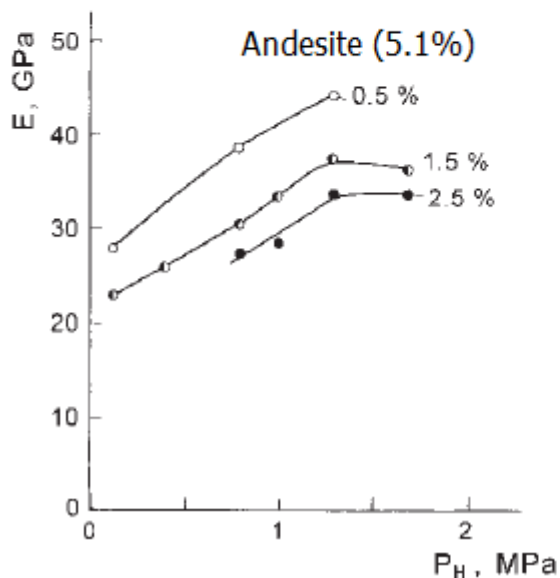
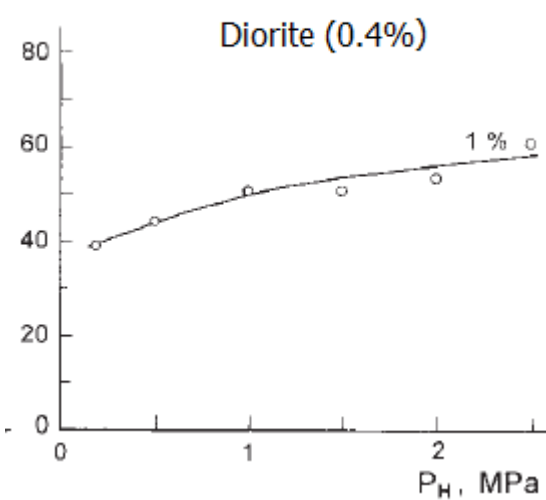
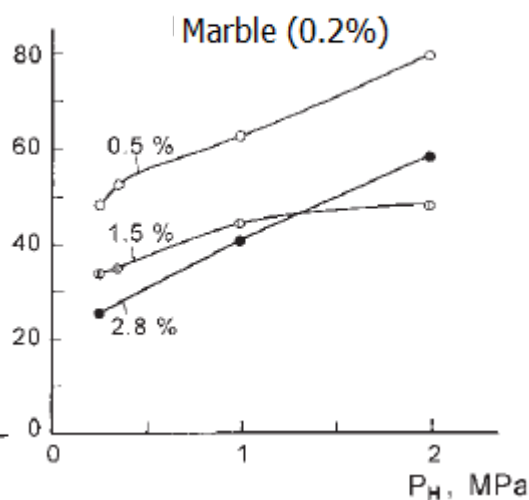
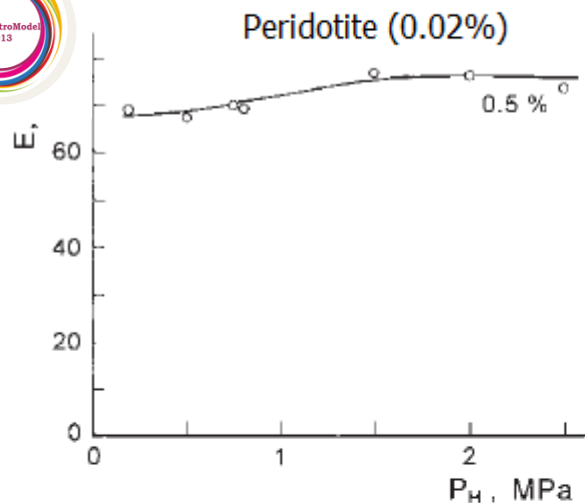
□ In compact rocks, Young's modulus is nearly constant until their fracture.

□ In porous rocks, it generally decreases with increasing strain due to the micro fracturing.

□ In ductile rocks (tuffs and marble) begins to increase with strain under high P just after the yielding, that can be attributed to compaction.

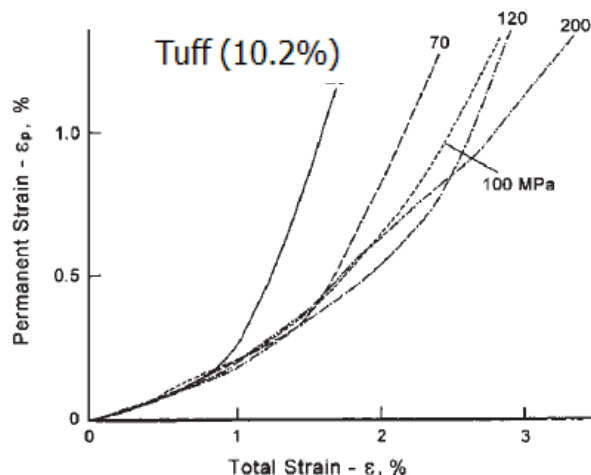
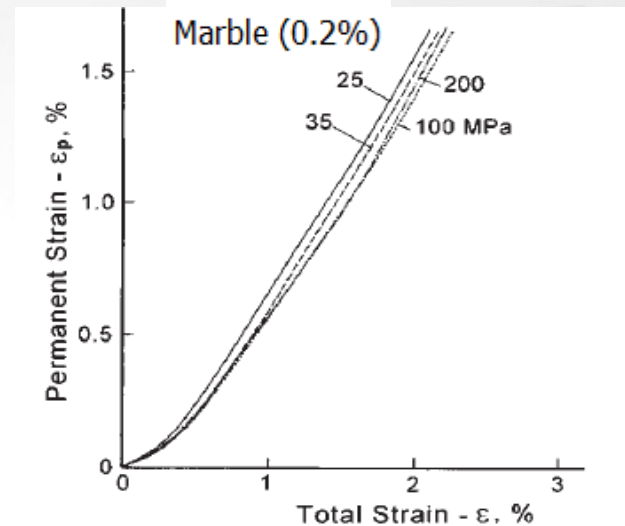
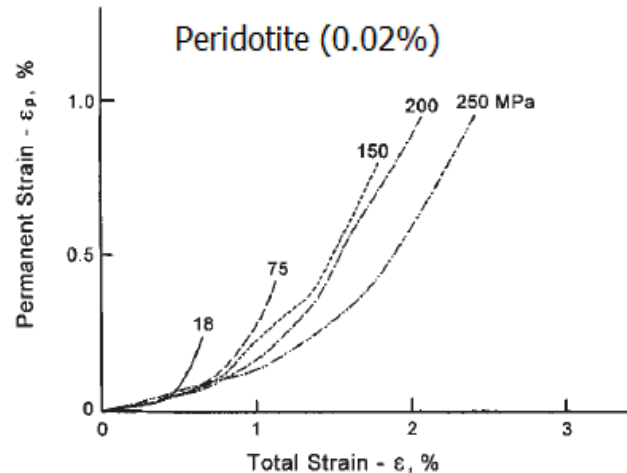


Experimental Rock mechanics. Modulus of elasticity vs confining pressure



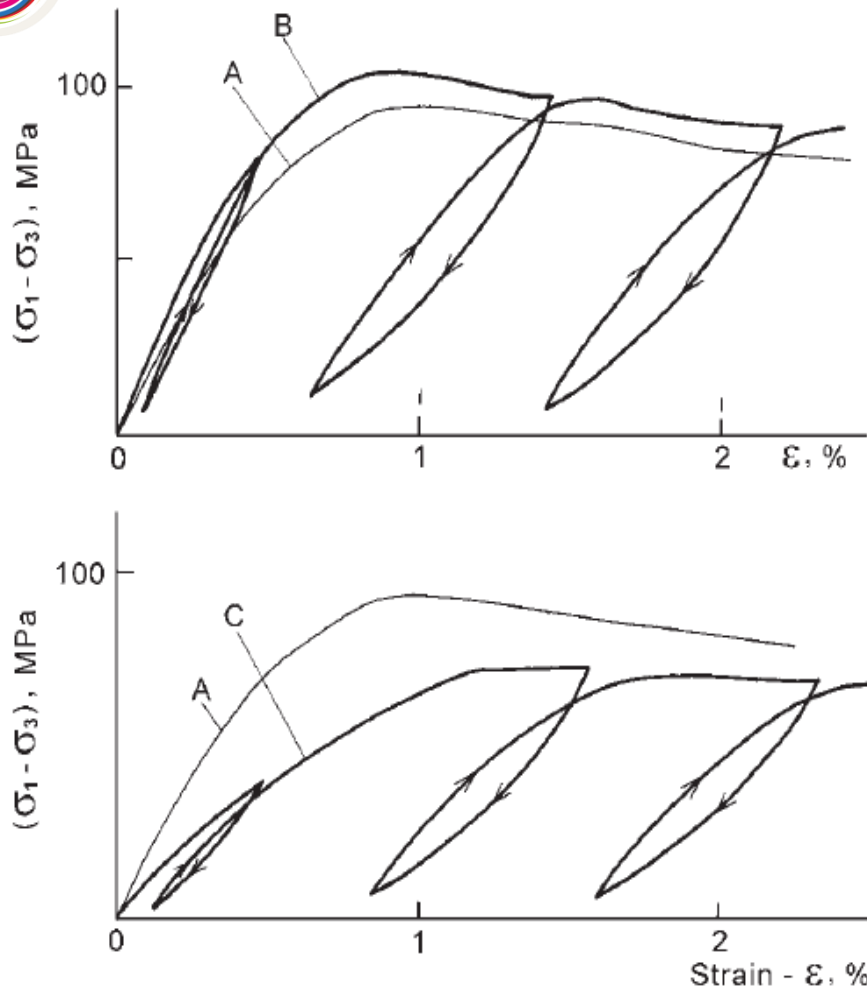
- ☐ Normally, E increases with P .
- ☐ The modulus of peridotite, is largest and increases slightly.
- ☐ The modulus E of diorite is lower than that of peridotite and its increase is more appreciable

Experimental Rock mechanics. The permanent strain



- ❑ In compact brittle rocks, the permanent strain appears slightly in the nearly linear part of stress-strain curve and increases abruptly from a proportional limit.
- ❑ In porous silicate rocks, the appreciable permanent deformation increases linearly at the initial stage and the slope of this curve increases suddenly at some stage.
- ❑ In carbonate rocks, the ratio ϵ_p/ϵ is very large (60–80%) except for the initial stage of deformation, and nearly independent of confining pressure.

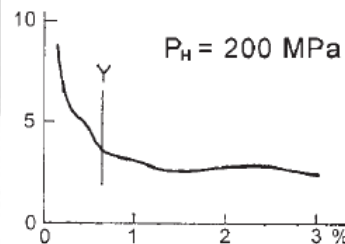
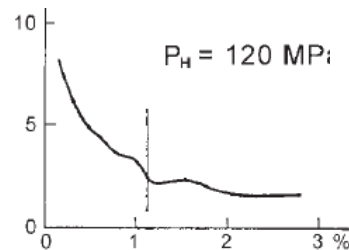
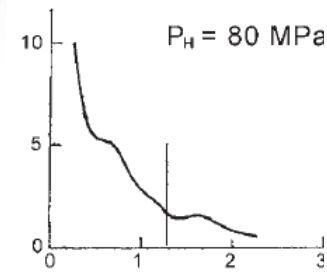
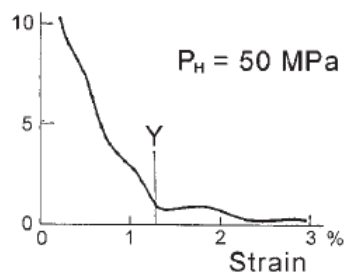
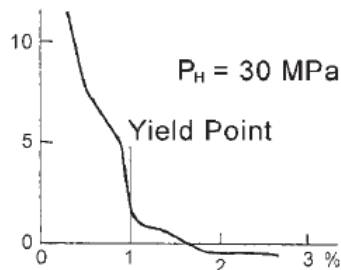
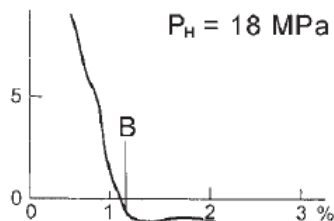
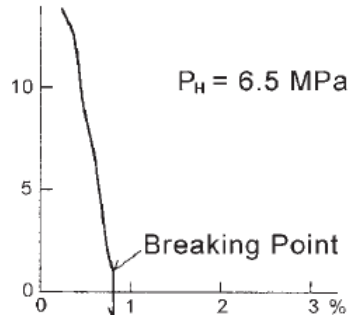
Experimental Rock mechanics. Effects of previous loading



- ❑ A Tuff (10.2%) specimens were initially exposed to hydrostatic pressure of 80 MPa and 200 MPa. Thereafter, these specimens (B and C) and a virgin specimen (A) were tested under conventional triaxial compression conditions at 18 MPa confining pressure.
- ❑ The Young's modulus in the case B are not significantly different from the case A. On the other hand, the compressive strength and Young's modulus in the case C are markedly lower than in the case A.
- ❑ Effects of previous loading may be complex. However, a history of deformation of rock specimen is more or less preserved in mechanical properties - elasticity, plasticity, fracture strength, deformation characteristics in repeated loadings etc

Experimental Rock mechanics.

Yield stress

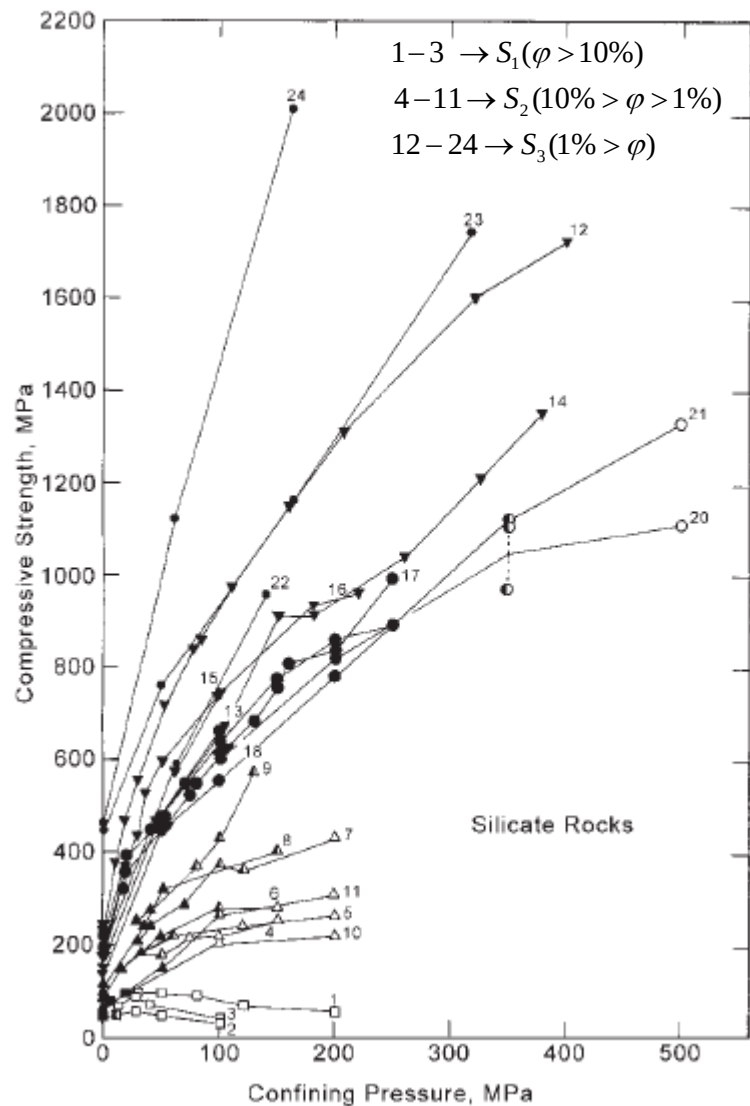


Relation between $d(\sigma_1 - \sigma_3)/d\varepsilon$ and strain ε for Tuff (10.2%).

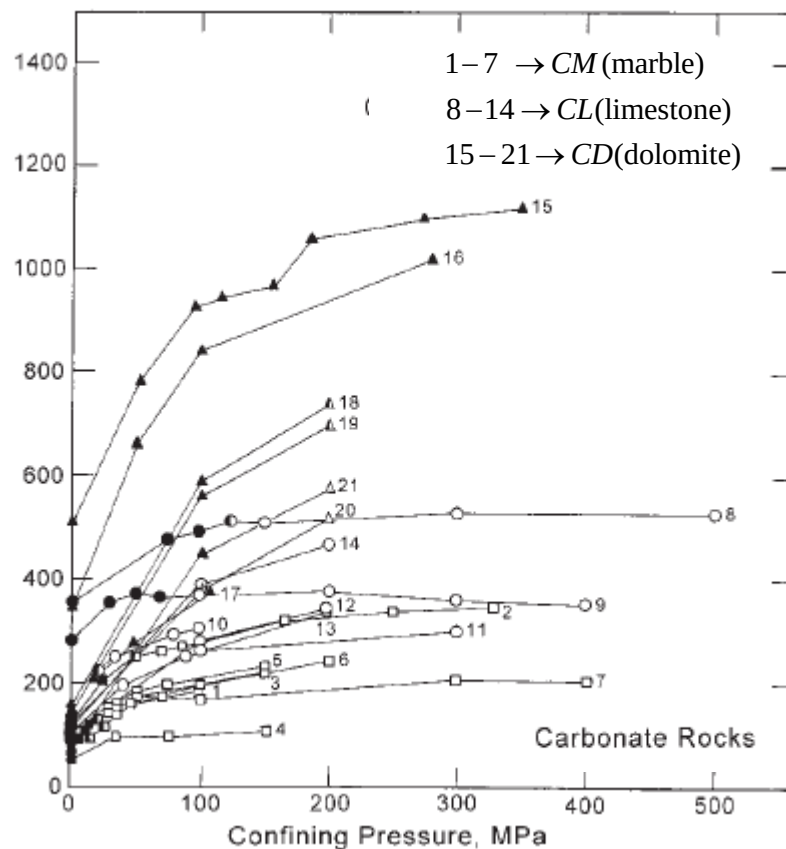
- ❑ Strength in the brittle state is definitely determined.
- ❑ Strength in the ductile state - yield stress, is the stress at which the sudden transition from elastic to plastic deformation state takes place.
- ❑ Some rocks show a gradual transition from elastic to plastic stage, that is, the slope of the stress-strain curve varies gradually and the determination of yield stress is difficult
- ❑ The local yielding begins to occur even under low stress, because the stress in the rock specimen having nonuniform structures distributes around structural irregular points.
- ❑ The yielding in such takes place gradually in a wide range of stress values, and the macroscopic yielding may be completed at a certain stress value, which is treated as yield stress.
- ❑ The slope of the stress-strain curve decreases gradually with the increase of strain and becomes nearly constant at the yield stress.

Experimental Rock mechanics.

Relation between strength and confining pressure



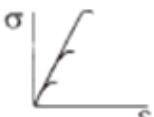
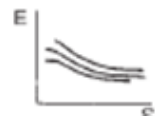
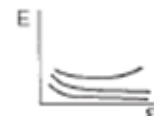
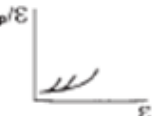
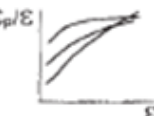
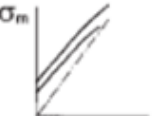
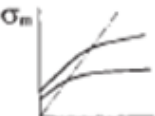
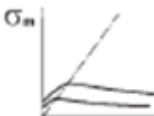
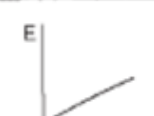
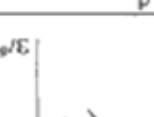
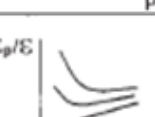
Closed symbols: brittle behavior;
Semi-closed: transitional behavior;
Open: ductile behavior.



Experimental Rock mechanics.

