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DYNAMICS OF SPACE ELEVATOR ON ASTEROID

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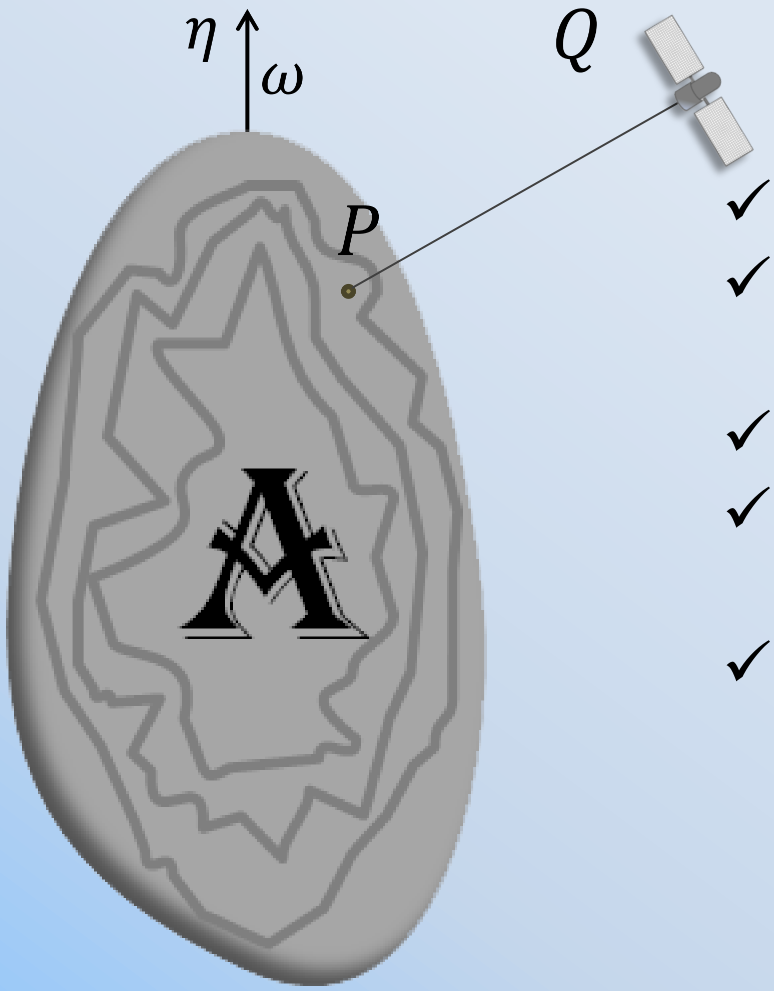
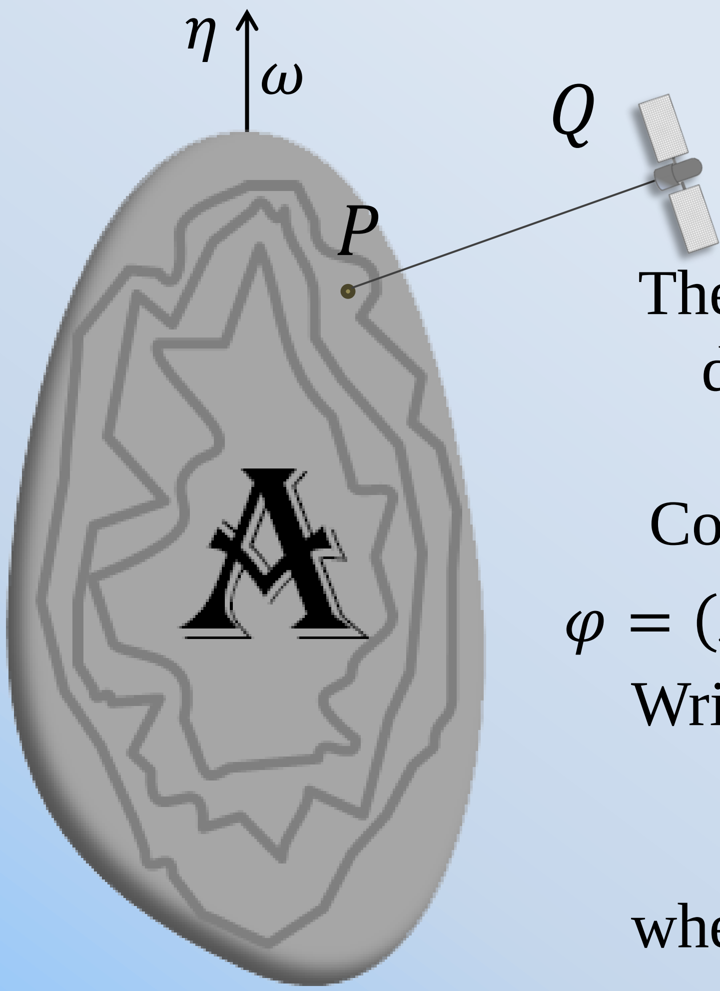
ASTEROID WITH ATTACHED PENDULUM