Summary of mechanical properties for silicate rocks



Rocks	Peridotite Diorite	Tuff	Andesite
Porosity (%)	0 - 1	5 - 10	10 - 25
Stress - Strain			
Young's Modulus - Strain			
$\left(\frac{\text{Permanent Strain}-\epsilon_p}{\text{Total Strain}-\epsilon} \right)$ - Strain			
Strength - Pressure			
Young's Modulus - Pressure			
$\left(\frac{\text{Permanent Strain}-\epsilon_p}{\text{Total Strain}-\epsilon} \right)$ - Pressure			

- ❑ The tested silicate can be divided into three groups according to their mechanical properties.
- ❑ Porosity is the most important factor for mechanical properties in silicate rocks.
- ❑ The mechanical properties of the marble are significantly different from those of silicate rocks.
- ❑ Although the porosity of the marble is very small (about 0.2%), its properties are nearly similar to the third group in appearance.
- ❑ This shows that the mineral composition is also an important factor in mechanical behavior in rock deformation under pressure

Experimental Rock mechanics. The Coulomb-Mohr fracture criterion



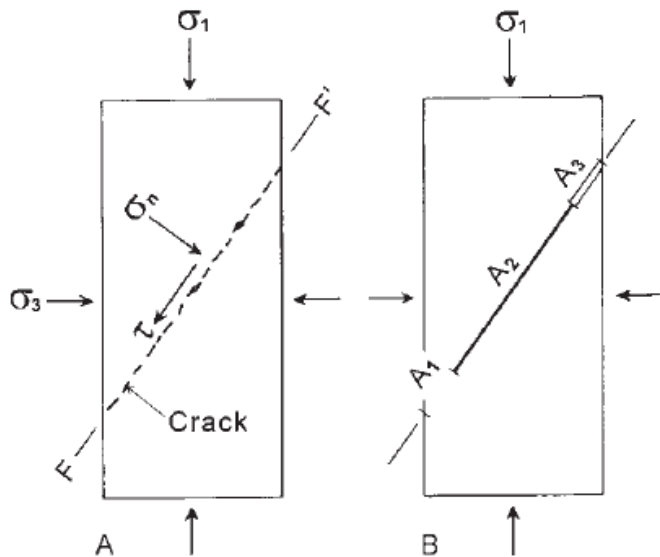
- The criterion states that shear fracture takes place when the shear stress becomes equal to the sum of the cohesive shear strength and the product of the coefficient of internal friction and the normal stress across the fracture plane:

$$\tau = \tau_0 + \mu_i \sigma_n$$

- Until the cohesion of rock is broken, no sliding surface exists in the intact body, and thus, the Coulomb equation seems to be contradictory, or the concept of internal friction is unsatisfactory *Handin (1969), Scholz (1990), Paterson (1978)*



- *Mogi (1974)*: According to acoustic emission measurements a number of micro-fractures occur during the deformational process prior to the macroscopic fracture. They are located along the future fault plane, which area is divided into parts, A1, A2, and A3



- A1 is still intact, $T=T_1$, shearing resistance
- A2 is the total area of closed crack surfaces, $T=T_2$
- A3 is the total area of open crack surfaces, $T=T_3=0$.

$$\tau = \tau_0 + \mu_i \sigma_n = \frac{T}{A} = \frac{T_1}{A} + \frac{T_2}{A} = \tau_1 \frac{A_1}{A} + \mu \sigma_n \frac{A_2}{A}$$

τ_1 is the inherent shearing strength of the intact part
 μ is the coefficient of external friction

Experimental Rock mechanics. The Coulomb-Mohr fracture criterion. Validation



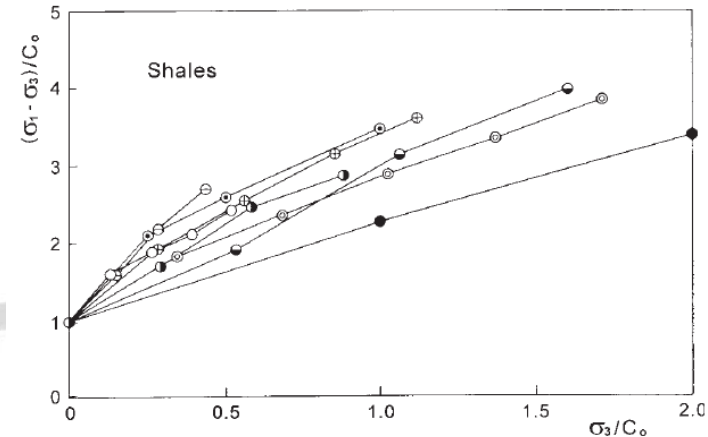
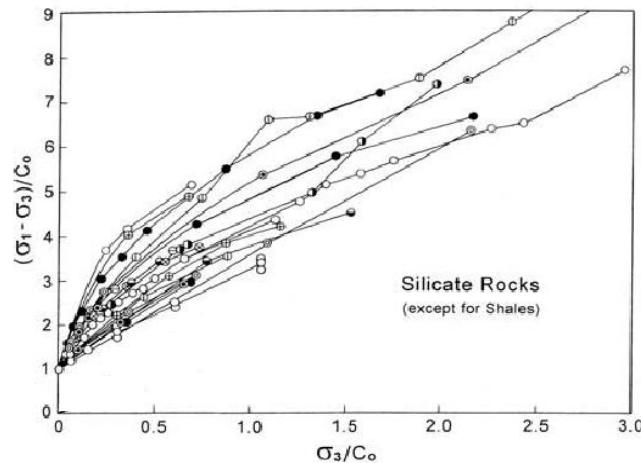
- The criterion can be reformulated in normalized with respect to UCS terms. If confining pressure is zero, brittle rock is fractured at

$$\sigma_1 = UCS = C_0$$

- Confining pressure provides additional support to a specimen, so

$$\sigma_1 - \sigma_3 = C_0 + \alpha \sigma_3, \quad (\sigma_1 - \sigma_3) / C_0 = 1 + \alpha \sigma_3 / C_0$$

- If the criterion is valid, α should be constant depending only on μ_i

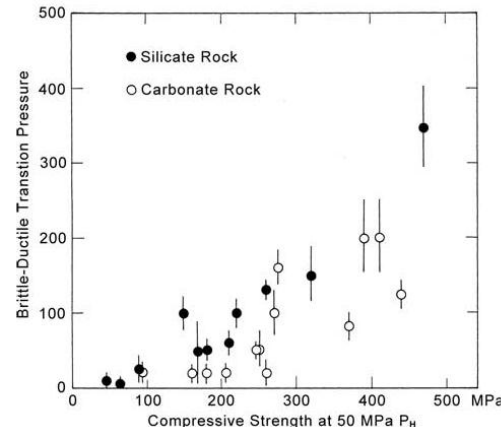
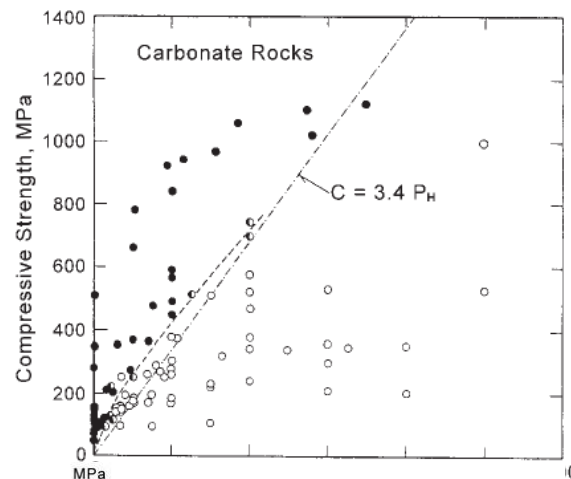
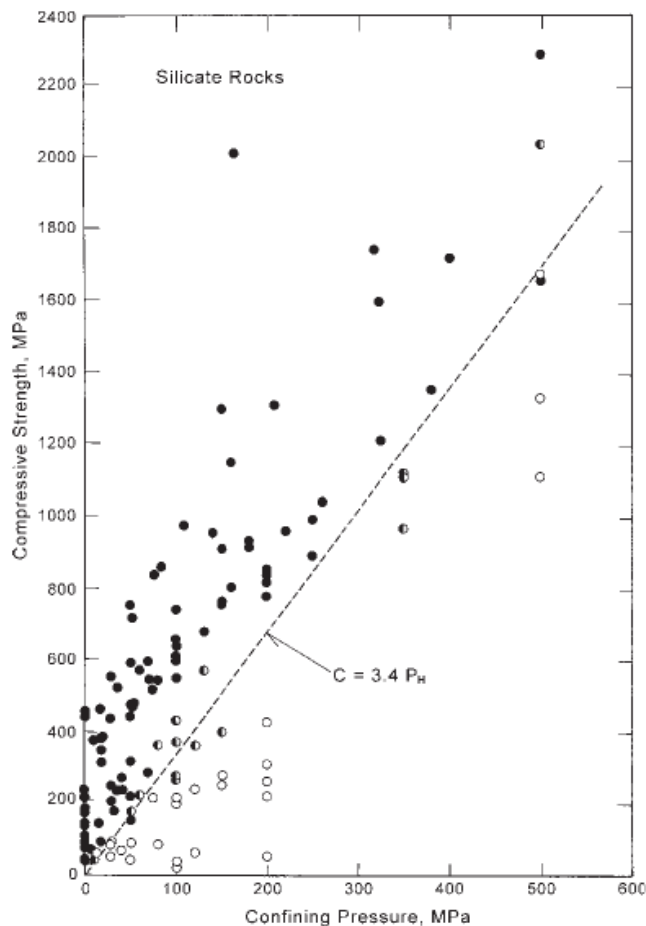


- The curves are limited to a very brittle region (high-pressure region near the brittle-ductile transition) pressure is not included. Linear parts are parallel to each other.
- The criterion appears to apply to brittle fracture of many rocks, except for a low-pressure region and near the brittle-ductile transition pressure.

Experimental Rock mechanics. Brittle-ductile transition. Additional observations



- With increase of confining pressure, ductility, which is defined as the ability to undergo large permanent deformation without fracture, increases markedly and a transition from the brittle to the ductile state takes place at some confining pressure.

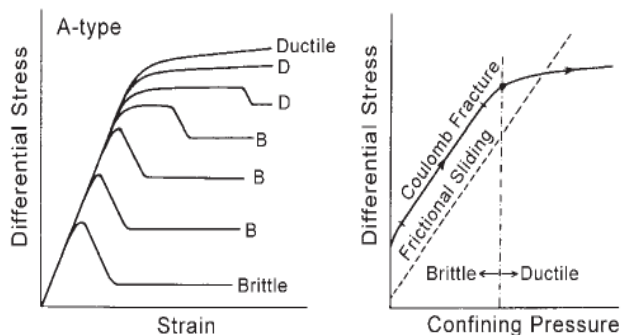


$$\sigma_1 - \sigma_3 = 3.4\sigma_3$$

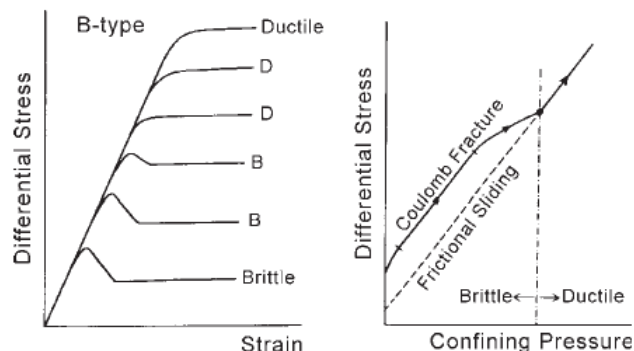
- The brittle-ductile transition pressures of silicate rocks are appreciably higher than those of carbonate rocks.
- This suggests that there are different mechanisms of the brittle-ductile transition in different rock types

Experimental Rock mechanics.

Brittle-ductile transition. A-type and B-type rocks



- ❑ In this case of A-type rocks , the brittle-ductile transition may take place when the fracture strength is equal to the yield strength.

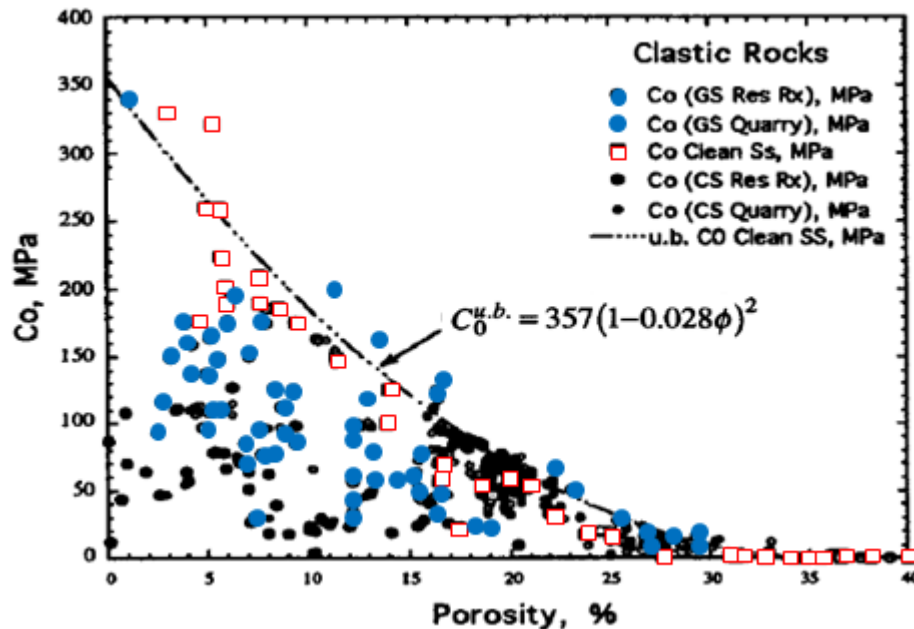


- ❑ In the case of B-type, the brittle-ductile transition pressure is the pressure at which the strength of rock at faulting is equal to the strength due to frictional resistance after faulting .
- ❑ Some carbonate rocks, particularly at high temperature, are A-type and silicate rocks are B-type.

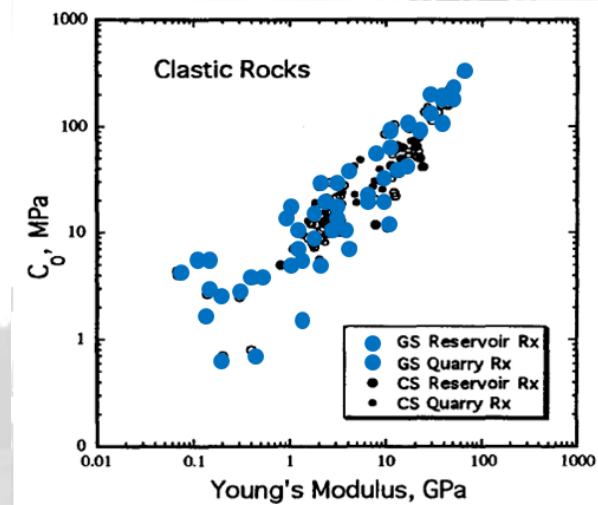
Experimental Rock mechanics. Classification quiz



- ❑ In the above, we followed the classification of rocks by belonging to mineral *group* (Mogi, 2007)
- ❑ A question appears so far: why silicates are so “good”, but carbonates are so “bad”? ?
- ❑ *Plumb (1994)* tested 784 clastic rocks classified according to their dominant load-bearing solid phase: the volume fraction of clay minerals, V_{clay}



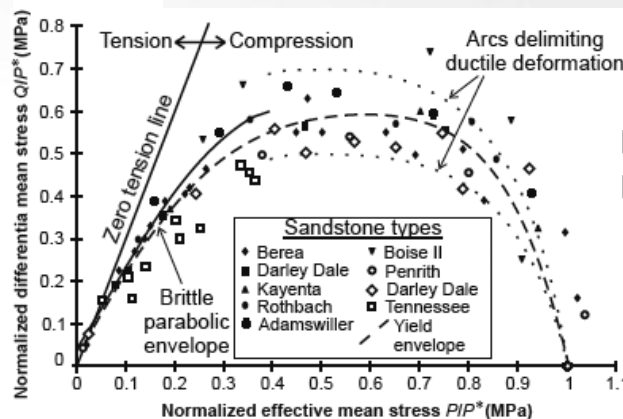
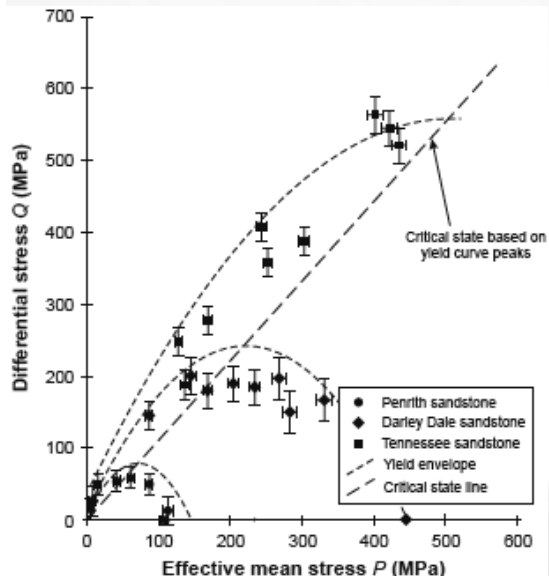
- ❑ *Clean sandstones* – well cemented sandstones with $V_{clay} < 5\%$ behaves as described above and as observed in *Hoshino et al 1972; Dunn 1973; Logan 1986; Perkins & Weingarten 1988; Scott & Nielson 1991; Vernik et al 1993*



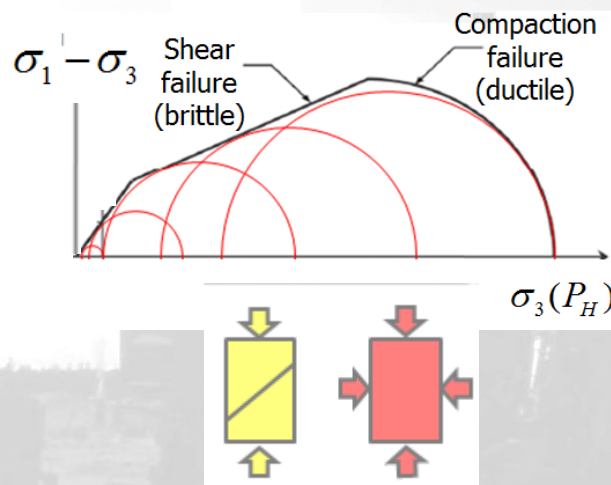
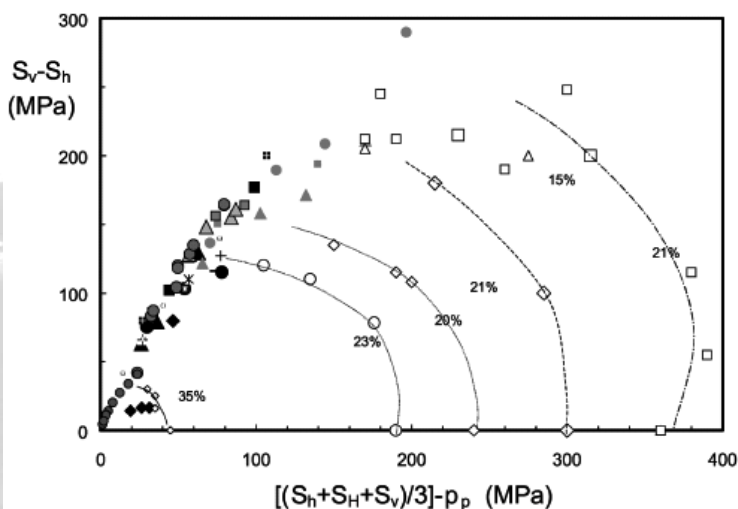
- ❑ One has to classify materials with respect to mechanical properties, not to chemical composition

Experimental Rock mechanics.

Further experiments with sandstones. Elliptical cap for compaction



□ Cuss et al (2003)
□ Schutjens et al (2004)

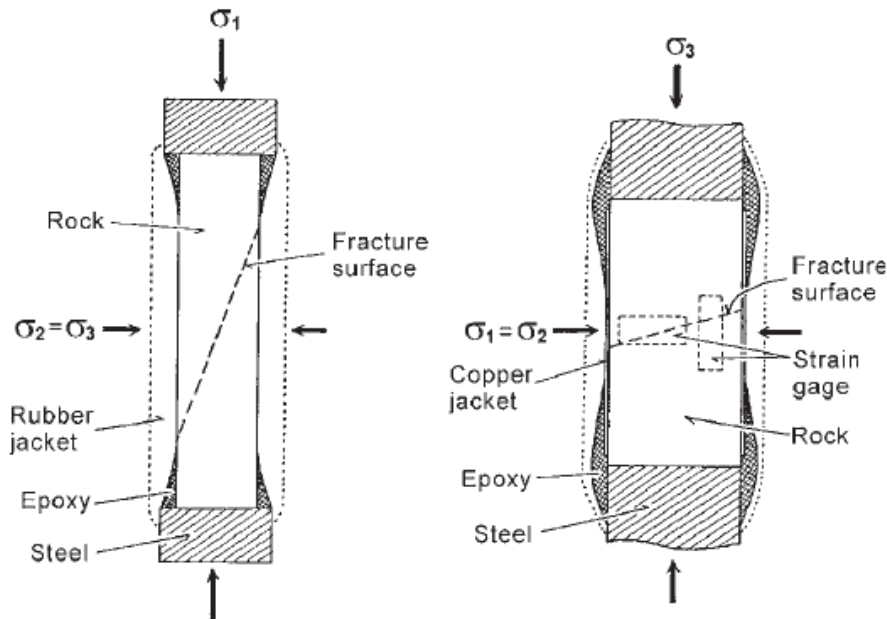


Experimental Rock mechanics.

Compression and extension under confined pressure



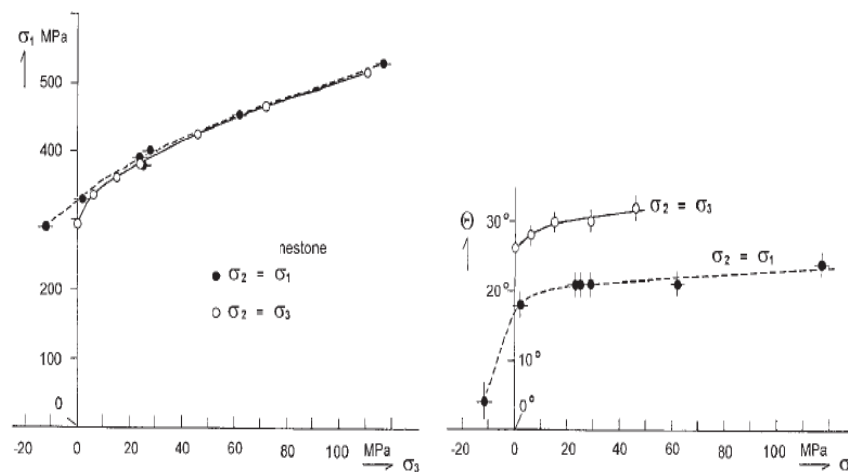
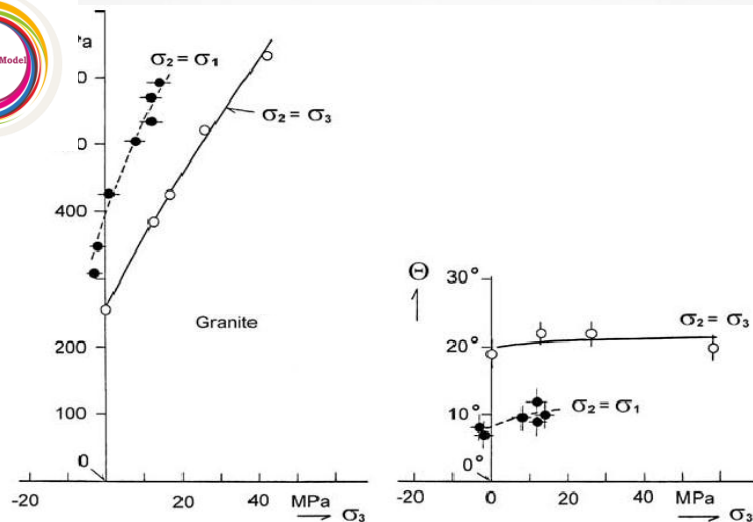
- ❑ All previous results were obtained under the condition $\sigma_1 > \sigma_2 = \sigma_3$. The question appears thus what is the influence of the intermediate stress. The simplest way to check whether it has any influence is to make extension test under the same experimental set up and compare the results.



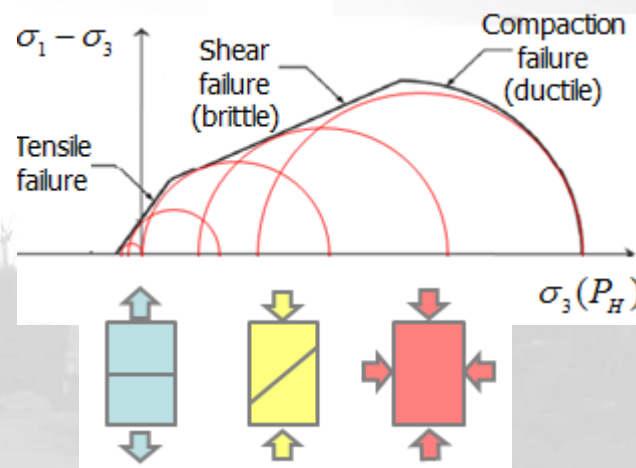
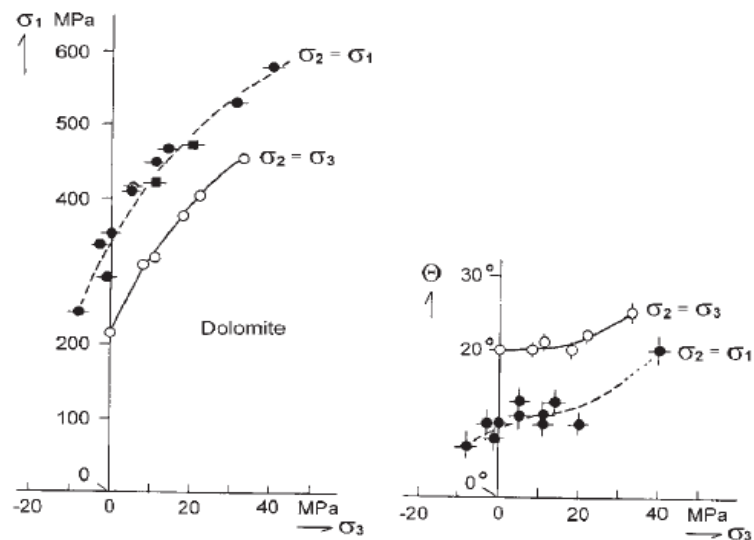
- Since $\sigma_1 = \sigma_2 > \sigma_3$ in the extension test, additional epoxy supply was required to avoid stress concentration.
- Special examination on the chosen material (granite, dolomite and limestone) with respect to their homogeneity has been carried out.
- All specimens were taken from a single block of each rock.
- Compression specimens were circular cylinders 1.6 cm in diameter and 5.0 cm long.
- Extension specimens were from 2.30 to 2.54 cm in diameter and 5.0 cm long.

- ❑ If the intermediate principal stress is without influence, the curve of σ_1 versus σ_3 for extension should coincide with the curve for compression tests

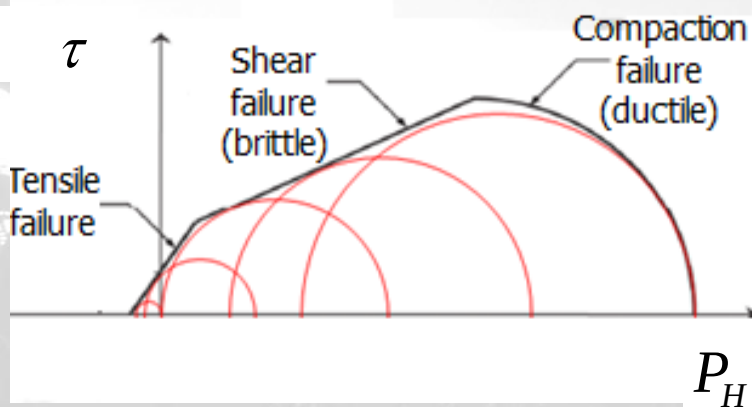
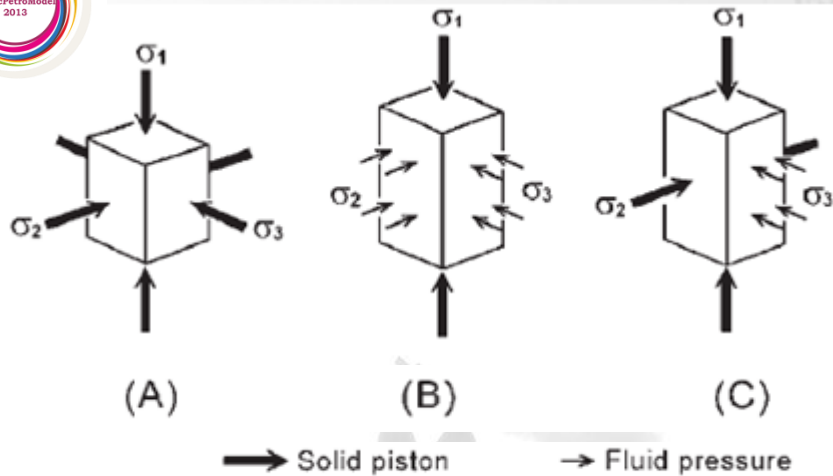
Experimental Rock mechanics. Compression and extension under confined pressure



- The intermediate principal stress has marked influence on failure conditions in these rocks. Failure stress (σ_1) and friction angle depends on conditions (except limestone)



Experimental Rock mechanics. True triaxial compression experiments



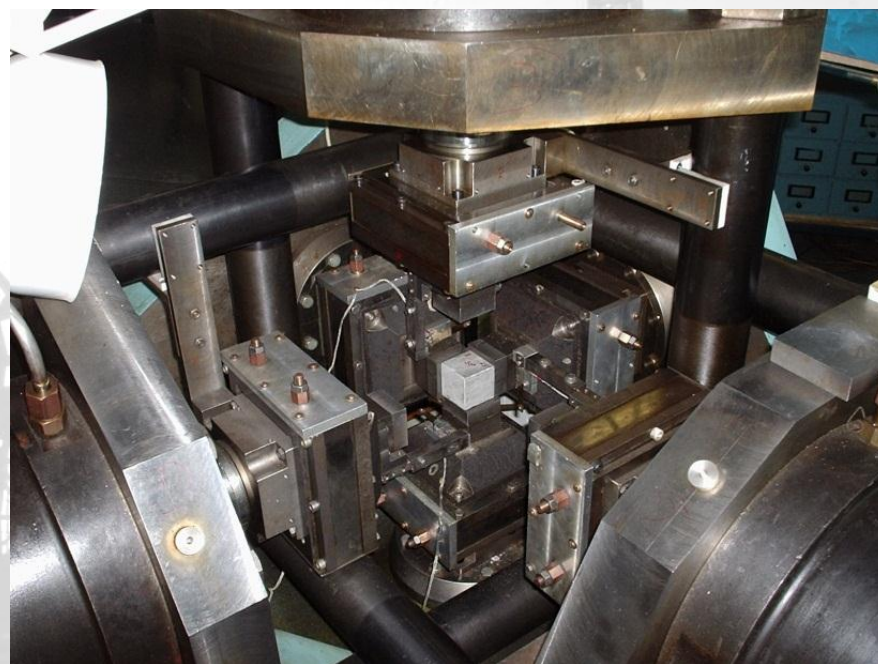
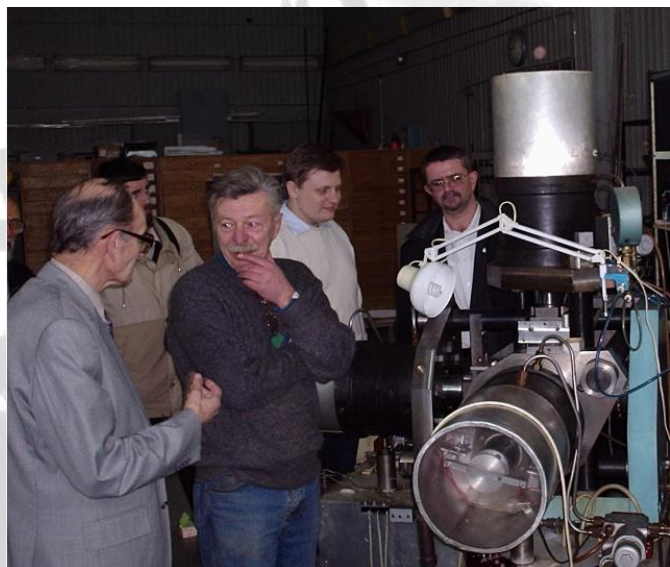
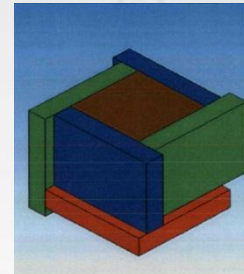
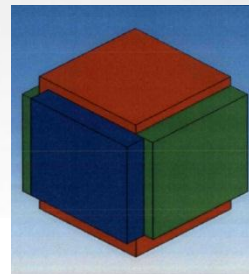
- ❑ A-type experiment: Triaxial compression by three solid pistons
- ❑ B-type experiment: Axial compression by a solid piston and lateral compression by two different fluidal pressures
- ❑ C-type experiment: Triaxial compression by two pairs of solid pistons and fluid-confining pressure
- ❑ Octahedral shear stress vs mean rock pressure (*Mogi, 2007*):

$$\tau = \frac{1}{3} \left((\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 \right)^{1/2}$$

$$P_H = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3)$$



Experimental Rock mechanics. True triaxial compression experiments at the IPMech RAS





Part 4. Building and calibration of a MEM



Building and calibration of a MEM. Smart field – the ultimate goal for next decades



- ❑ The general aim is to develop efficient software tools for the optimization of oil field development and operations.
- ❑ This includes data assimilation, fast simulation, model updating, and optimal control.
- ❑ Techniques being developed should be essential for the success of Smart Fields, also known in industry by names such as i-fields, e-fields, Field of the Future, etc.
- ❑ Optimal control would be implemented in existing fields at any stage of their development and in new fields.

How to develop such approach?

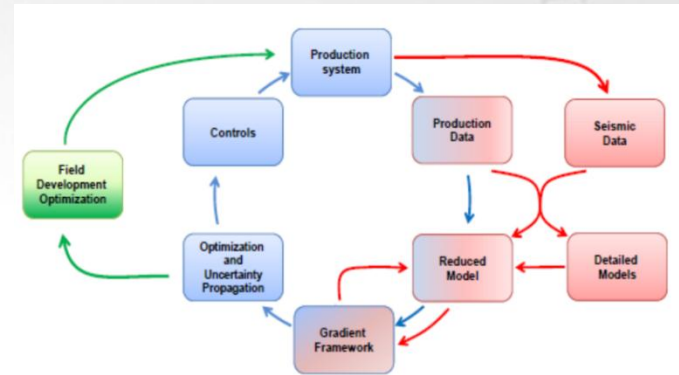
“Reservoir Simulation System for the Next Decades”, 9th International Forum on Reservoir Simulation, Dec 9-13 2007, Abu Dhabi, UAE by CMG, Shell, Petrobras/Cenpes:

1. Unified approach for black-oil, compositional, thermal and chemical simulation;
2. The ability of perform full field (Wellbore/Reservoir) simulation of thermal, EOR or unconventional processes with multiscale physics;
3. The potential to analyze recovery mechanisms with coupling of fluid flow with geomechanics

Traditional approaches for developing and operating oil and gas fields which are based on combinations of commercial software (ECLIPSE, Roxar etc) are rarely optimal. The positional gains of deploying these new technologies are very significant.



Building and calibration of a MEM. Smart field Consortium at Stanford University



- ❑ "Optimization techniques for initial and continuous development over the life of the field. This includes determining where and when to drill, the design of the wells to be drilled and the type of monitoring needed. Risk assessment and decision making techniques will be investigated for aiding management decisions".
- ❑ "Efficient and reliable techniques for data filtering and assimilation, because huge quantities of data are produced in real-time monitoring".
- ❑ "Continuous model updating, since real-time model calibration could greatly improve future predictions. The models involved refer, for example, to geological aspects, fluid description (relative permeability, thermodynamic parameters, etc.), and multiphase flow in wells and facilities".
- ❑ "Very fast reservoir simulators and adequate proxies for spanning the search space in the optimization".
- ❑ "Effective integration of repeated geophysical remote-sensing data (e.g. seismic, gravity, electro-magnetic, etc.) into the reservoir updating and optimization process".



Building and calibration of a MEM.

Schlumberger and Chevron Announced Release of INTERSECT



Schlumberger



INTERSECT (2000-настоящее время)

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❑ Schlumberger and Chevron Energy Technology Company, announced in 2009 the release of INTERSECT* next generation reservoir simulation software to enable simulation of large complex reservoirs and highly heterogeneous systems.
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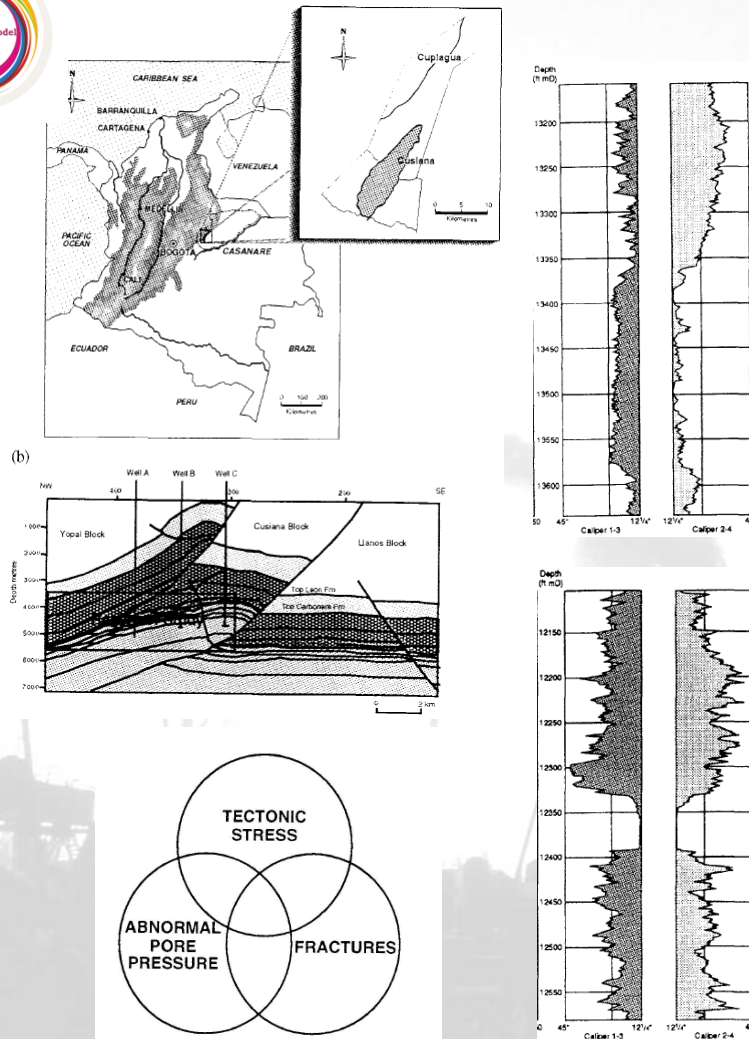
❑ "Going beyond capabilities offered by the current generation of simulators, INTERSECT runs large and heterogeneous models—simulating tens of millions of cells—fast. It also accurately models complex geology and wells with minimal or no upscaling. Supporting sophisticated field management, it handles thousands of wells in mature fields, as well as simulating all fluid types and recovery processes in a single simulator. Advanced production controls enable flexible operational field management".
- 

❑ "The technology is the result of extensive research collaboration and a joint development effort between the two companies beginning in 2000. Both companies have been developing and using simulators for several decades and have accumulated significant knowledge as a result"
- 

❑ "To provide open extensibility, future releases of INTERSECT will include Ocean* development tools—enabling oil and gas companies, independent software providers and universities—to create plug-ins to customize simulation workflows for specific reservoir and recovery challenges".