- ✓ Suppose $\boldsymbol{PQ} = l$
- ✓ Introduce a frame $O\zeta\xi\eta$ (RF) uniformly rotating with the body about the axis $O\eta$.
- ✓ This rotations is assumed to be stable.
- ✓ Suppose RF is composed by the asteroid's central principal axes of inertia.
- ✓ Positions of points P and Q with respect to RF are given by they radius-vectors: $\boldsymbol{OP} = \left(p_z, p_x, p_y\right)$ and $\boldsymbol{OQ} = (z, x, y)$.

AUGMENTED POTENTIAL AND ROUTH FUNCTION



The potential of centrifugal force is

$$U_C = -\frac{1}{2} m\omega^2 (z^2 + x^2)$$

The gravitational potential depends on mass distribution of asteroid $\mathfrak{A}$. Denote it as

$$U_N = U_N(z, x, y).$$

Constraint:

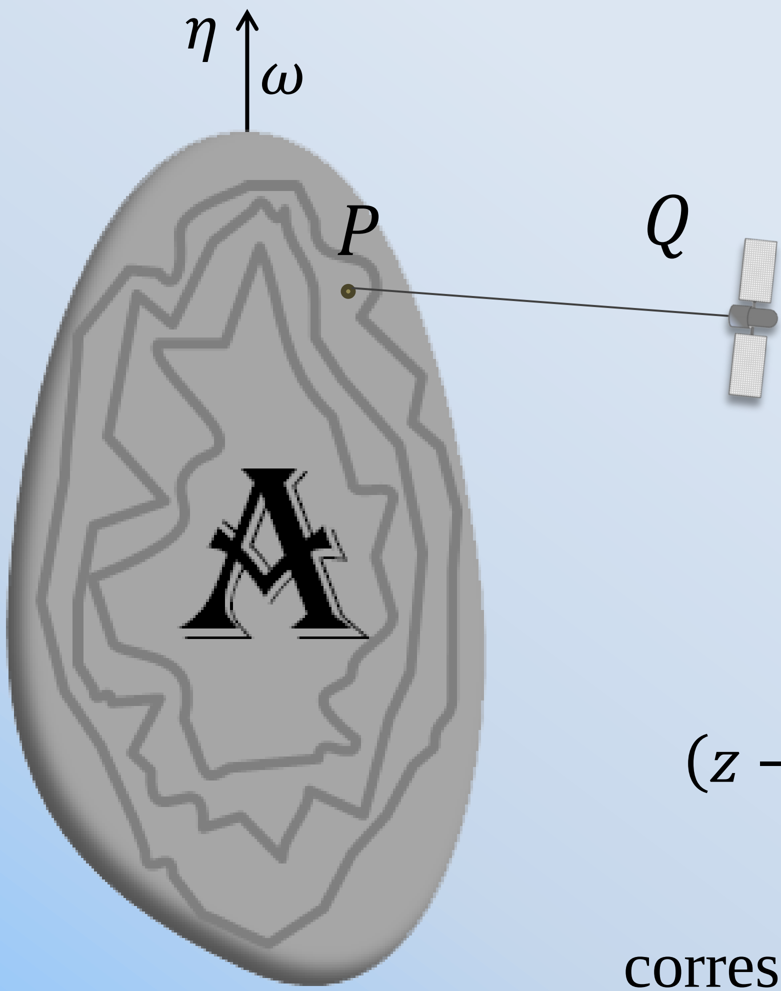
$$\varphi = (z - p_z)^2 + (x - p_x)^2 + \left(y - p_y\right)^2 - l = 0$$