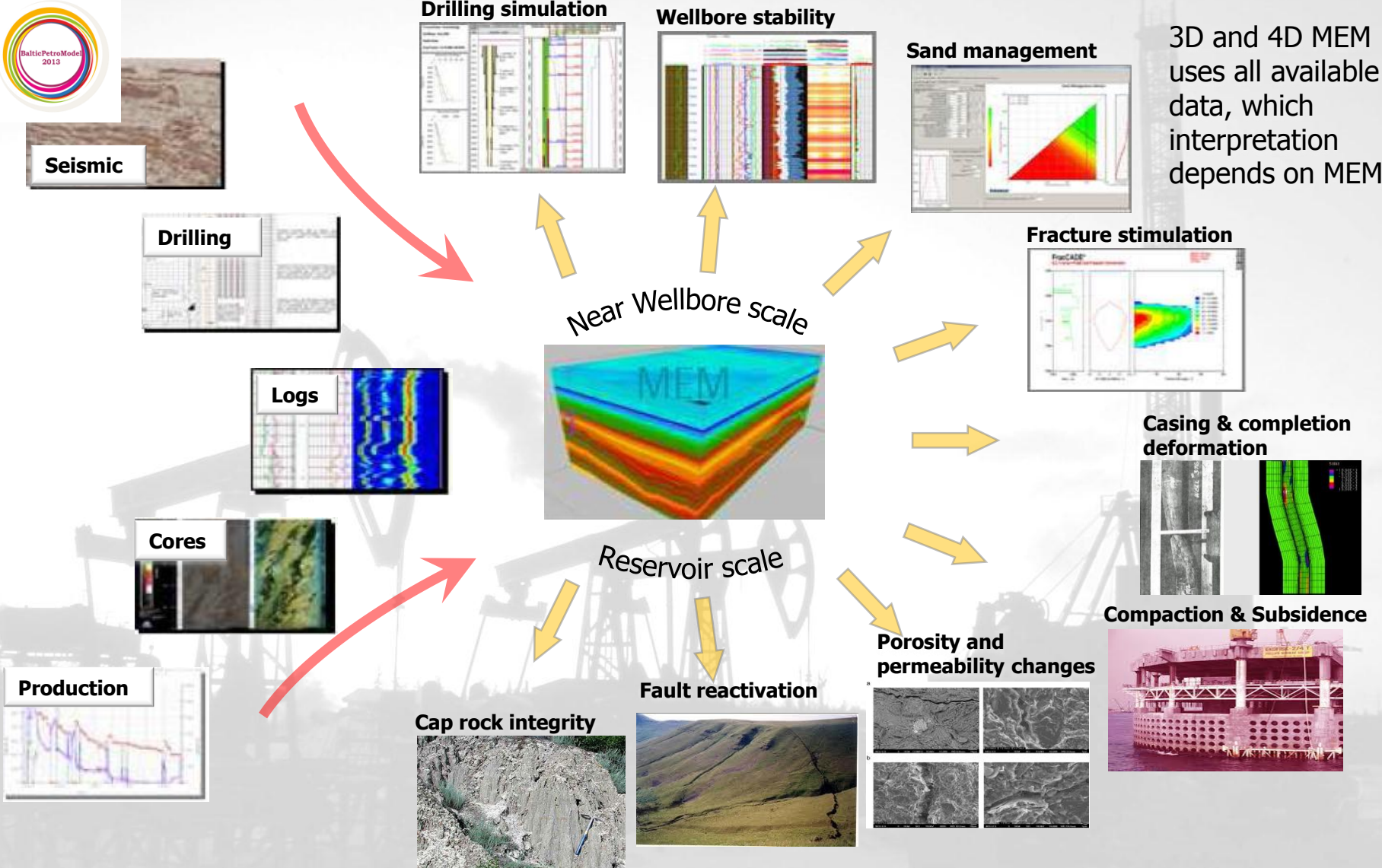
Building and calibration of a MEM.

Cusiana field – a godfather of MEMs

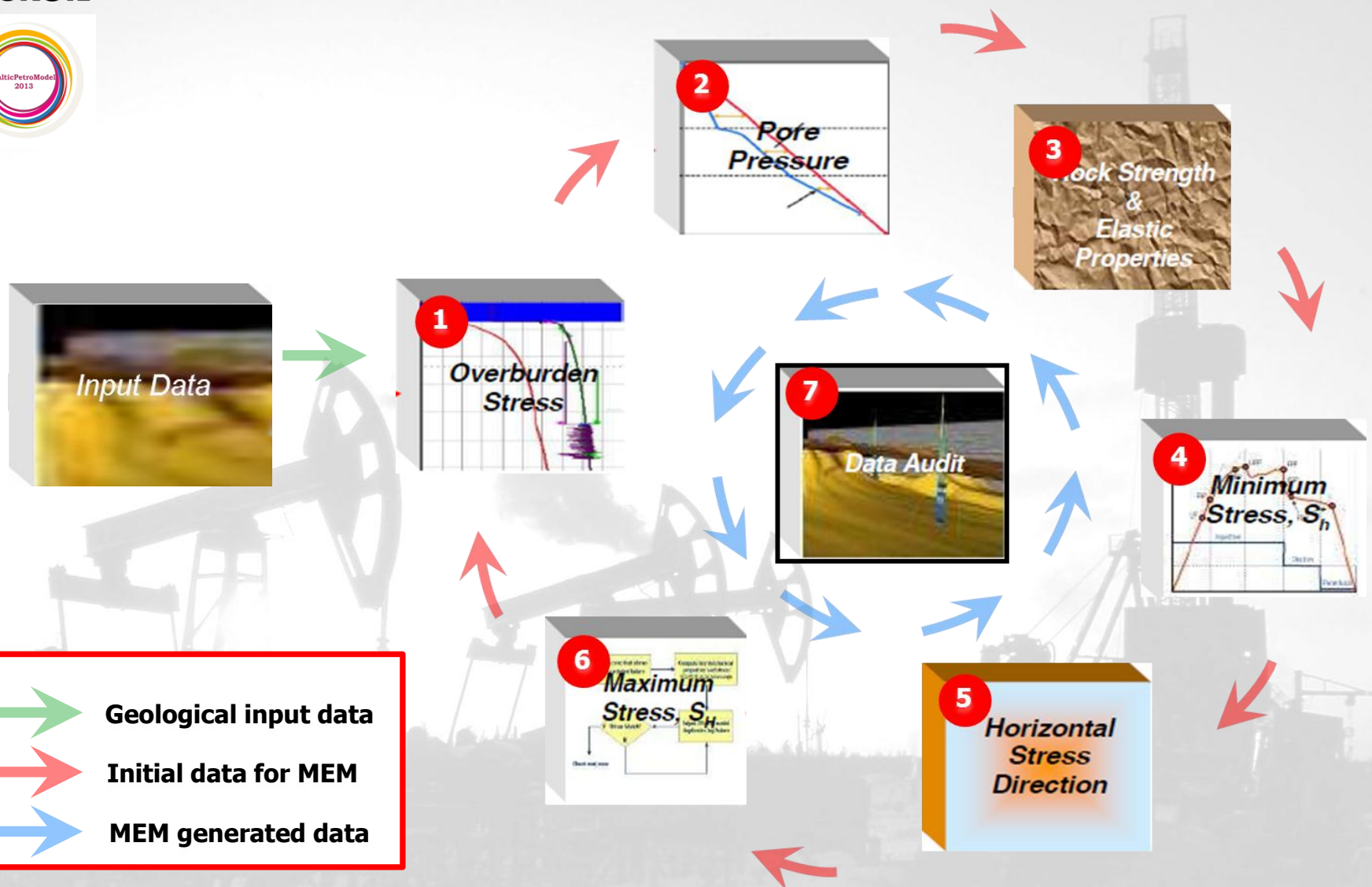


- ❑ Major drilling problems has been encountered by BP in the Cusiana field, mostly related to hole enlargements, which was unsolvable by using conventional solutions (*Addis et al., 1993, Last et al., 1995*)
- ❑ The field is located in a tectonically active fractured region with abnormally high pore pressure, so all three factors for wellbore instability development enforced each other
- ❑ An integrated multidisciplinary team of geologists, geophysicists, reservoir engineers from several oil and service was joined and forked together for a year to collect enough geomechanical information and find a new approach for solution
- ❑ This experience let Schlumberger experts to generate a new vision on the integrated field optimization procedure. As a result of study Cusiana field basic principles of so called MEM were developed (*Plumb et al., 2000*)

Building and calibration of a MEM. Iterative loop of initiation



Building and calibration of a MEM. Three stages of the iterative process



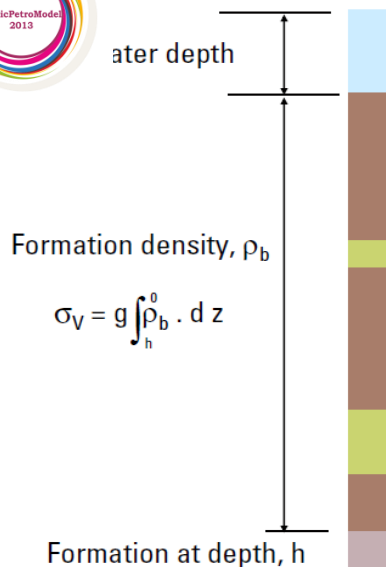
Building and calibration of a MEM.

1. Overburden stress



Building and calibration of a MEM.

Overburden stress (1/2)



- At any point in the earth, the overburden stress is equivalent to the weight of the overlying formation, so it might be integrated with possible taking into account of the acceleration dependence on depth.

$$\sigma_{zz} = \int_0^z \rho(z') g(z') dz'$$

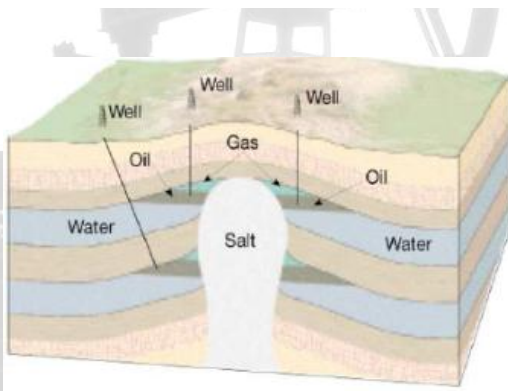
- Weight of water becomes significant in deep water

- For porous rocks $\rho(z') = \phi(z') \rho_f(z') + (1 - \phi(z')) \rho_s(z')$

- The overburden or vertical stress is usually considered as a principal stress $\sigma_{zz} = \sigma_v$, so the two other principal components are horizontal. This is true for large depth in tectonically relaxed region, but this may not be true:

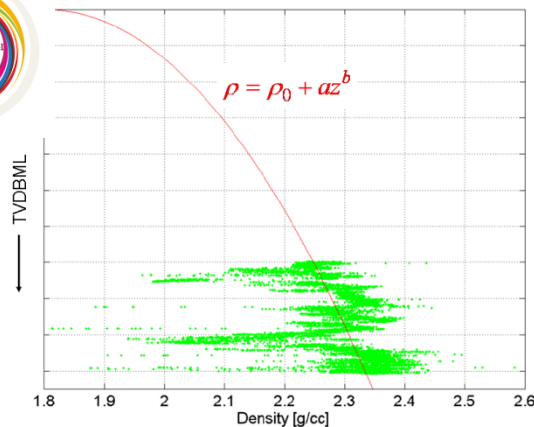
- near surface
- under strongly sloping surface, even at depth
- near heterogeneities such as inclusions or faults
- near underground openings such as boreholes
- near depleting reservoirs

- Check whether the vertical stress is a principal stress by observations of drilling-induced tensile fractures *Zobac M. (2007), Soliman M. (2008)*



Building and calibration of a MEM.

Overburden stress (2/2)

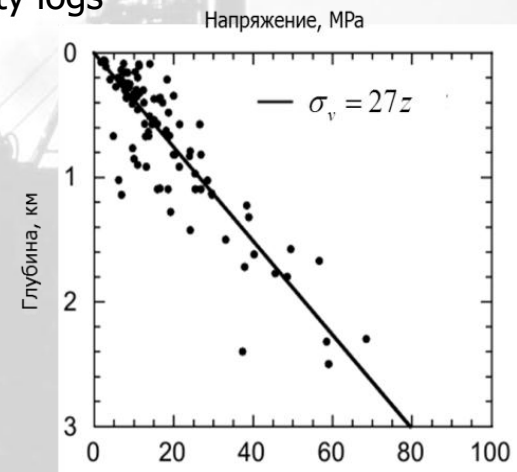
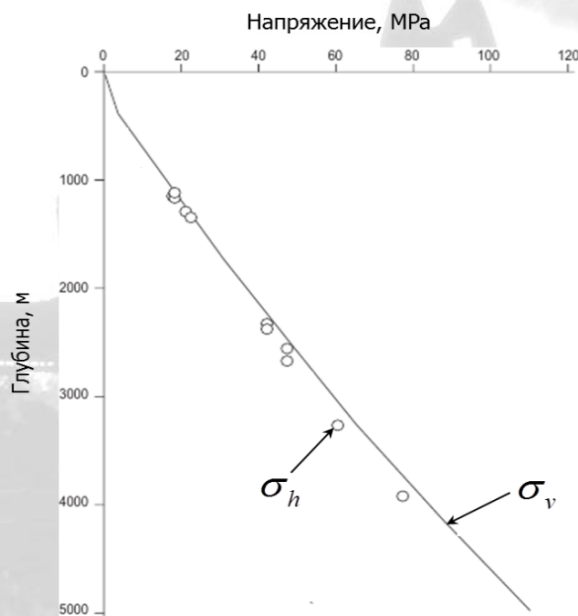


- Variability of the density logs, as well as the fact that they are often not obtained over the total depth of interest, leads to uncertainty in the calculated overburden *Sayers et al. (2009)*

- It may happen that the exact measurements of vertical stress is required for correct HF design, if it is very close to the least principal stress, as shown in a Figure (*Wiprut, Zoback et al. 2000*). The horizontal stress is determined from extended leak off tests in the Visund field, the overburden stress is determined from integration of density logs

- If the accuracy is not the pivotal point, it is convenient to make use of the established in (*Brown & Hoek, 1978*) trend

$$\sigma_v = 27z$$



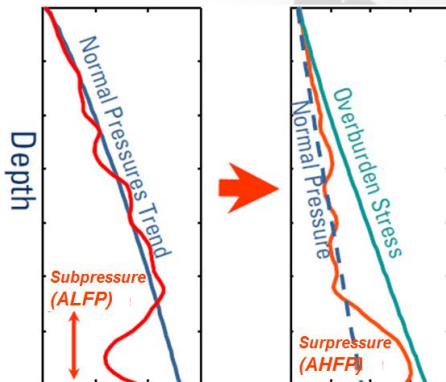
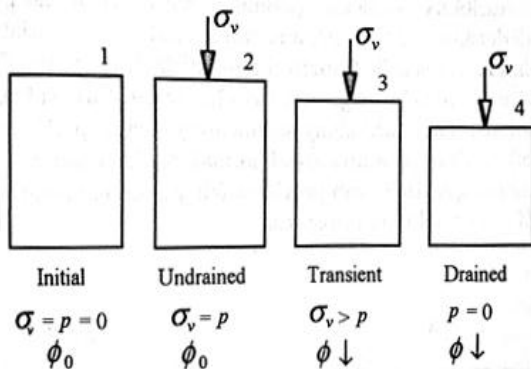
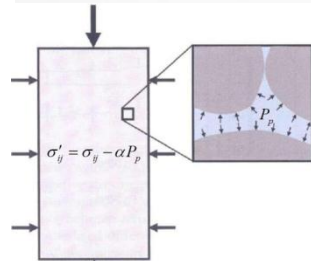
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2. Pore pressure



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Pore pressure (1/3): Normal and abnormal PP



- The simplest way to define pore pressure would be to integrate fluid density over the depth (hydrostatic pore pressure)

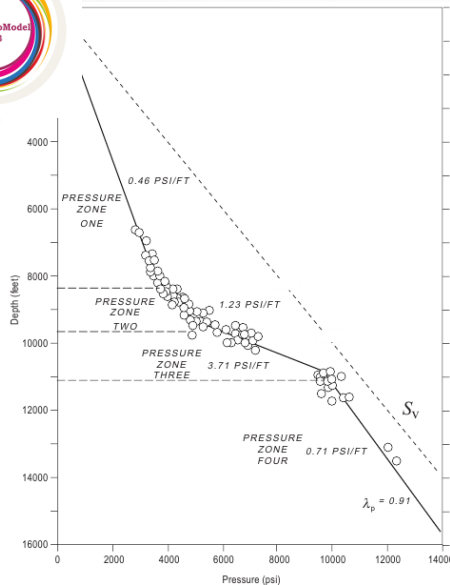
$$P_p^{hydro} = \int_0^z \rho_f(z')g(z')dz'$$

- Depends on
 - salinity of the water,
 - temperature,
 - content of dissolved gases
- Hydrostatic pore pressure is defined at salinity of 80,000 ppm of sodium chloride at 25 C = 0.465 psi/ft
- What generates abnormal pore pressure - normal hydrostatic pressure indicates a fluid communication (vertical) between the formations. Abnormal pressure occurs if one or more of the formations are impermeable to the vertical hydraulic communication
 - Subpressures (or ALFPs) have been encountered far less than the surpressures, mainly artificially induced by production
 - Surpressures (or AHFPs) have been encountered in all of the continents of the world where exploratory drilling for hydrocarbons has been conducted. It is limited by overburden stress



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Pore pressure (2/3): Compartmentalization



- Pore pressure measurements in the Monte Cristo field, located onshore near the Gulf of Mexico in south Texas (*Engelder & Leftwich 1997*)

- The observation that a given reservoir can sometimes be compartmentalized and hydraulically isolated from surrounding formations has received a lot of attention over the past decades. The economic reason for this interest is obvious as production from distinct compartments has a major impact on the drilling program required to achieve reservoir drainage. (*Ortoleva 1994*)

- Analysis of Norwegian North sea wellbore data (*Grollmund, Zoback et al. 2001*) indicated significant variations of $\lambda_p = P_p / \sigma_v$ at each of considered depth 1500, 2000 and 3000 meters. So one has to laterally interpolate pore pressure data with caution but consider the compartmentalized zones in view of physical and chemical mechanisms of their formations (*Chilingaryan et al, 2002*):

- Undercompaction – the rate of compaction is higher than the rate of fluid migration
- Tectonics – loading that leads to shear stress with associated pore pressure development
- Diaperism
- Thermal and chemical processes



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Pore pressure (3/3): Measurements



□ Pore pressure can be predicted before drilling:

- Information from nearby wells: Penetration rates, lithological sequence, measured densities, initial test pressures
- Seismic data: Velocities obtained from processing seismic reflection data are related to pore pressure by *Eaton (1975)* or *Bowers (1995)* approximations, which correlate vertical effective stress and interval velocity with normally pressurized stress and velocity of unconsolidated sediments

□ During drilling

- Change in penetration rate: Based on faster drill through overpressured rock than through normally pressured rock. When drilling in clean shales this fact can be utilized to detect the presence of abnormal pressure, and even to estimate the magnitude of the overpressure).
- MWD & LWD
- Cutting and mud characterization

□ After drilling

- PTA
- acoustic & resistivity logs (for shale)
- Correlation with fracture and overburden gradients



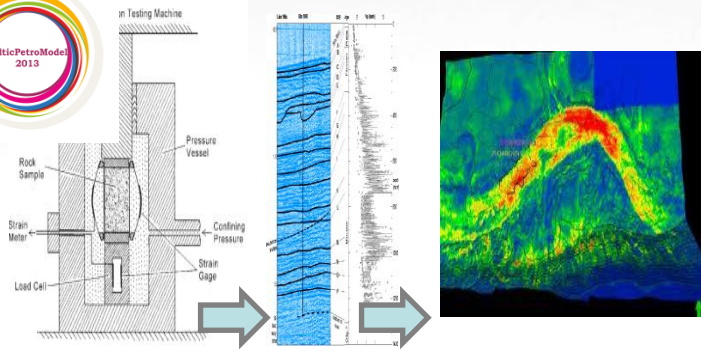
Building and calibration of MEM.