Write down the Routh function:

$$W(x, y, z, \lambda, \omega, l) = U + \frac{\lambda}{2}\varphi$$

where $U = U_C + U_N$

EQUILIBRIA POINTS OF THE PENDULUM



$$\frac{\partial U}{\partial z} + \lambda(z - p_z) = 0,$$

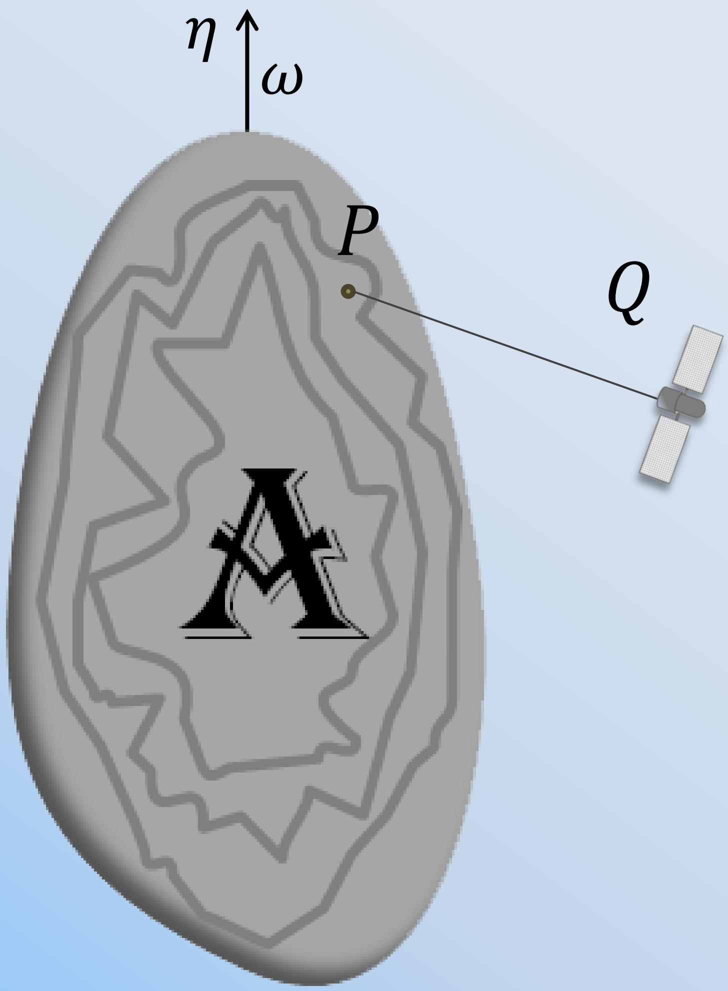
$$\frac{\partial U}{\partial x} + \lambda(x - p_x) = 0,$$

$$\frac{\partial U}{\partial y} + \lambda(y - p_y) = 0.$$

$$(z - p_z)^2 + (x - p_x)^2 + (y - p_y)^2 = l$$

Critical points of this system correspond to the equilibria of the pendulum with respect to the rotating coordinate frame

EXCLUDING LAGRANGIAN MULTIPLIER



$$\frac{\partial U}{\partial x}(y - p_y) - \frac{\partial U}{\partial y}(x - p_x) = 0,$$

$$\frac{\partial U}{\partial y}(z - p_z) - \frac{\partial U}{\partial z}(y - p_y) = 0,$$

$$\frac{\partial U}{\partial z}(x - p_x) - \frac{\partial U}{\partial x}(z - p_z) = 0,$$

It expresses that the moment of force relative to the point P is equal to zero.

POSSIBLE EQUILIBRIA OF THE SPACECRAFT

$$\frac{\partial U}{\partial x}(y - p_y) - \frac{\partial U}{\partial y}(x - p_x) = 0,$$

$$\frac{\partial U}{\partial y}(z - p_z) - \frac{\partial U}{\partial z}(y - p_y) = 0,$$

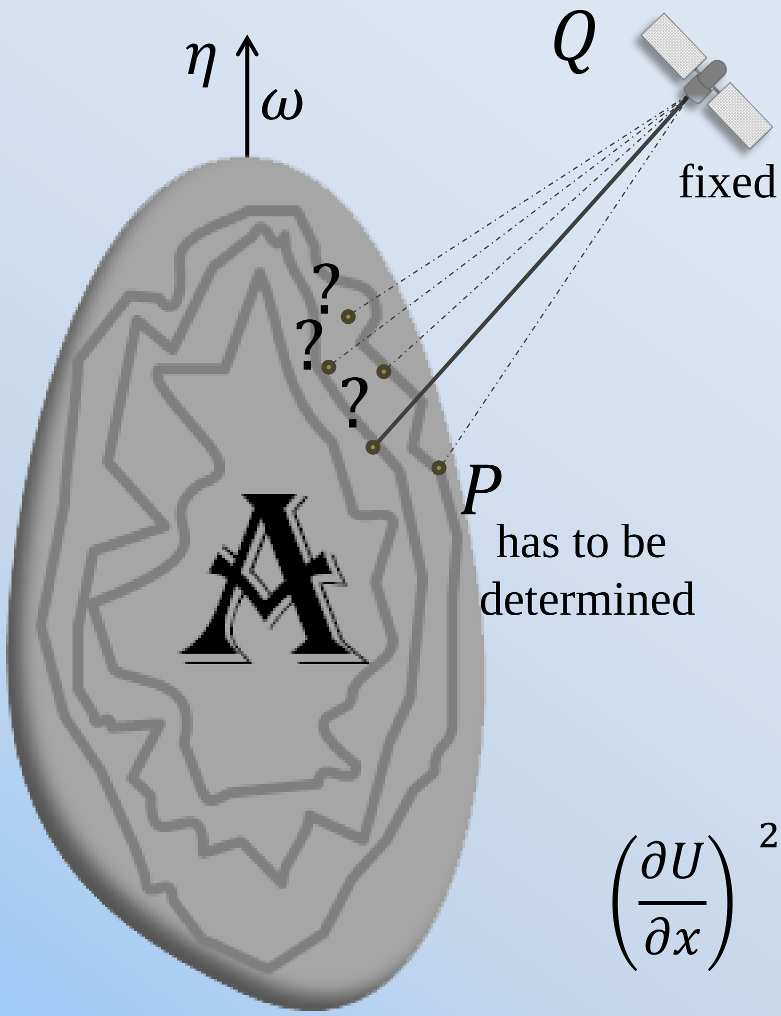
$$\frac{\partial U}{\partial z}(x - p_x) - \frac{\partial U}{\partial x}(z - p_z) = 0,$$

- ✓ Suppose anchor P is fixed.
- ✓ Γ corresponds to possible equilibrium of point Q
- ✓ Curve Γ passes through the anchor P and points where

$$\frac{\partial U}{\partial x} = \frac{\partial U}{\partial y} = \frac{\partial U}{\partial z} = 0$$

- ✓ When spacecraft Q is located in the libration point, then the tether is slack: $\lambda = 0$.

SOLUTION OF THE INVERSE PROBLEM



✓ Suppose l is pre-defined

The problem is to determine the anchor point for a given length of the tether if spacecraft Q is in equilibrium in a given point of the rotating reference frame.