3. Rock strength and elastic properties



Building and calibration of a MEM:

Rock strength and elastic properties (1/7): Dynamic and static moduli



□ Input data for elastic properties:

- Core – static elastic properties
- Log – sonic and bulk density
- Seismic – low resolution but large volume

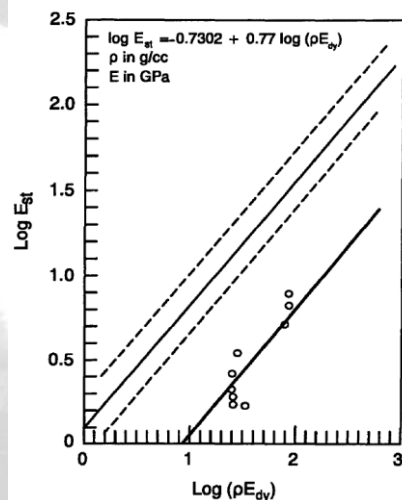
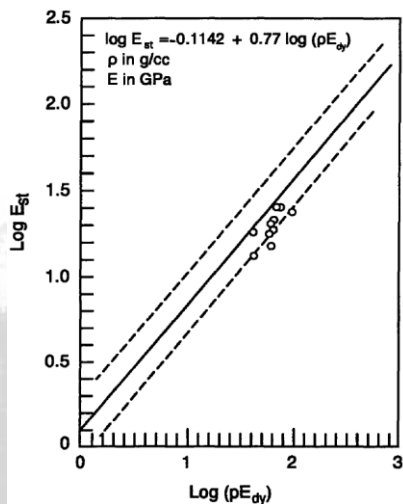
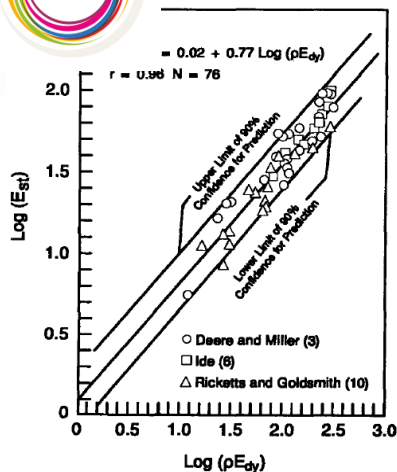
$$E_{dyn} = \rho V_s \frac{3V_p^2 - 3V_s^2}{V_p^2 - V_s^2} \gg E_{st} = \frac{\sigma_z}{\epsilon_z}$$



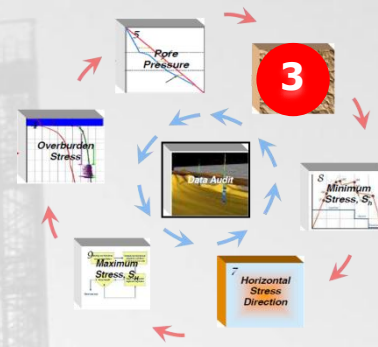
- The static moduli of rock determined from static stress- strain testing are generally different from and characteristically lower than dynamic moduli determined from acoustic sound speeds.
- There are numerous explanations for the differences ranging from viscoelastic behavior to strain amplitude effects. These differences are still a current area of research
- The dynamic elastic moduli are subsequently converted to static moduli using a three-step procedure:
 - Application of a fluid correction accounts for the difference of elastic properties between oil- or water-filled pore space and compressible gas-filled pore space by applying the Gassman equation to compute the drained (or dry) modulus.
 - Lithology classification specifies the rock type and porosity by using petrophysical log analysis or lithology classification of seismic inversion volumes.
 - Correlation of dynamic to static moduli by applying lithology-specific correlation functions

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Rock strength and elastic properties (2/7): Correlations for moduli



- There is a wide body of literature showing correlations between dynamic and static moduli in different lithologies.
- Eissa and Kazi (1988)* use published data from a range of lithologies (consolidated samples, porosity <15%) and combine these datasets and test a number of different functional forms for the correlation.
- Morales and Marcinew (1993)* have shown low and moderately permeability rocks (15-25%) are fitted by the correlation, but high-permeability sandstones (i.e. good-quality reservoir rocks) have a very different correlation function from that of Eissa and Kazi (1988)
- Note that it is easy to apply a correlation function and derive static elastic properties from dynamic elastic properties. By far the more difficult task is to quality control the resulting properties.
- Often, an unsuccessful project is not caused by the failure of geomechanical principles to work, but by a combination of insufficient data, application of unsuitable correlations and poor judgement on the errors introduced by poor data and use inappropriate correlations (*Herwanger & Koutsabeloulis, 2011*)

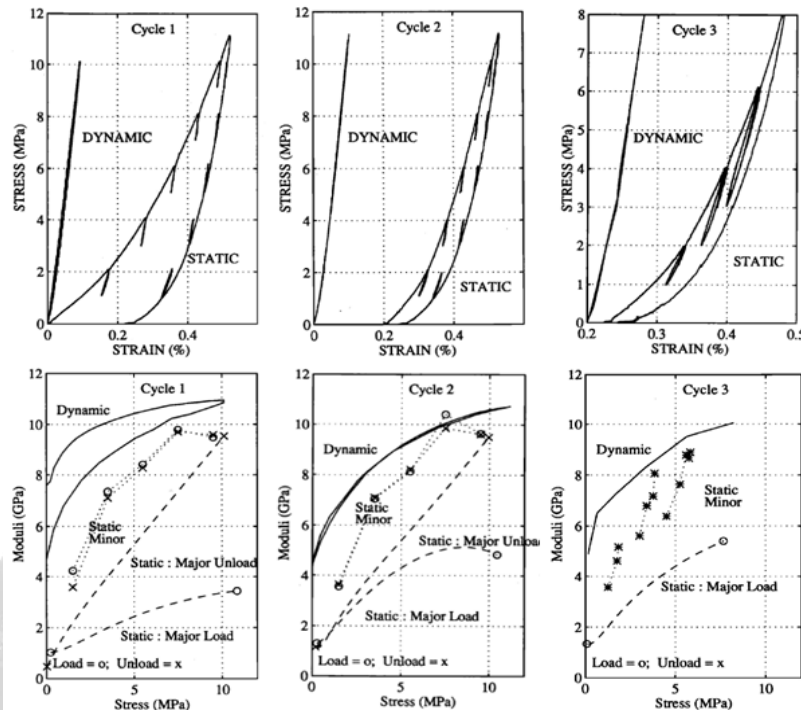


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Rock strength and elastic properties (3/7): Plona & Cook experiment



- Plona and Cook (1995) run series of stress-cycling measurements on room-dry Castlegate sandstone (UCS = 16 MPa) where both the static stress-strain behavior and the dynamic sound speeds on the same sample of rock during unconfined, uniaxial compression were investigated



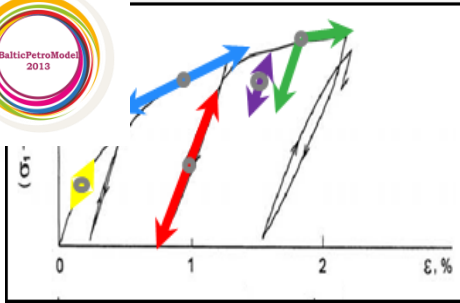
- Two type of cycles:
 - 3 large load-unload (major) cycles
 - 5 pairs of minor cycles within each major one
- Dynamic strains are calculated from measured stresses divided by dynamic young modulus
- Results of cycles 1 and 2:
 - The slope of the static major cycle is significantly less than the slope of the dynamic curve except for unloading at high stress.
 - At high stress, the slope of the static minor cycles is similar to the slope of the dynamics tress-strain curve.
- In cycles 3, additional minor cycles with decreasing amplitude were conducted. The moduli strive close to dynamic at large stresses



- Plona and Cook (1995): "We demonstrated that with a consistent definition of the static elastic modulus in terms of small load-unload cycles, the difference between static and dynamic moduli is negligible for Castlegate sandstone".

Building and calibration of a MEM.

Rock strength and elastic properties (4/7): Again, what is measured?



- Previously we mentioned different formulation on elastic moduli (**yellow** and **red**).
- **Violet** formulation is advocated by *Plona T., Cook J. (1995)* as having the most success in lessen the dynamic-static confrontation (although questions still remain)



- **Blue** line corresponds to nonlinear elastic part just before a yield point. The question gets rise: do we have any experiments related to dynamic and static moduli behavior at this branch? The answer could give clear estimation of the load-unload amplitude role if loaded material is still before the yield point.

- *Fjaer E. (1999)* considered **blue** formulation as the most reliable for interpretation static moduli and introduced two parameters:

$$F = \frac{\Delta \varepsilon_z - \Delta \varepsilon_z^e - \Delta \varepsilon_z^p}{\Delta \varepsilon_z}, \quad \Delta \varepsilon_z^p = P \Delta \sigma_z$$

$$E_{dyn} = \Delta \sigma_z / \Delta \varepsilon_z^e \quad \Rightarrow \quad E_{st} = \frac{E_{dyn} (1 - F)}{1 + P E_{dyn}}$$

$$E_{st} = \Delta \sigma_z / \Delta \varepsilon_z$$

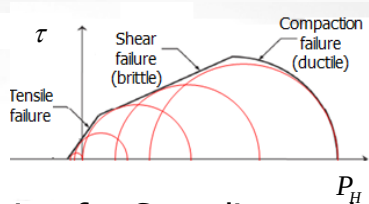
- F is a measure of a process which involves crushing of asperities at the grain contacts
- P is associated with friction controlled sliding along contact points or closed micro-cracks in the material during shear loading.

- **Green** line represent real loading-unloading process in wave propagation beyond the yield point
 - *Abramtsumyan (1982), Myasnikov V & Oleinikov A (2007):*

$$\rho V_{p1}^2 = \lambda + \mu - 2(\nu - \alpha - \beta) \operatorname{sgn} \varepsilon, \quad \rho V_{s2}^2 = \rho V_{s3}^2 = \mu - 1/2(\nu + \alpha - 2\beta) \operatorname{sgn} \varepsilon$$

Building and calibration of a MEM.

Rock strength and elastic properties (5/7): Planning for sampling



- Laboratory modeling of rock strength properties was discussed in detail above

□ Planning for Sampling and Core Tests (*Khaksar et al 2009*):

- Is core planned to be cut in the new well or is core from offset wells required?
- Is there suitable preserved core available for testing from offset wells?
- Can preserved samples be used for rock mechanics testing?
- If unpreserved core is all that is available, has it degraded on exposure to air or moisture?
- If unpreserved core is acceptable, can representative plugs be cut?
- Will it be necessary to scan the core/preserved core before cutting plugs?
- Should a plug fail, can a replacement plug of similar rock properties be taken?
- What is the availability and schedule of the rock mechanics testing laboratory?

□ Sample Selection(*Chang et al., 2006*):

- In unpreserved core, is the state of preservation representative for strength tests?
- Avoid the temptation to only sample strong looking sections of core. Avoid choosing only "interesting looking" sections of core.
- Micro-fractures, natural fractures and faults are usually should, in the main, be avoided
- Debris from wide fault or fracture zones such as breccia, should be sampled too since they can range from loose to well cemented and can be tested for rock strength, grain size distribution.
- Reservoir rocks should be examined for cement mineralogy. This helps in typing the range in rock strengths and for a cements propensity for dissolution.



Building and calibration of a MEM.

Rock strength and elastic properties (6/7): Strength correlations



- ❑ For practical applications, correlation functions between log-derived properties and strength properties are commonly used. The correlation functions need to be calibrated with laboratory rock mechanics tests *Khaksar et al (2009)*
- Establish the classification of the geological column into units with similar mechanical behaviour (mechanical stratigraphy). Within each class, the rocks are classed by the dominant way of supporting stress. In clastic rocks, the subdivision is according to the dominant load-bearing solid phase (Plumb, 1994). For carbonates, it is determined by the nature of the pore space, which dictates the failure mode of the rock (Vajdova et al., 2004)



Table 1: UCS Models for Sandstones

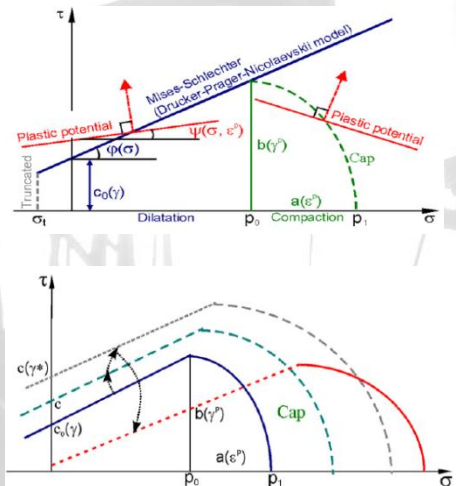
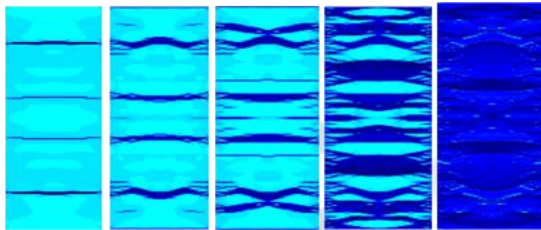
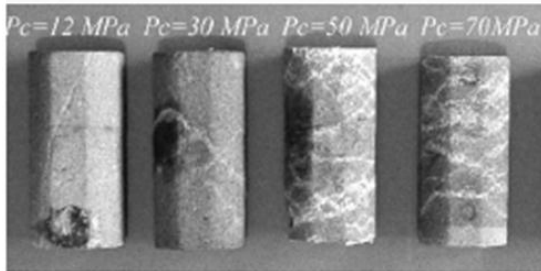
Model and Reference	Equation	Remarks
Dt-McNally (McNally, 1987)	$C_0 = 185213e^{-0.037Dt}$	Low to medium porosity sandstones, 65 < Dt < 100 μs/ft and UCS > 3000 psi, Permo-Triassic age SE Australia
Dt-Mod McNally (Modified McNally)	$C_0 = 838825e^{-0.057Dt}$	A modified McNally equation for unconsolidated and high porosity sandstones with UCS less than 3000 psi
Dt-HRDS (Rahman et al. 2008)	$C_0 = 40847e^{-0.0268Dt}$	Tertiary sandstones, offshore gas field, South Asia
Dt-FORMEL (Raaen et al. 1996)	$C_0 = 145 \times (140 - 2.1Dt + 0.0083Dt^2)$	90 < Dt < 140 μs/ft
φ-FORMEL (Raaen et al. 1996)	$C_0 = 145 \times (43 - 140\phi + 63\phi^2)$	0.2 < φ < 0.35
Dt Cubed-Sand (Chang et al. 2006)	$C_0 = 2.05 \times 10^9 Dt^{-3}$	Gulf of Mexico, weak and unconsolidated rocks
Dt-Freyburg (Freyburg, 1972)	$C_0 = 1.55 \times 10^6 / Dt - 4567.5$	Consolidated Thuringia sandstones, Germany
φ-Sarda (Sarda et al. 1993)	$C_0 = 16172e^{-11.6\phi}$	Germigny-sous-Coulombs reservoir, with the φ < 0.35
φ-Vernik (Vernik et al. 1993)	$C_0 = 36830 \times (1 - 2.7\phi)^2$	Reasonable for consolidated sandstones with φ < 0.30
φ-V _{clay} -Vernik	$C_0 = 145 \times (254 - 204 \times V_{clay}) \times (1 - 2.7\phi)^2$	Modified Vernik equation with V _{clay} for shaly sandstones with φ < 0.30
φ-Literature1 (Chang et al. 2006)	$C_0 = 40165e^{-10\phi}$	UCS between 300 and 52000 psi and φ less than 0.33

- Establish correlation functions and their application to each mechanical unit.
- Try to establish a physics-based approach to link strength properties with log-derived properties.

❑ This example with silicates above shows that blind application of published correlations, without well-considered judgement of applicability to a specific situation, can and will lead to unsatisfactory results.

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Rock strength and elastic properties (7/7): Numerical rock lab?



- ❑ Note, as was mentioned above, the influence of the intermediate principal stress has to be taken into consideration.
- ❑ True triaxial testing machines are not a routine practice in petroleum industry
- ❑ In 3D numerical models, where all three principal stresses take on distinct values, true triaxial failure models can be employed. Note further, that this the rock behaviour after failure state (or yield) can be modeled and predicted.
- ❑ Exact reproduction of experimental setup is a challenge, so one can presently dream on a understanding the qualitative reasons for specific sample behaviour (formation of compaction bands in high porosity sandstone is unable to mimic within the concept of associated law, *Stefanov et al, 2011*)



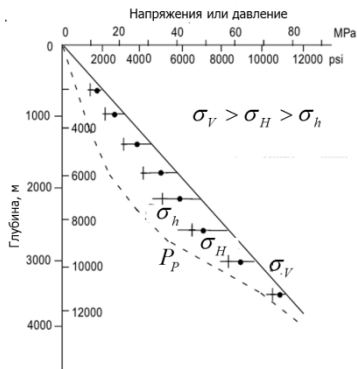
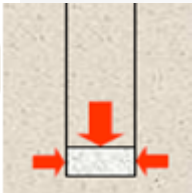
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4. Minimum horizontal stress



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Minimum horizontal stress (1/4): Logs and empirical correlations



- ❑ If a rock is constrained horizontally, vertical loading squeezes the rock and generates horizontal stress
- ❑ Knowledge of the least principal stress is not only a critical step in determining the full stress tensor, it also provides important information for drilling stable wells. During drilling, mud weights must be kept below to prevent accidental hydraulic fracturing and lost circulation
- ❑ A number of empirical techniques have been proposed for estimating the least principal stress in the absence of direct measurements
- ❑ If lateral strain is confined to zero, and two horizontal stresses are equal (bilateral constrain)

$$\sigma_h - \alpha P_p = K_0 (\sigma_v - \alpha P_p), \quad K_0 = \frac{\nu}{1 - \nu} \Rightarrow K_0 \rightarrow K$$

$$K(z) = \nu(z) / (1 - \nu(z))$$

Eaton (1969)

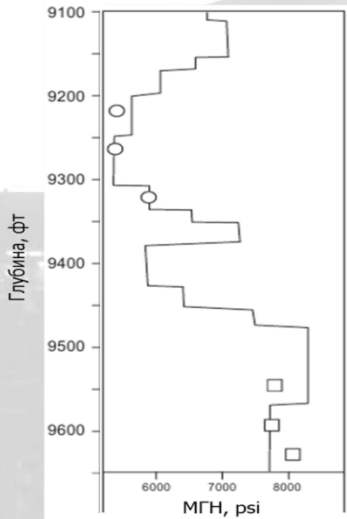
$$K(z) = ((1 + \mu^2(z))^{1/2} + \mu(z))^{-2}$$

Zoback & Healy (1984)

$$K(z) = 1 - \phi(z)$$

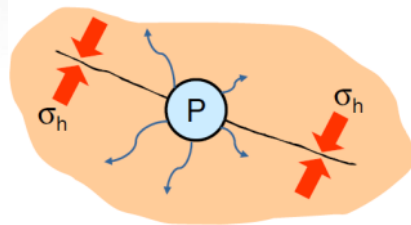
Holbrook et al., (1993)

- ❑ The result of comparative study (*Zoback, 2007*): The best is the "stress log" with thoroughly calibrated Vp, Vs and Gamma logs in the environment where bilateral constrain is valid (*Whitehead et al. 1986*)



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Minimum horizontal stress (2/4): LOT and XLOT



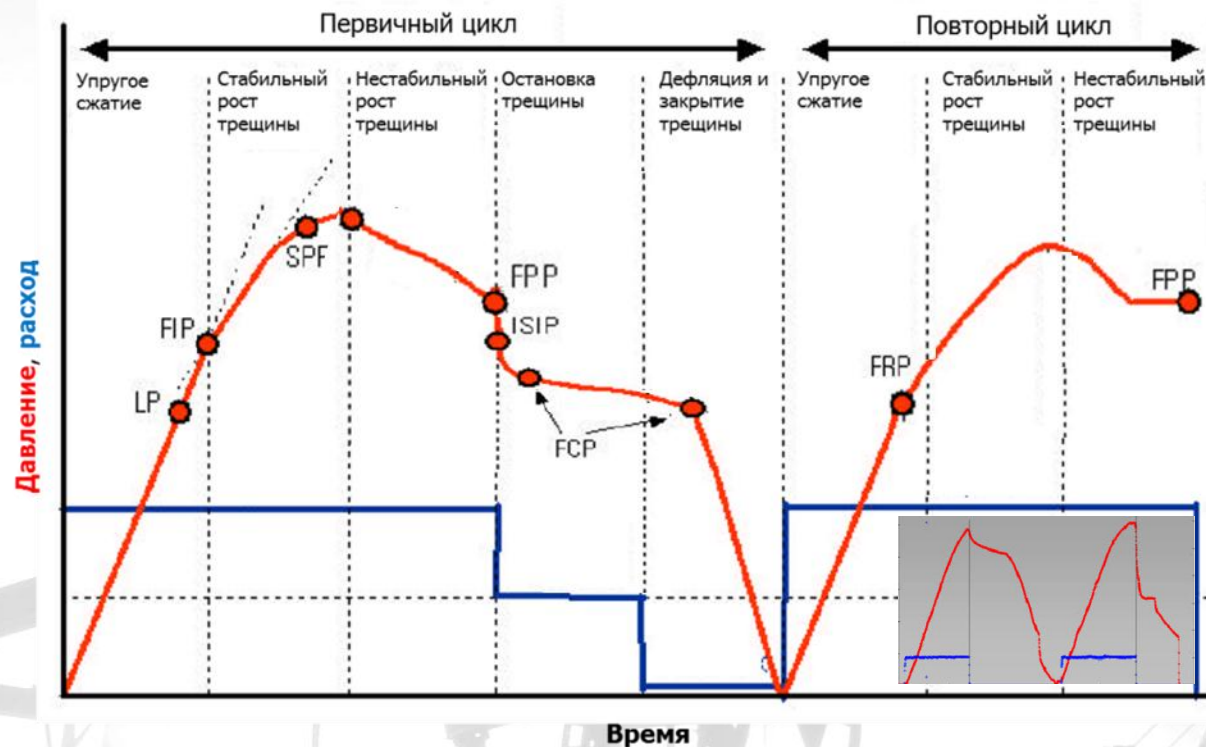
- A number of tests (pump-in/shut-in, pump-in/flowback tests,) are based on the fact that HF propagates in the maximum stress direction and after shut-in pressure falls and HF closes when pressure equals to minimal stress



- A pump-in/shut-in and pump-in/flowback tests are also called leak-off test (LOT) and extended leak-off test (ELOT, XLOT) if are performed during the drilling phase of a well, in the formation immediately below each casing shoe.
- If tests run just before stimulation works, they are called mini-frac tests (*Fjaer et al. 2008*)
- Anyway, HF without proppant is performed with recording pressure in the wellbore-HF-reservoir system
- One area in the analysis of pressure tests that often causes confusion is the nomenclature of leak-off testing events, generated by obsolete internal corporate manuals (*Lee et al, 2004*)
- An example is fracture pressure:
 - the maximum-recorded pressure just prior to unstable fracture growth
 - fracture closure pressure.

Building and calibration of a MEM.

Minimum horizontal stress (3/4): LOT and XLOT



van Oort E. (2008)

- ☐ LP - limit pressure
- ☐ FIP - fracture initiation pressure
- ☐ SPP - stop pump pressure
- ☐ UFP (FBP)- uncontrolled fracture pressure
- ☐ FPP - fracture propagation pressure
- ☐ ISIP - instantaneous shut-in pressure
- ☐ FCP - fracture-closure pressure
- ☐ FRP - fracture-reopening pressure



Building and calibration of a MEM.

Minimum horizontal stress (4/4): FIP or FCP?



- ❑ A manager has to make a decision which value of measured pressures should be considered as a good enough estimation for a minimum stress value.
- ❑ On one hand, precise information of mechanical properties is of undoubted importance important for drilling management
- ❑ On the other hand, FIP could be considered as a good enough if to neglect the influence of dependence of elastic properties on stresses, temperature effects, overflow behind a casing shoe etc during the interpretation *Fjaer et al., (2008), Zoback, (2007)*.
- ❑ Additionally, XLOT requires installing additional surface pump out equipment providing constant flow back rate.
- ❑ So, it is not completely out of sense to use FIP as an initial data for MEM with further measurements improvement with MEM recommendations accounted.



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5. Horizontal stress directions

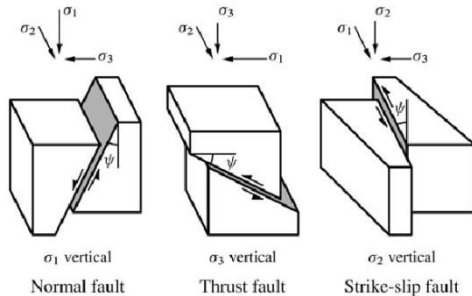


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Horizontal stress directions (1/3): Geological indications



- If the current stress state is tectonic, the principle axes can be restricted by geological indications.



- Fault classification by *Anderson (1951)*:

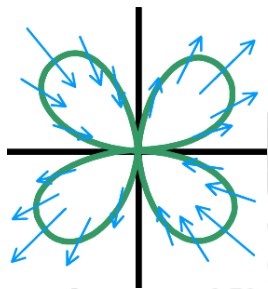
- Fracturing will take place in a plane which is parallel to the direction of the intermediate principal stress, at angles of less than 45° to the direction of the maximum principal stress.

- Earthquake focal mechanism

- Plot the direction of the vectors from the focus to different seismic station are on a stereographic projection of a unit reference hemisphere
- P-waves radiate relative to the focus with compressional or dilatational initial motion. Two regions appear at the projection
- Three orthogonal axis associated with dilatational and extension regions correspond to $\sigma_1 > \sigma_2 > \sigma_3$ (McKenzie, 1969: σ_1 is within DQ)

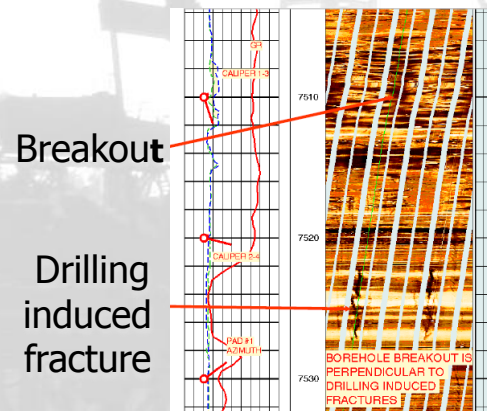
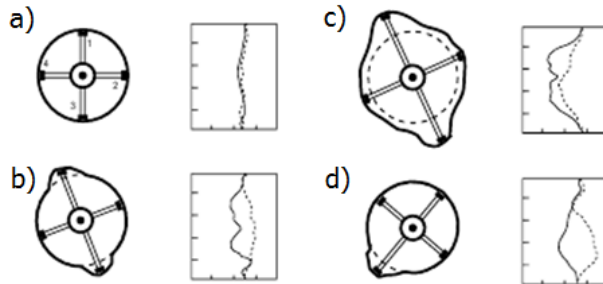
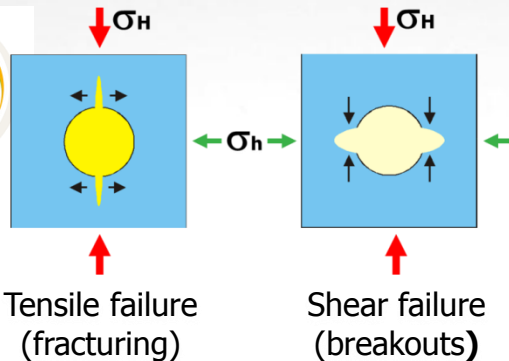
- Dikes and flank volcanos around larger craters

- Some dikes represent HF, so they lie perpendicular to σ_3
- Perpendicular to a radius from a master crater to a flank volcano identifies σ_3 (Nakamura, 1977)



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Horizontal stress directions (2/3): Borehole methods

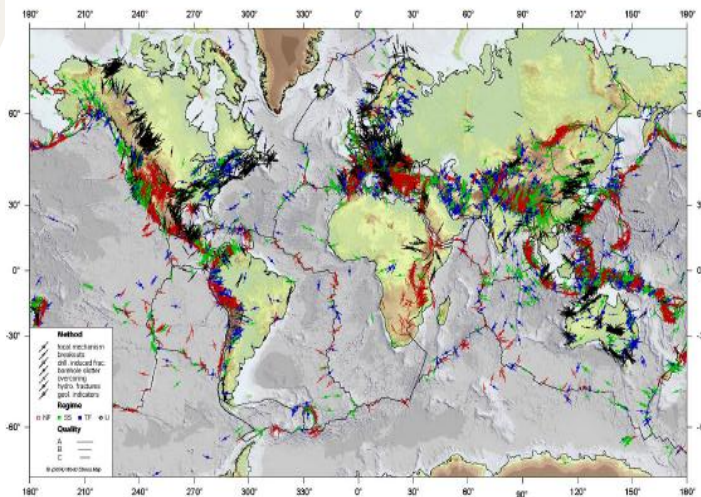


- ☐ Borehole methods of determination of horizontal stress directions are based on the observations of failure at the borehole wall failure
- ☐ Two distinct failure modes can be detected in vertical wells
- ☐ The caliper log (four-arm and 6-arm) has commonly been used to estimate horizontal stress directions from breakout orientations.
 - Several criteria (*Plumb and Hickman, 1985*) would help to distinguish between breakouts (b), washouts (c) and keysets (d)
 - Difficulties even for vertical wells (*Fejerskov & Bratli, 1998*).
 - 6-arm caliper requires sophisticated algorithms for interpretation (*Jarosinski, 1998*).
- ☐ Image logs include both electrical and acoustical borehole imaging log
 - In electrical tool, electrodes are in contact with the formation, well suited for investigation of fine structures like natural or drilling induced fractures.
 - The acoustical imaging tool is based on reflection of waves from the wall, recording the travel time and amplitude of the reflected pulses.
 - Better for detection of breakouts, as fractures do not significantly changes borehole radius or reflectivity. No problems with washouts



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Horizontal stress directions (3/3): World stress map



- ❑ *Zoback and Zoback (1980)* showed that a variety of different types of stress-related data could be used to produce comprehensive maps of stress orientation
- ❑ A stress measurement quality criterion for different types of stress indicators was later proposed (*Zoback and Zoback, 1989, 1991*) to avoid non-tectonic, surface-related sources of stress



- ❑ Zoback and Zoback criterion was subsequently utilized in the International Lithosphere Program's World Stress Map Project, a large collaborative effort of data compilation and analyses by scientists from 30 different countries (*Zoback M, 1992*).
- ❑ Today, the World Stress Map (WSM) database has almost 10,000 entries and is maintained at the Heidelberg Academy of Sciences and the Geophysical Institute of Karlsruhe University, Germany (<http://www-wsm.physik.uni-karlsruhe.de/>).

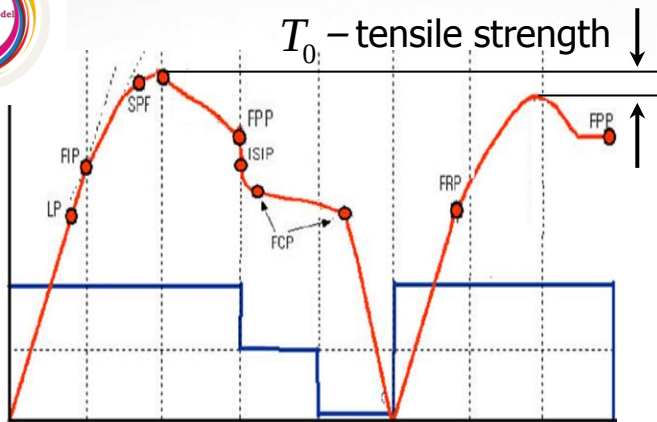
Building and calibration of a MEM.

6. Maximum horizontal stress

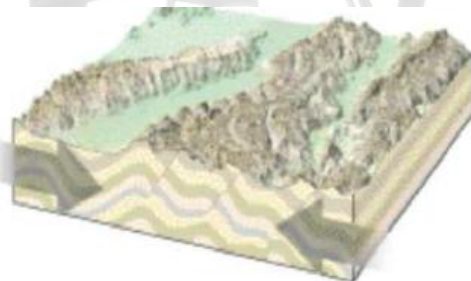


Building and calibration of a MEM.

Maximum horizontal stress (1/3): Estimation from XLOT and AHFP



$$\sigma_H = 3\sigma_h - P_b - P_p + T$$



$$\sigma_H = \sigma_h + \frac{1}{AB} \Delta P_p$$

- One would derive maximum horizontal stress from LOT, if tensile strength is known (Hubbert & Willis, 1957), (Haimson & Fairhurst, 1970), or XLOT, if not (Bredehoeft et al. 1976).

- Indeed, from classical solution on stress distribution around a hole (Kirsch, 1898), HF occurs if

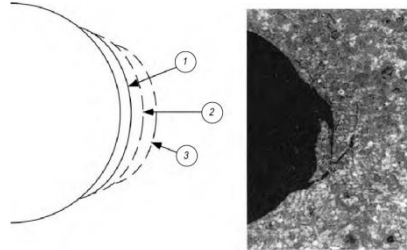
$$P_b = 3\sigma_h - \sigma_H - P_p + T_0, \quad \text{or,} \quad P_b|_{T_0=0} = 3\sigma_h - \sigma_H - P_p$$

- Applicability is limited by any factor which makes the Kirsch solution invalid: shallow depth, low in-situ stress and temperature, open holes
- One would derive maximum horizontal stress measured overpressure and the least stress if vertical stress is not changed, and maximum stress is much larger than minimum
 - Skempton's parameter to be measured in triaxial test
 - Tectonics is not the unique reason for overpressure
 - Mechanism valid preferably in young sediments since in well cemented low porosity rocks displacement cause dilatancy which, in turn, decrease pressure

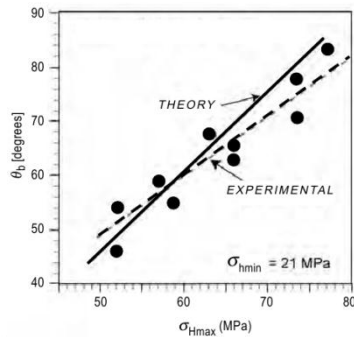


Building and calibration of a MEM.

Maximum horizontal stress (2/3): Detailed observations of breakouts

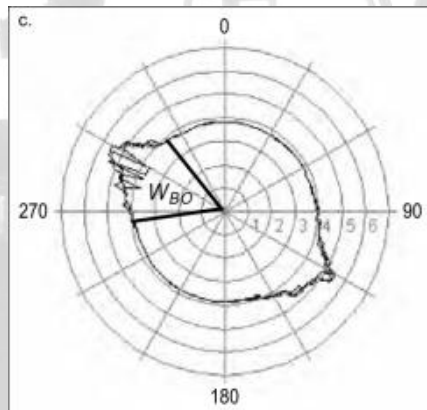


- After the formation of wellbore breakouts, they are expected to increase in depth, but not width. This is as shown theoretically (*Zoback, Moos et al. 1985*) and experimentally (*Haimson and Herrick 1989*)



- Measured breakout widths compare very well with those predicted by the simple theoretically predicted (*Zoback, Moos et al. 1985*) from simple equilibrium condition of the stress concentration at the edge of a breakout with the rock strength

$$\sigma_H = \frac{(C_0 + 2P_p + \Delta P) - \sigma_h(1 + \cos 2w_{BO})}{1 - 2 \cos 2w_{BO}}$$



- The width of breakout can be precisely detected by acoustical borehole imaging log
- The technique is intensively validated versus all alternative robust data
- The shapes of the breakouts obtained numerically in terms of a total plastic strain failure criterion (*Willson et al. 1999*) are quite similar to the shape predicted using the elastic stress concentration predicted by the Kirsch equations (*Zoback 2007*)

Building and calibration of a MEM.

Maximum horizontal stress (3/3): Schlumberger Sonic Scanner



- In addition to axial and azimuthal measurements, the tool makes a radial measurement to probe the formation for near-wellbore slowness and far-field slowness. Typical depths of investigation equal two to three times the borehole diameter.



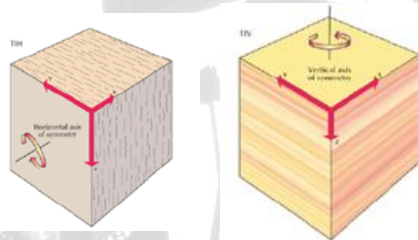
- Interpretation of measured velocity field is based on the fact, that velocity of elastic waves increases in the direction of applied stress, while in other direction remain almost unchanged (*Herwander & Koutsabeloulis, 2011*). Anisotropic stress induces anisotropic velocity field in initially isotropic media

- Thus, having a measured field $V_{r,\theta,\varphi}^{P,S}$ one could recover stress field if a rheology model $\sigma = \Lambda : \varepsilon$ is given

- Presently quasi-linear model $\Lambda = \mathbf{C} = \lambda \varepsilon$ (so called TOE, third order elasticity *Thurston & Brugger, 1964*), which is a limitation of an approach (*Hornby, 1998*)

- In a VTI symmetry the model depends on 6 parameters which measurement is a routine procedure (*Vernik & Liu 1997, Wang 2002*). In orthorhombic case one has to measure 9 parameters, which is much more expensive (*Tsvankin, 1997*).

- The main problem is the applicability of the approach to media where intrinsic and induced anisotropies do not coincide (*Sayers et al., 2009*)



Orthorhombic (9 parameters)

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

Building and calibration of a MEM.

7. Data audit



Building and calibration of a MEM.

Data audit (1/12): Data processing by origin



- ☐ Collection, processing and analysis of all the input data which differ each other **by origin**
- Estimation of adequacy, sufficiency and consistency of the data for the problem under consideration
 - ❖ quality estimation
 - ❖ multichannel verification
 - ❖ development and maintenance of automation devices for data gathering and storage



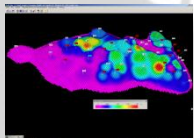
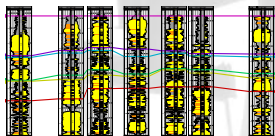
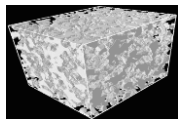

- Classification of the data and determine free parameters in the space of solutions to be obtained.
- Identification of the data which can be iterated in the iterative MEM construction process

Building and calibration of a MEM.

Data audit (2/12) : Data processing by scale range



- ❑ Collection, processing and analysis of all the input data which differ each other **by scale** (pore, core, wellbore reservoir)



- Data must satisfy the basic upscaling rule (*Kosterin 1986*) :
 - ❖ to be homogeneous at small scale and heterogeneous at large scale;
 - ❖ large-scale processes should depend on small-scale phenomena;
 - ❖ large-scale and small-scale modeling should be consistent
- In conventional approaches upscaling is often considered that small-scale degrees of freedom are negligible and large-scale and small-scale models are postulated (*Bazant & Cedolin, 2005, Berg, 2006, Das & Hassanizadeh 2005*). Results of the modeling at these scales may be inconsistent, (*Steinhauser 2008*).
- Mesoscale theory (*Dinariev & Mikhaylov, 2007,2008,2013*):
 - ❖ small-scale properties are retained as internal degrees of freedom at large-scale cells
 - ❖ statistical distribution of internal properties
 - ❖ mesoscale parameters are obtained from small-scale description



Building and calibration of a MEM.