From system of equations for equilibria of the pendulum one can obtain:

$$\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial z}\right)^2 =$$

$$= \lambda^2 \left((z - p_z)^2 + (x - p_x)^2 + \left(y - p_y\right)^2 \right)$$

Taking into account constraint $\varphi = 0$, one arrives at $\lambda = \pm l^{-1} |\mathrm{grad} U|$

SOLUTION OF THE INVERSE PROBLEM

✓ Suppose $\lambda \neq 0$

From system of equations for equilibria of the pendulum one can obtain:

$$\begin{cases} \dfrac{\partial U}{\partial z} + \lambda(z - p_z) = 0 \\ \dfrac{\partial U}{\partial z} + \lambda(z - p_z) = 0 \\ \dfrac{\partial U}{\partial z} + \lambda(z - p_z) = 0 \end{cases} \Rightarrow \quad \begin{aligned} p_z &= z + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial z} \\ p_x &= x + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial x} \\ p_y &= y + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial y} \end{aligned}$$

Thus the location of the anchor is found

POSITION OF ANCHOR ON THE ASTEROID SURFACE

- ✓ Take into account that the anchor should be located on the asteroid surface.
- ✓ Let this surface be described in the $O\zeta\xi\eta$ as $f\left(p_z, p_x, p_y\right) = 0$
- ✓ Suppose Q is fixed (for a while)

$$p_z = z + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial z};\ \ p_x = x + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial x};\ \ p_y = y + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial y}$$

The system is not self-contained:
3 equations for determination 4 unknowns: p_z, p_x, p_y and l

The equation of the surface $f\left(p_z, p_x, p_y\right) = 0$ closes this system.

SPECIAL CASE OF ASTEROID SURFACE

Suppose that asteroid surface is a sphere of radius R centered in O:

$$p_z^2 + p_x^2 + p_y^2 - R^2 = 0$$

$$p_z = z + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial z}; \quad p_x = x + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial x}; \quad p_y = y + \frac{l}{|\mathrm{grad}U|}\frac{\partial U}{\partial y}$$

$\lambda = l^{-1}|\mathrm{grad}U|$

$$p_z^2 + p_x^2 + p_y^2 - R^2 = 0$$

$$(z + ln_z)^2 + (x + ln_x)^2 + \left(y + ln_y\right)^2 = R^2$$

where $\left(n_z, n_x, n_y\right) = \frac{\mathrm{grad}U}{|\mathrm{grad}U|}$

SPECIAL CASE OF ASTEROID SURFACE

Suppose that the pre-defined equilibrium does not coincide with a libration point: $|\mathrm{grad}U| \neq 0$.

$$(z + ln_z)^2 + (x + ln_x)^2 + \left(y + ln_y\right)^2 = R^2$$

$$\Downarrow$$

$$l^2 + 2p_1 l + p_0 = 0$$

$$p_1 = zn_z + xn_x + yn_y, \qquad p_0 = z^2 + x^2 + y^2 - R^2$$

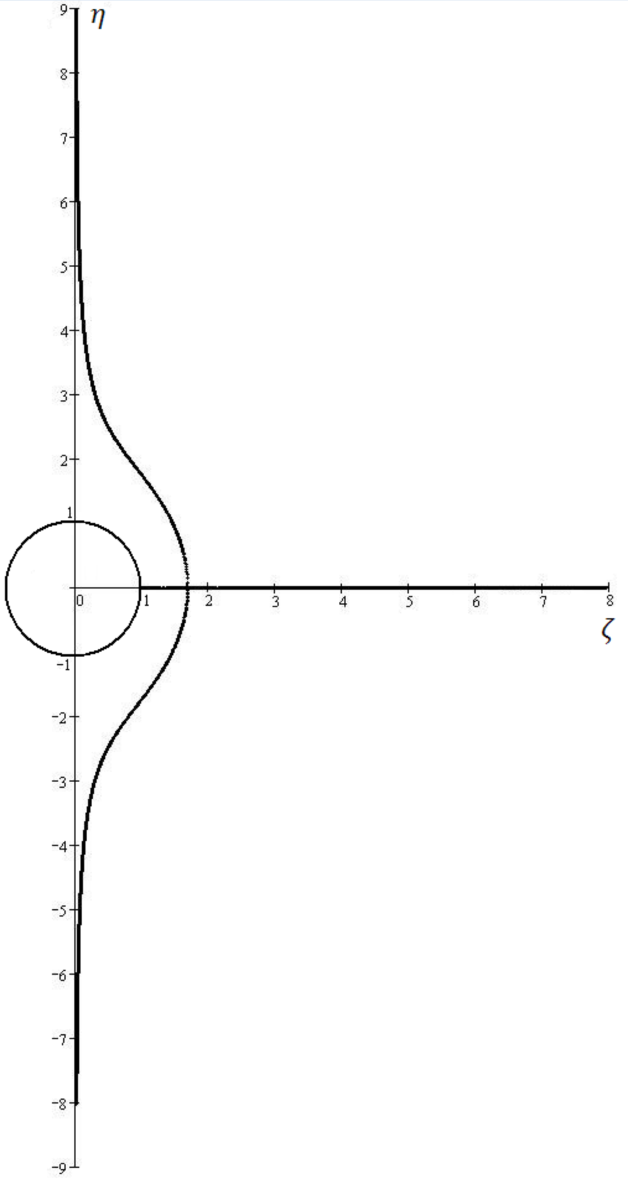
For its discriminant D the following inequality holds true:

$$\frac{D}{4} = p_1^2 - p_0 = \left(zn_z + xn_x + yn_y\right)^2 - (z^2 + x^2 + y^2 - R^2) =$$

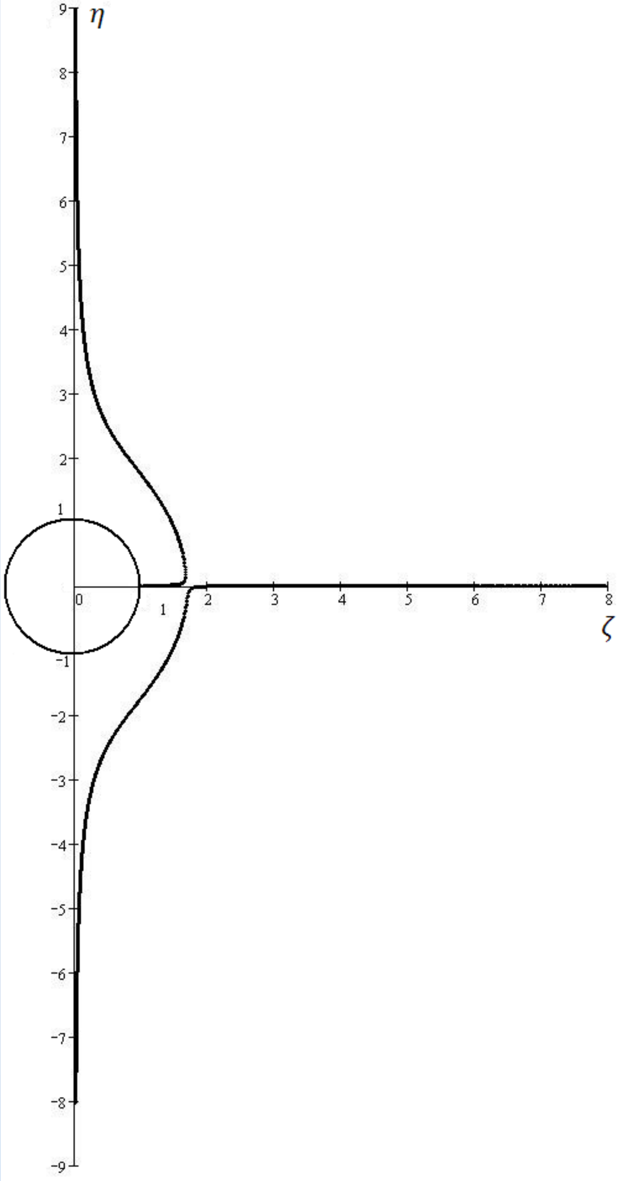
$$= \left(n_x y - n_y x\right)^2 + \left(n_y z - n_z y\right)^2 + (n_z x - n_x z)^2 + R^2 > 0$$

Thus both roots of this equation are real.