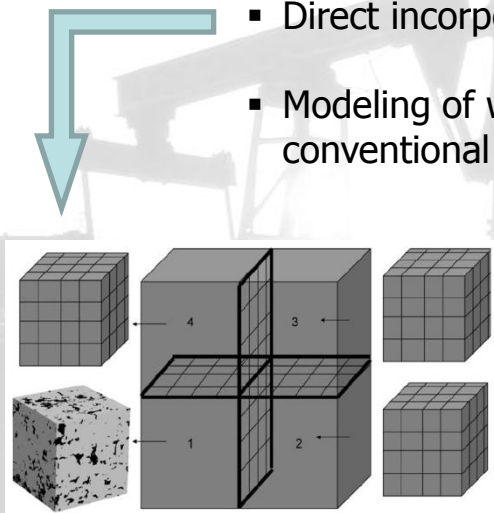
Data audit (3/12): Chose of physical model for data incorporation



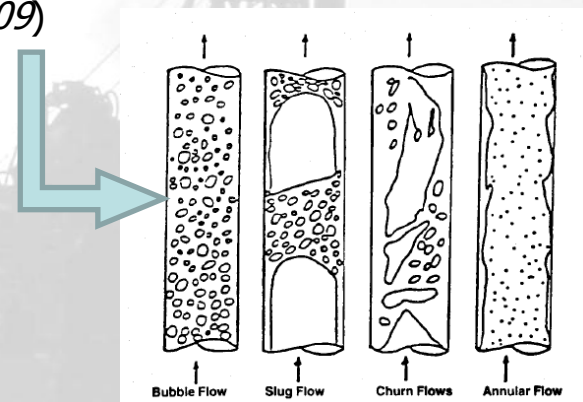
- ❑ A concept of “physical model for data incorporation” is a topical subject today
- ❑ Heavy capital investments are now made not in what is potentially profitable, but in what is potentially possible
- ❑ Modern HPC technologies make it possible to incorporate any “crazy” idea into reservoir simulation loop (*Boreskov & Kharlamov 2010*) if it decrease of uncertainty of the predictions
- ❑ Two bright examples can be shown with this relation:



- Direct incorporating of NCL with reservoir simulations (*Balhoff et al., 2007,2008*)
- Modeling of wellbore processes with multiphase Navier-Stokes equations instead of conventional multisegment well model (*Bonizzi et al., 2009*)



- ❑ These examples have one common feature: uncertainty related with nonunique solution of fundamental hydrodynamic problem – multiphase flow in pipes

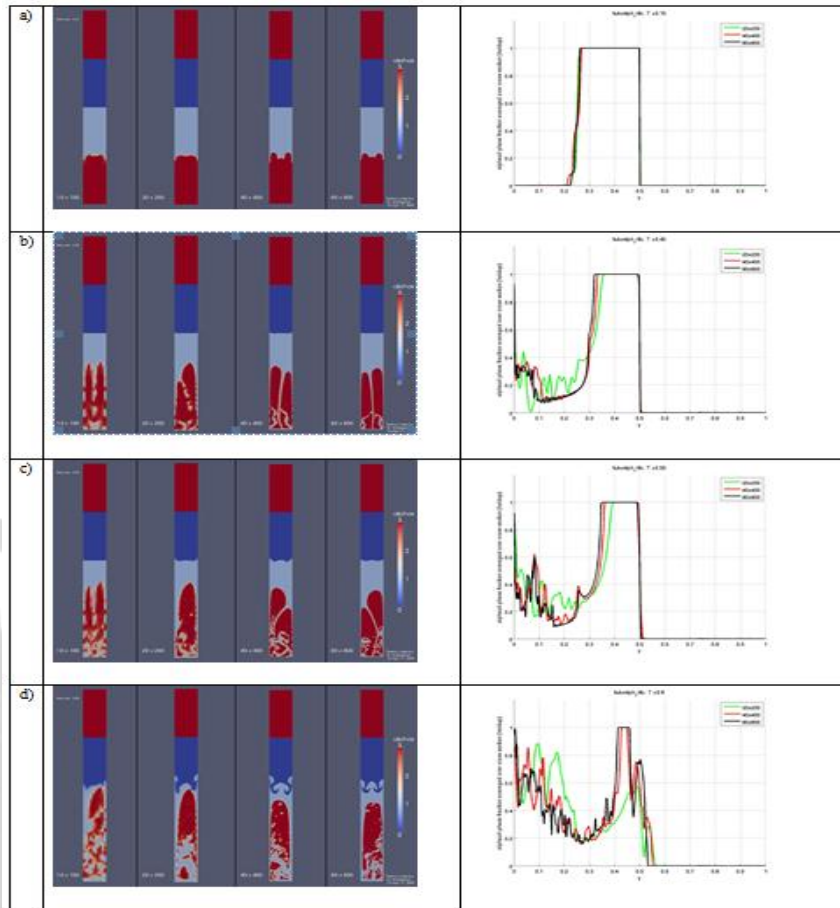
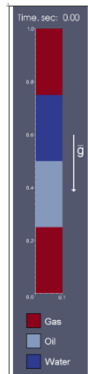


Building and calibration of a MEM.

Data audit (4/12): Chose of "physical model for data incorporation"



- ❑ Multiphase solvers already present in OpenFOAM – an open source CFD toolbox (<http://www.openfoam.com>) has been tested for the ability to produce reliable solutions in *some* sense (Aidagulov & Myasnikov, 2011)



- ❑ 3-phase gravity segregation test was considered on different grid resolutions with different initial velocity perturbations.
- ❑ One can see that neither in spatial or in averaged solutions we have not obtained converged distributions
- ❑ If to introduce the average solutions in a *smart* way, one could obtain: an average solution of 3D equation coincide *in some sense* with 1D solution of the averaged 3D equations
- ❑ This result is valuable for wells but not for NCA, where we can't relate the process with the result directly!

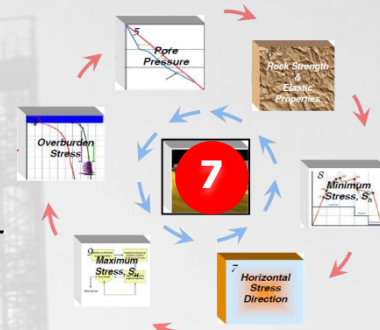


Building and calibration of a MEM.

Data audit (5/12): Computational model for chosen physical model



- One has to decide if it is possible to make use of methods or models with not proven feasibility
- The example here is a streamline technology (*Bratvedt et al., 1992*), which is used today for the wide range of applications (*Datta-Gupta 2000, Kozlova et al., 2007, Andrianov et al., 2007,*) although, theoretically is unproved in a general case (*Rykov, 2008*):



$$\tau = \tau(t, x, y, z), \quad \varphi = \varphi(t, x, y, z), \quad \psi = \psi(t, x, y, z)$$

$$\nabla \cdot \{f \nabla F\} \rightarrow \frac{1}{\sqrt{g}} \frac{\partial}{\partial \tau} \left(\sqrt{\frac{g}{g_{11}}} \cdot \frac{f}{\sqrt{g_{11}}} \frac{\partial F}{\partial \tau} \right) +$$

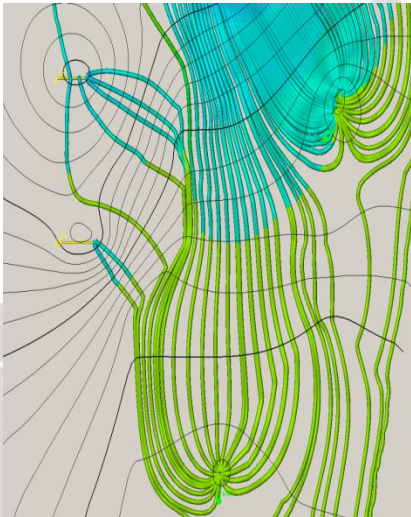
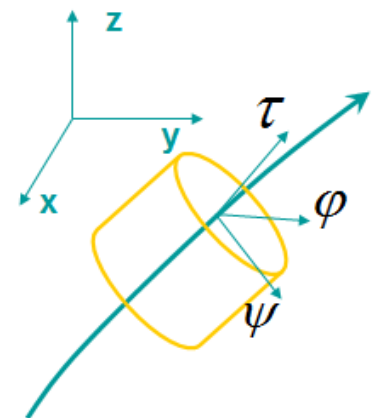
$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial \varphi} \left(\sqrt{\frac{g}{g_{22}}} \cdot \frac{f}{\sqrt{g_{22}}} \frac{\partial F}{\partial \varphi} \right) + \frac{1}{\sqrt{g}} \frac{\partial}{\partial \psi} \left(\sqrt{\frac{g}{g_{33}}} \cdot \frac{f}{\sqrt{g_{33}}} \frac{\partial F}{\partial \psi} \right); \quad \nabla \tau \parallel \nabla P$$

$$\frac{\partial h}{\partial t} \rightarrow h_t + h_\tau \tau_t + h_\varphi \varphi_t + h_\psi \psi_t$$

If then $g_{11} = 1$

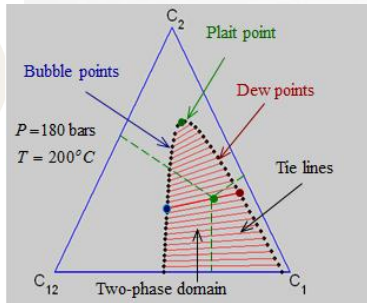
and $g_{22} \cdot g_{33}$ does not depend on τ

$$\nabla \cdot \{f \nabla F\} \rightarrow \frac{\partial}{\partial \tau} \left(f \frac{\partial F}{\partial \tau} \right), \quad \frac{\partial h}{\partial t} \rightarrow h_t$$

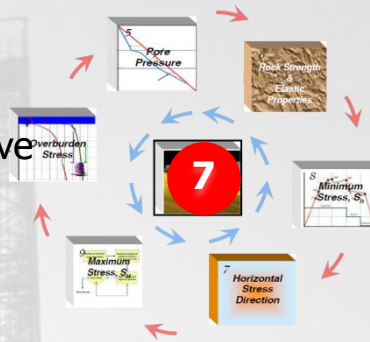


Building and calibration of MEM.

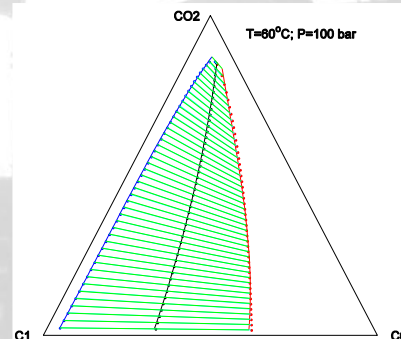
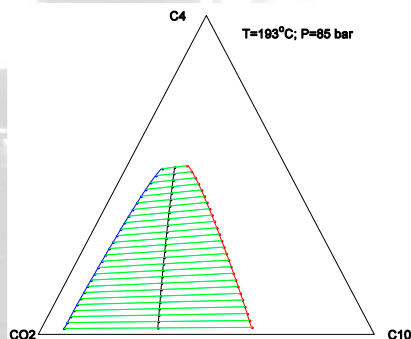
Data audit (6/12): Computational model for chosen physical model



- For gas condensate reservoir, where phase transition processes between liquid and gaseous phases could significantly impact on recovery optimization, the effective procedures for phase equilibrium description is required
- The procedure itself is routine if the chosen equation of state describes adequately the mixture behaviour (not often the case)



- Being implemented into reservoir simulator, the procedure of the phase equilibrium problem at each time step for any spatial point makes the simulations unacceptably slow
- Each composition vector of a N component mixture can be presented by a point within the (N-1) dimensional regular polyhedron of unit height; it is a triangle in the case under consideration. The interior of the triangle is divided into single-phase and two-phase domains. Those are separated with (N-2) dimensional binodal surface, which is the locus of bubble points and dew points.



$$C_1, C_2, \dots, C_{N_C-1} \Rightarrow C_1, \gamma_1, \gamma_2, \dots, \gamma_{N_C-2} \equiv C_1, \gamma$$

$$C_k = A_k(\gamma)C + B_k(\gamma),$$

- Varying polynomials degree one can obtain phase approximations with any requested accuracy.
- Can approximate PVT field data, no necessity in specific equation of state

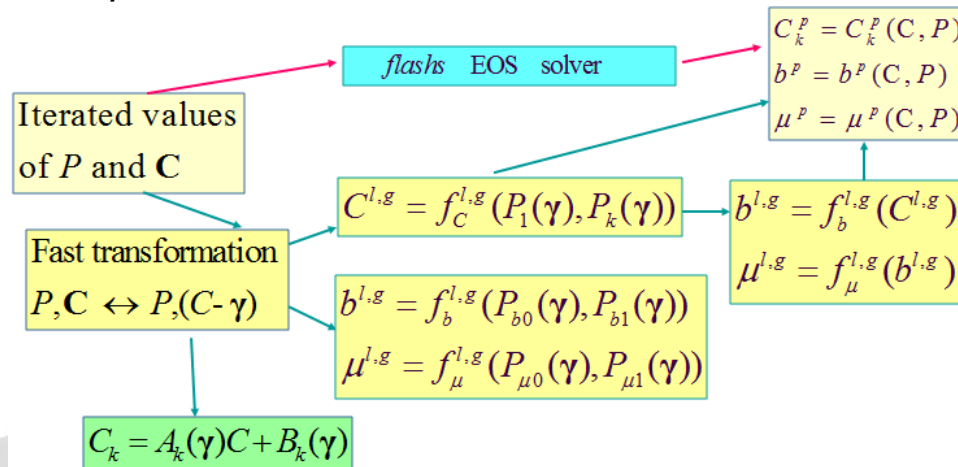
(Orr 2000, Entov & Voskov 20011, 2002, Belov & Myasnikov, 2007)

Building and calibration of a MEM.

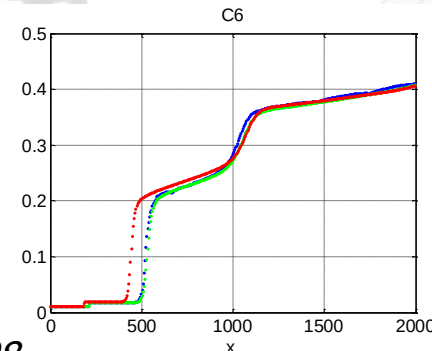
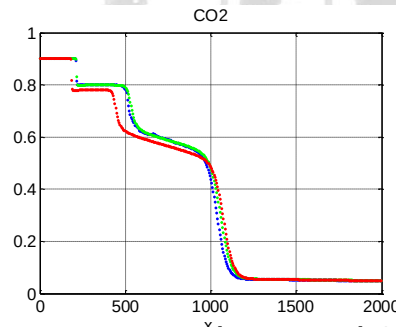
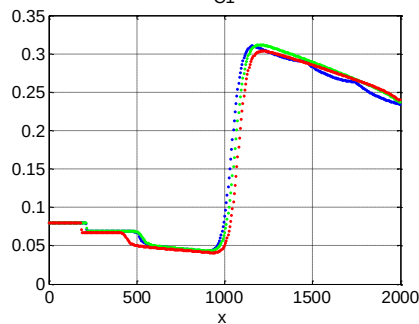
Data audit (7/12): Computational model for chosen physical model



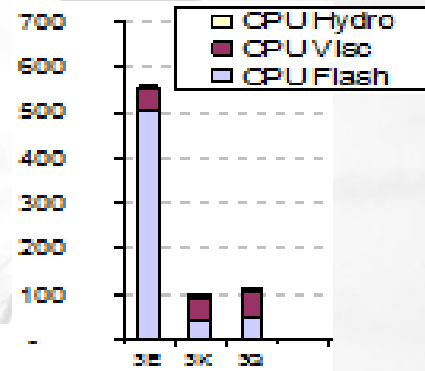
- Being implemented into reservoir simulator, the procedure can drastically reduce CPU time (observed record is 20 times), depending on required accuracy



$C_1CO_2C_6C_{10}$, $T = 193^\circ C$, $P_{in} = 100$ bar, $P_{out} = 50$ bar



Lysov, Myasnikov, et al 2008



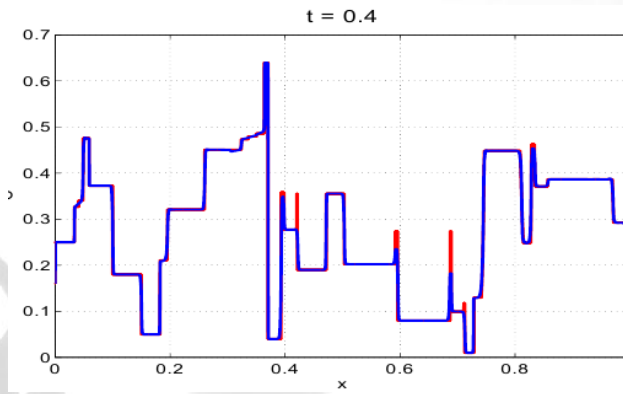
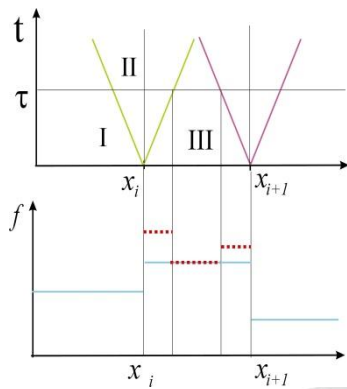
E300 flash
 $(n_{p1}, n_{pk}) = (2, 2)$
 $(n_{p1}, n_{pk}) = (0, 0)$

Building and calibration of a MEM.

Data audit (8/12): Numerical method for the computational model



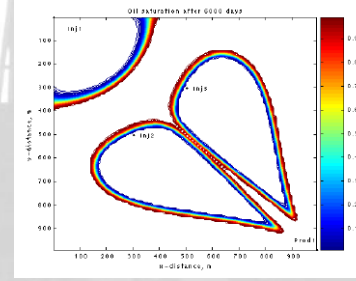
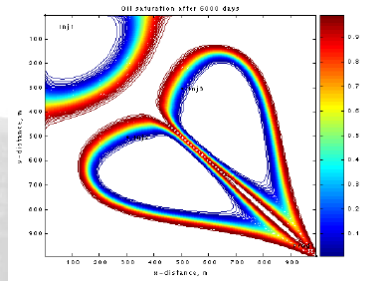
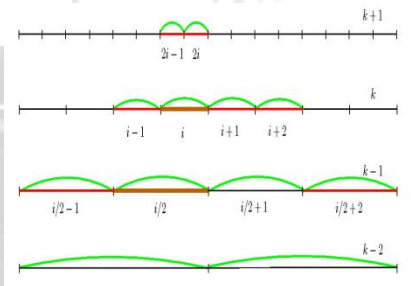
- ❑ If to continue with streamlines, the most bright example in the oil-gas industry of the proper computational technology is a Front Tracking technique



- Appeared initially for 2 phase black oil incompressible flow and became the leader in the market
- Best achievement: 2 phase 3 component incompressible flow (*Belov & Myasnikov, 2007*)



- ❑ If Front Tracking is not developed yet, AMR technique could certainly be applied for the 1D transport across streamlines (*Andrianov, Bratvedt 2007*)



- ❑ The resolution is evidently higher, but the technique may be much slower than original one

Building and calibration of a MEM.

Data audit (9/12): Numerical method for the computational model



- ❑ One of the first coupled fluid + GM solutions (if not the first) in reservoir simulations was presented by *Koutsabeloulis & Hope* in 1996 by coupling VISAGE and FRONTSIM solutions
- ❑ Two pivotal points in the coupling procedure:
 - Which parameter couples the solutions and how
 - How many updates of fluid fields should be made for one stress field update



- ❑ As we saw in the Part II of this lecture, quite complicated relations could exist between phases, although porosity is a “standard” agent up to date even for dual porosity reservoirs. *Khalili and Valliappan (1996)* showed that the condition, derived in the Part II for two phase thermodynamical consistence can be extended for the dual porosity linear poroelastic case in the way:

$$\boldsymbol{\varepsilon}_S^* \equiv \alpha_f \mathbf{U}, \quad [p] = \frac{\partial u_S}{\partial \boldsymbol{\varepsilon}_S^*} : \mathbf{U} = \frac{\partial u_S}{\partial \alpha_f} \Rightarrow \begin{cases} \frac{\partial \phi_f}{\partial t} = (\alpha_f - \phi_f) \left(\frac{\partial \text{tr}[\boldsymbol{\varepsilon}]}{\partial t} + \frac{1}{K_m} \frac{\partial \bar{p}_f}{\partial t} \right) + C_{mf} \left(\frac{\partial \bar{p}_f}{\partial t} - \frac{\partial \bar{p}_m}{\partial t} \right) \\ \frac{\partial \phi_m}{\partial t} = (\alpha_m - \phi_m) \left(\frac{\partial \text{tr}[\boldsymbol{\varepsilon}]}{\partial t} + \frac{1}{K_m} \frac{\partial \bar{p}_m}{\partial t} \right) + C_{mf} \left(\frac{\partial \bar{p}_m}{\partial t} - \frac{\partial \bar{p}_f}{\partial t} \right) \end{cases}$$

where

$$C_{mf} = \frac{1}{K} \left(\alpha_m \alpha_f - \phi_m \phi_f \frac{\alpha_m + \alpha_f}{\phi_m + \phi_f} \right),$$

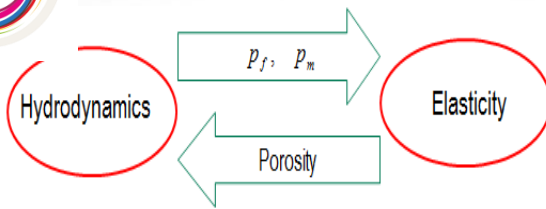
and $\alpha_{f,m}$ are the modified Biot-Willis parameters.

Building and calibration of a MEM.