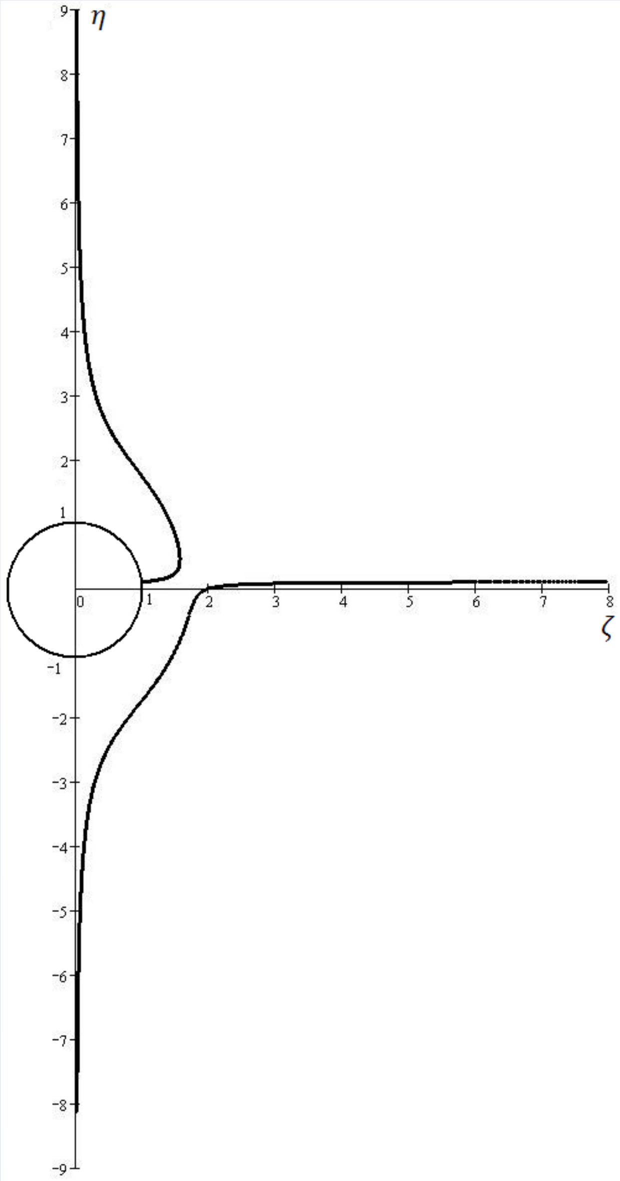
POSSIBLE EQUILIBRIA IN SPECIAL CASE



$\boldsymbol{OP} = (0, \sin(0.0001), \cos(0.0001))$

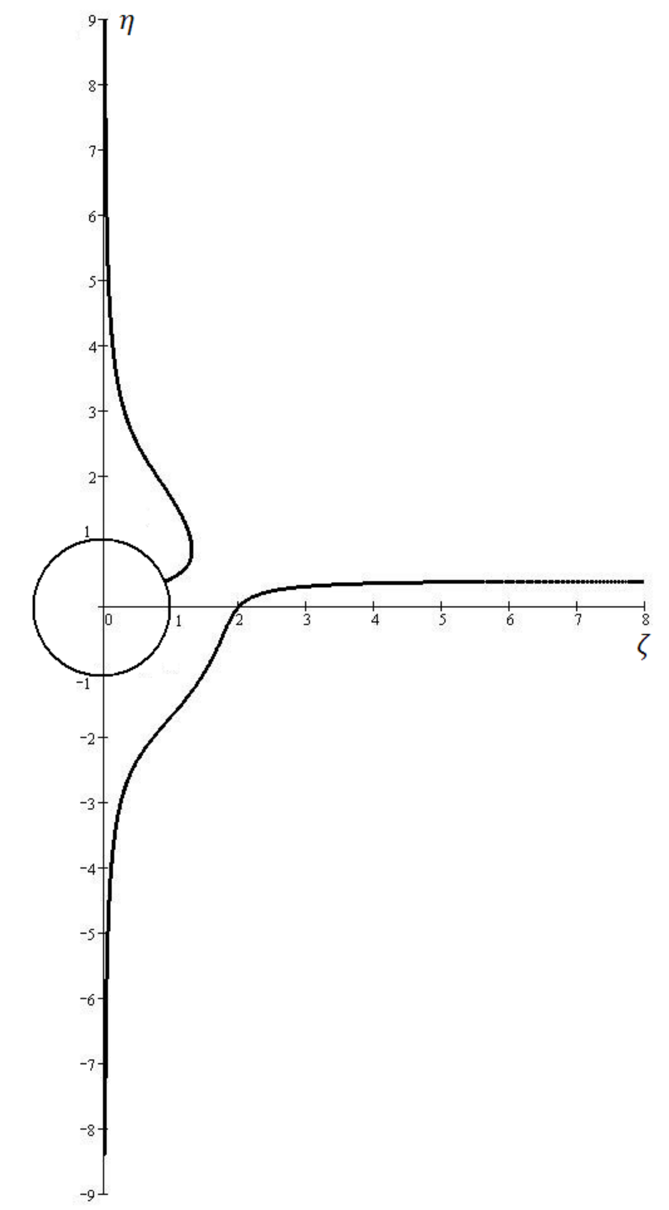


$\boldsymbol{OP} = (0, \sin(0.001), \cos(0.001))$

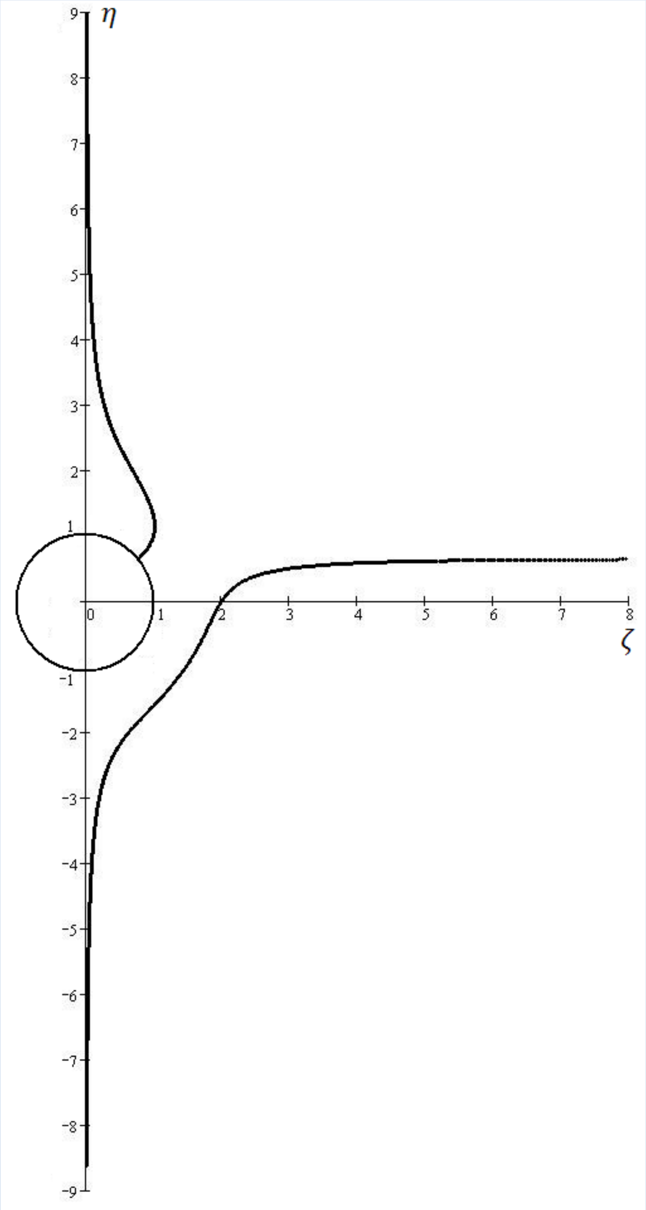


$\boldsymbol{OP} = (0, \sin(0.1), \cos(0.1))$

POSSIBLE EQUILIBRIA IN SPECIAL CASE



$\boldsymbol{OP} = (0, \sin(0.4), \cos(0.4))$



$\boldsymbol{OP} = (0, \sin(0.7), \cos(0.7))$

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