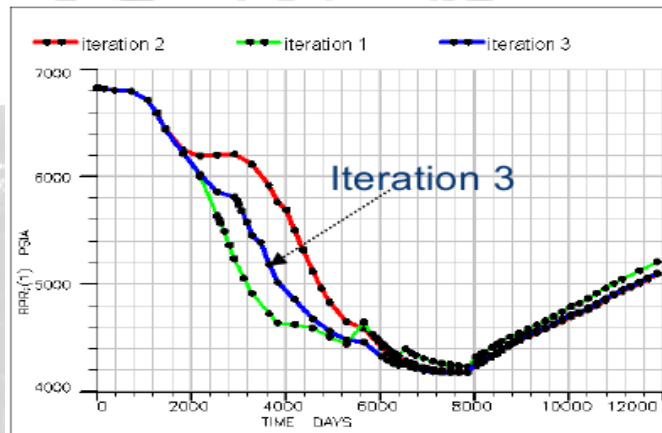
Data audit (10/12): Numerical method for the computational model



- Here we consider two algorithms of coupling.

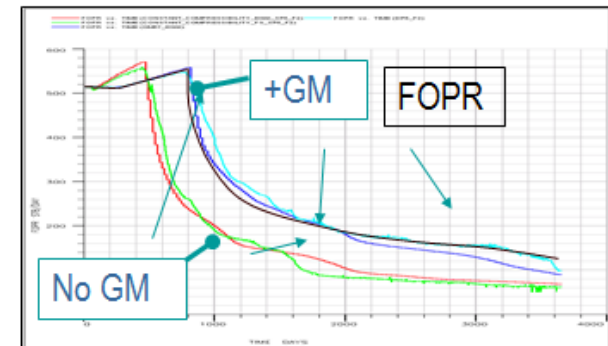


T_k	$p_f^k, p_m^k, \epsilon^k, s_{a,i}^k, \phi_f^k, \phi_m^k$	A set of streamlines is generated
$\left. \begin{matrix} t_1^k \\ t_2^k \\ \vdots \\ t_N^k \end{matrix} \right\}$	$(s_{a,i})_n^k$	The transport equation along streamlines is solved
T_{k+1}	$p_f^{k+1}, p_m^{k+1}, \epsilon^{k+1}, s_{a,i}^{k+1}, \phi_f^{k+1}, \phi_m^{k+1}$	Pressure and elasticity equations are solved. New values of porosity are obtained.



- "internal" coupling: geomechanical solver is called inside the basic reservoir simulator.
- Dealing with streamline-based fluid simulator, it is naturally to arrange "internal" coupling basing on Streamline simulator intrinsic sequential structure
- The other one will be called "external" because both reservoir simulator and geomechanical solver run completely separately (*Samier & Gennaro, 2007*).
- Sixth SPE Comparison test (*Firoozabadi & Thomas, 1990*)

FrontSim
ECLIPSE



Rybdylova & Myasnikov, 2008

Building and calibration of a MEM.

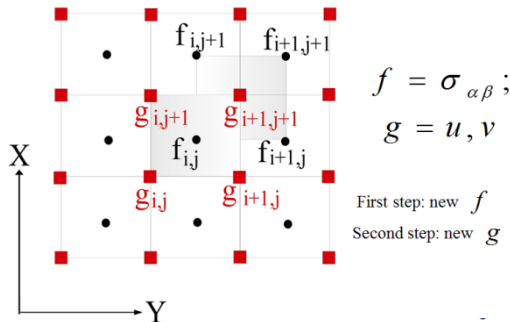
Data audit (11/12): Numerical method for the computational model



- For solution of governing equations for the solid phase, three general numerical technologies are used

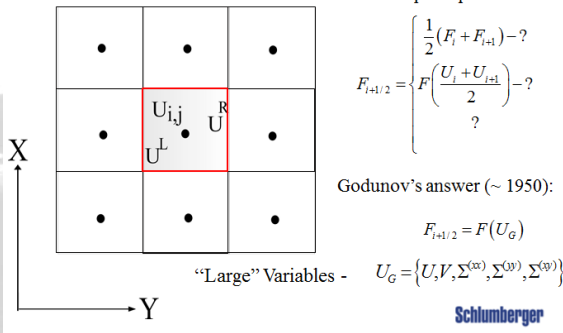
- Finite difference
- Finite volume
- Finite element

- For linear elasticity,

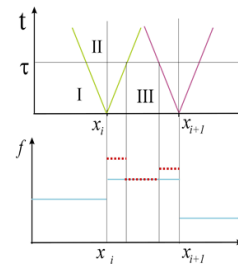
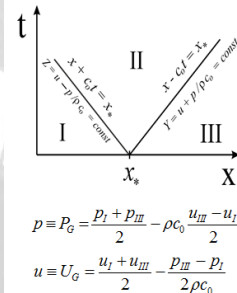


$$\begin{cases} \frac{\partial u}{\partial t} = \lambda_p \left[\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \right] \\ \frac{\partial v}{\partial t} = \lambda_p \left[\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} \right] \end{cases}$$

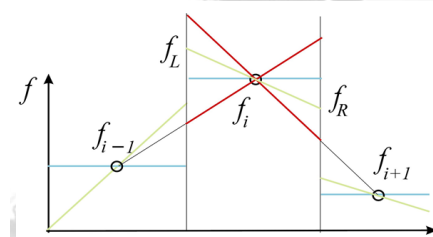
$$\begin{cases} \frac{\partial \sigma_{xx}}{\partial t} = \lambda_E \frac{\partial u}{\partial x} + \lambda_F \frac{\partial v}{\partial y} \\ \frac{\partial \sigma_{yy}}{\partial t} = \lambda_F \frac{\partial u}{\partial x} + \lambda_E \frac{\partial v}{\partial y} \\ \frac{\partial \sigma_{xy}}{\partial t} = \lambda_G \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right] \end{cases}$$



Solution of the Riemann problem



$$\frac{1}{h} \int_{x_i}^{x_{i+1}} u' dx = u_{i+1/2} - \frac{\tau}{h\rho} [(P_\phi)_{i+1} - (P_\phi)_i]$$



I order:

Riemann problem for f_i and f_{i-1}

II order (MUSCL approach):

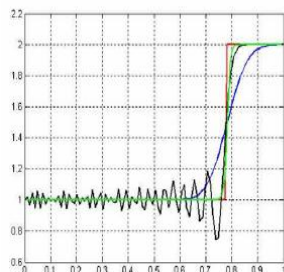
Riemann problem for f_i^L and f_{i-1}^R TVD, and, predictor-corrector technique to attain second order in time

Building and calibration of a MEM.

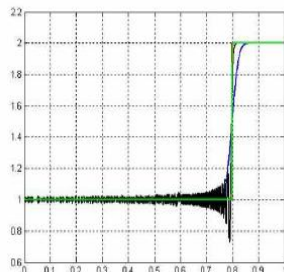
Data audit (12/12): Numerical method for the computational model



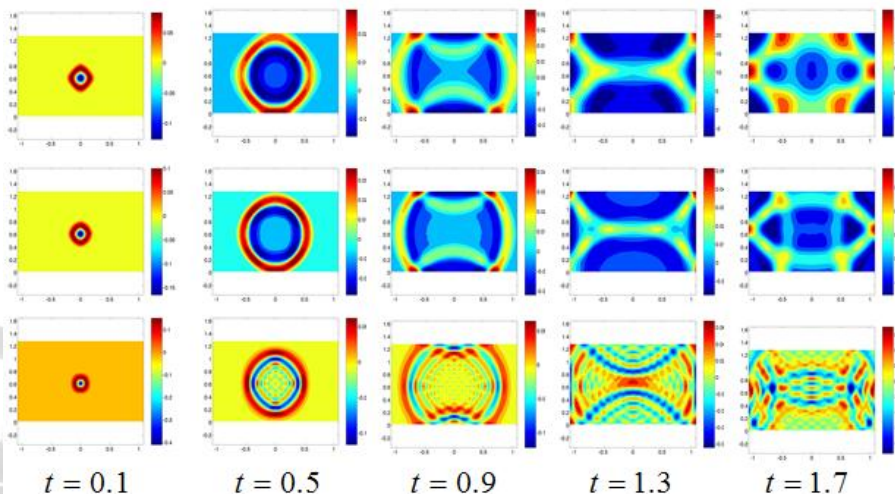
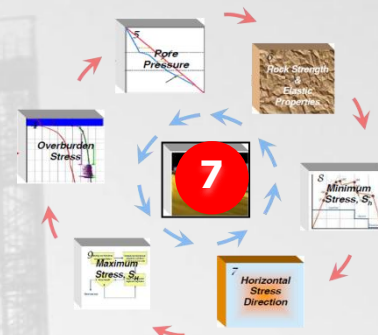
Grid=100 pt



Grid=1000 pt



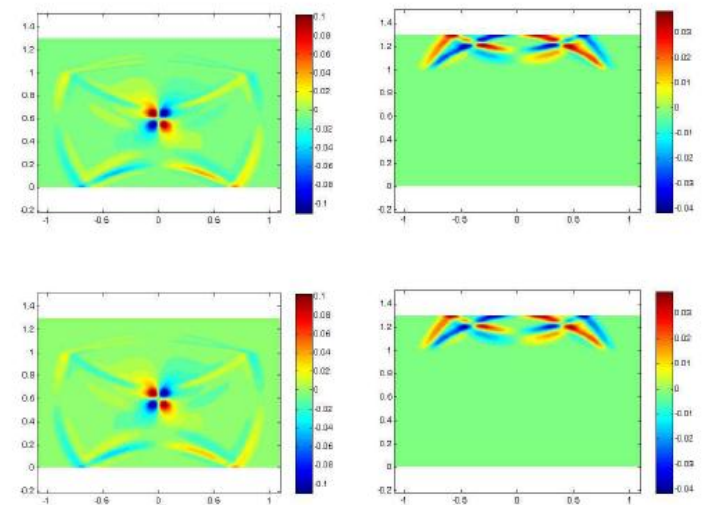
- ☐ FD is dispersive
 - ☐ Godunov is too slow
 - ☐ Rusanov is the best!
- (Myasnikov et al, 2007)



FOG

SOG

FLAC



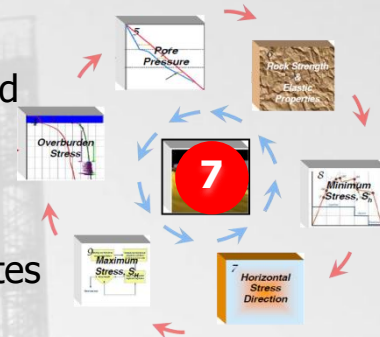
$$F_{i+1/2} = \frac{1}{2} \{ F(U_{i+1}) + F(U_i) - \lambda_{i+1/2} (U_{i+1} - U_i) \}$$

Building and calibration of a MEM.

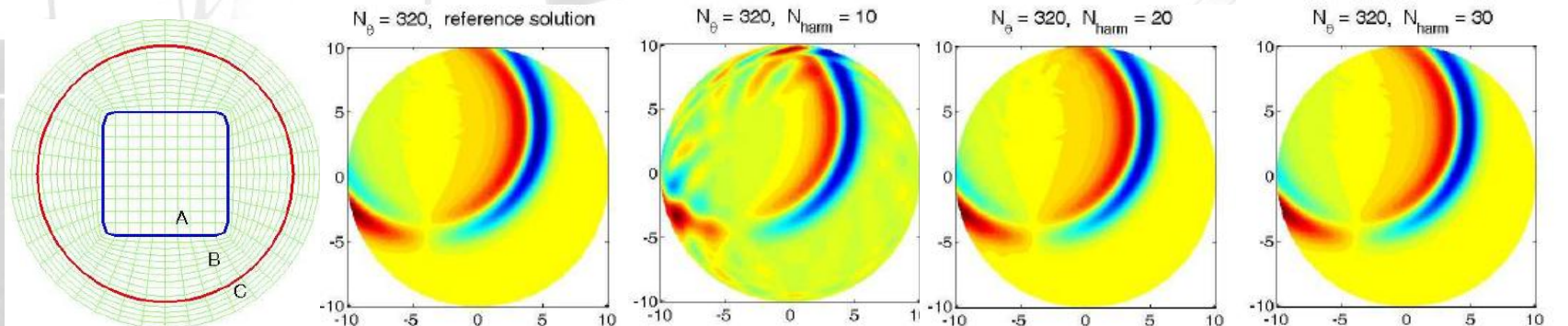
Data audit (13/12): Reservoir simulations and Computational mathematics



- ❑ One has to clearly understand the difference between reservoir simulations and computational mathematics applied to oil & gas industry problems



- ❑ SPE Comparison test project was dedicated to comparative study of computational technologies for reservoir processes modeling. The jury suggested competitors ten tests to exert the power of the software they try to sell
- ❑ All the suggested tests should have been run on extremely rough grids like 10x10 or 25x25x3 (*Odeh A 1981, Weinstein et al. 1986, Kenyon & Behie 1987, Aziz et al. 1987, Killough & Cossack 1987, Nghiem et al. 1991, Quandalle 1993, Killough 1995, Firoozabadi & Thomas 1990, Christie & Blunt 2001*). **WHY?**
- ❑ Another example is non-reflective boundary conditions, which problem is extremely important in computational mathematics (*van Joolen et al. 2003*) and farfetched in reservoir modeling process where all boundaries produce physically supported reflection (*Aziz & Settari, 2004*)
- ❑ Anyway, one should be able to produce such boundary conditions, if needed (*Myasnikov et al, 2007*):

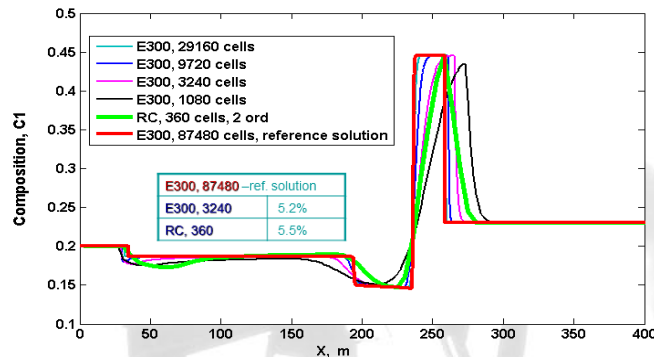


Building and calibration of a MEM.

Data audit (14/12): Optimization workflows



- To model properly gas condensate flows in near well-bore zones with taking into account non-isothermal and non-Darcy effects we combine the concept of the 2-parametric family of the 2nd order accuracy TVD scheme with the concept of approximation with arbitrary accuracy of phase equilibrium performed at the preprocessing stage (Melnikova & Myasnikov 2010, Zhabkina & Myasnikov 2010).

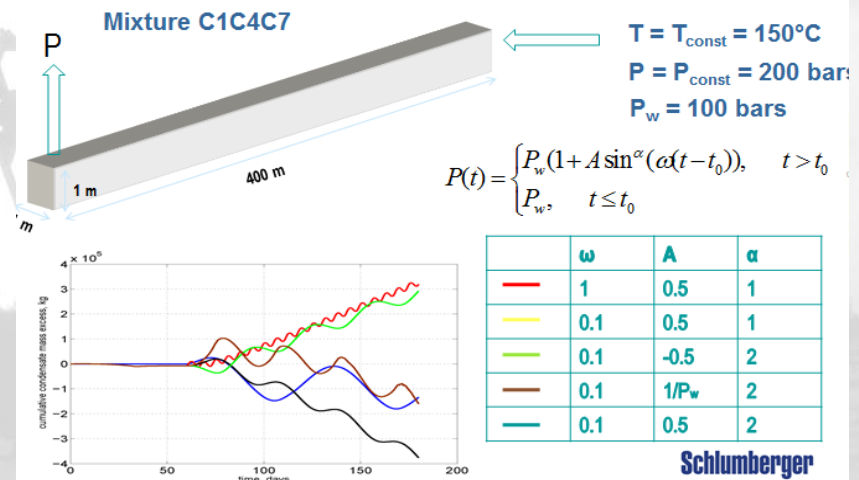


- Path #1: 2nd Order TVD Schemes with 2-parametric family of limiters
- Path #2: Phase equilibrium approximation at the preprocessing stage

Workflow:

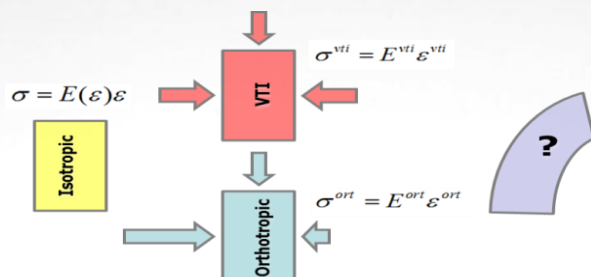
- Simplify the problem (3D to 1D, Non-stationary to steady state etc)
- For the demanded accuracy minimize the number of grid points and the polynomial degrees
- Generate recommendations / Go back to the initial problem

Optimization of gas condensate recovery. Model mixture

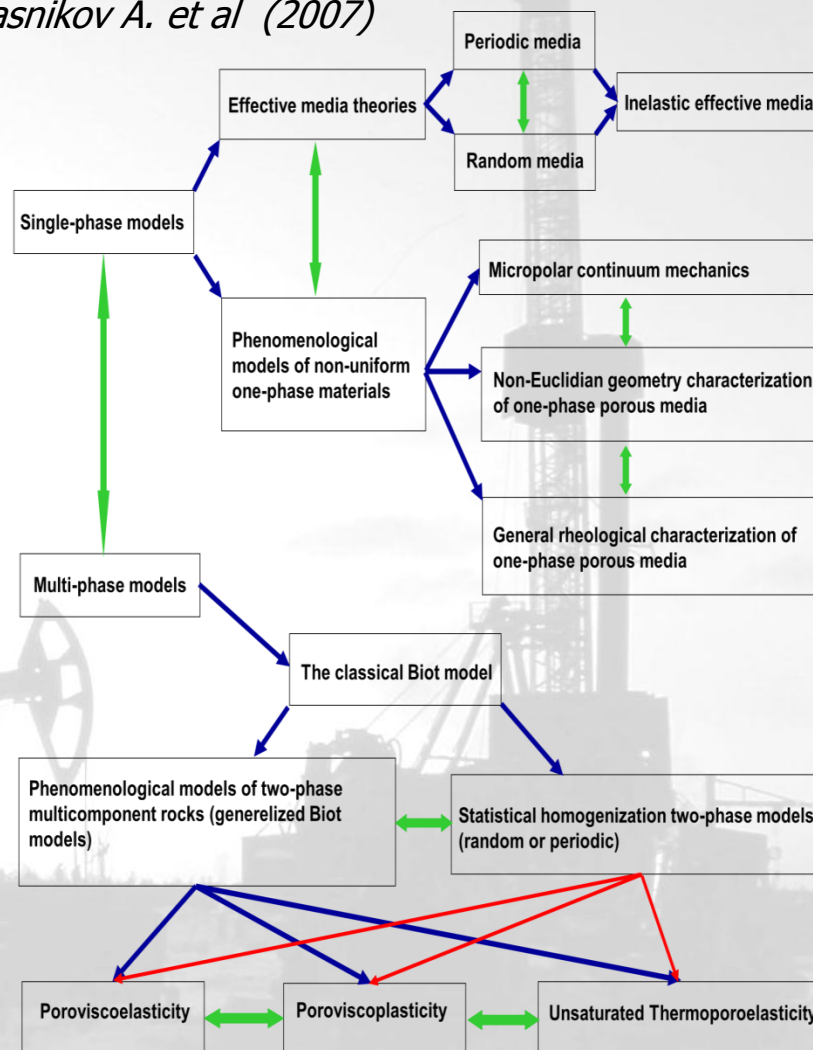


Building and calibration of a MEM.

Data audit (15/12): A concept of rheological monitoring



Myasnikov A. et al (2007)



- ☐ When consider VTI or Orthotropic loading of initially isotropic material no specific symmetry is prescribed in constitutive equations
- ☐ When consider wave propagations one has to identify the symmetry in linearized equations
- ☐ What to do if the loading history is not known?
- ☐ The suggestion is to construct a phenomenological model capable for description of large scale deformation of a rock .
- ☐ To construct a data base of wave propagation properties for each specific rheology
- ☐ To identify 3D and 4D seismic data by the rheology database

THANK YOU FOR YOUR ATTENTION



Introduction. Examples of geomechanical effects.

Seismic interpretation

