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NUMERICAL SOLUTION OF INTEGRAL EQUATIONS, DESCRIBING MASS SPECTRUM OF VECTOR MESONS

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1. Quasipotentional approach[1,2] is used intensively for description of two connected particles system such as $q\bar{q}$, e^+e^-, $\mu^+\mu^-$, etc. As the exact analytic solutions are known only for some simplest potential, wide application of this approach needs to develop the numerical methods of solving the quasipotentional equations with different potentials.

In general case, for two scalar particles connected system the quasipotentional equation in momentum space may be presented in the form

$$G^{-1}(p,E)\psi(p)= 1/(2\pi)^3\int V(p,k;E)\psi(k)dk/E_k, \qquad (1.1)$$

where $G(p,E)$ is a Green function, $\psi(p)$ is a wave function of relative motion of connected system[3], E is an energy of connected system.

Let us consider the equation (1.1)with the potential that describes a quark-antiquark interaction:

$$V(p,k;E)=V_1(p,k;E)+V_2(p,k;E), \qquad (1.2)$$

where the potential

$$V_1(p,k;E)=-\alpha/(p-k)^2 \qquad (1.3)$$

is an analog of Coulomb potential and describes interaction at a small distance, and the potential

$$V_2(p,k;E)=\beta/(p-k)^4 \qquad (1.4)$$

is locking and ensures limitless growth of eigenvalues of Eq. (1.1) and is more similar in concept to the phenomenological type.

When choosing the potential in such a manner,in the framework of the nonrelativistic case Eq.(1.1) has the form:

$$(E-p^2/2-V_0)\psi(p)=-1/(2\pi)^3\int(-\alpha/(p-k)^2+\beta/(p-k)^4)\psi(k)dk, \qquad (1.5)$$

where a shift of spectrum V_0 is introduced.If integrating Eq.(1.5) in angles, in central-symmetrical case we obtain the following equation:

$$(p^2/2+V_0-E)\varphi(p)=\alpha/\pi\int_0^\infty \ln\left|\frac{p-k}{p+k}\right|\varphi(k)dk-\beta/\pi\int_0^\infty((p-k)^{-2}+(p+k)^{-2})\varphi(k)dk. \qquad (1.6)$$

Here $\varphi(p)=p\psi(p)$, $\psi(p)=\psi(|p|)$. If we amplify the Eq.(1.6) by the condition $\varphi(0)=0$ and the norm condition

$$\int_0^\infty \varphi(p)^2dp=1 , \qquad (1.7)$$

we arrive at the eigenvalue problem.

2. The main requirement for computing mass spectrum and other characteristics of vector mesons by Eq.(1.6) is great computation accuracy of eigenvalue (EV) and eigenfunction (EF).Thus, for the numerical computations to be performed, we must solve the problem of computing the invariant subspace of great order matrix (with order N*N, where N~1000), which is not solvable in practice, if we do not take into account the special structure of the matrix.

In order to solve the arising algebraic problem a numerical algorithm has been devised utilizing the Multi-Grid subspace iteration method and efficient procedures for the processing of the Hankel and Toeptlitz matrices that emerge in the Galerkin discretization of an integral operator as well as the special algorithms for improving the accuracy of approximate solutions by the Richardson extrapolation. The computations have been performed on the CDC-6500 computer. In so doing the main memory capacity in use is $Q=(2p+O(1))N+O(p^2)$, where p is the number of computated EV.

Let us illustrate the basic characteristics of the algorithm by solving Shrödinger equation with the Coulomb potential, which we obtain from Eq.(1.6) by $\beta=0$

$$(p^2/2-\lambda)\psi(p)=-\alpha/\pi\int_0^\infty \ln\left|\frac{p-k}{p+k}\right|\psi(k)dk. \quad (2.1)$$

This equation has an exact solution. If $\alpha=1$, there exists the formula

$$\lambda_n=-1/(2n^2),\ n=1;2;3;... \quad (2.2)$$

The results of numerical computations for the first four EV depending on N discretisation order are tabulated in Table 1.

Table 1

N	λ1	λ2	λ3	λ4
2^7	-0.50000375	-0.12478302	-0.050508256	-0.0089176
2^8	-0.49999798	-0.12496973	-0.054147826	-0.0050906
2^9	-0.4999964	-0.12498776	-0,054654526	-0.00817317
2^{10}	-0.49999568	-0.12499518	-0.054965435	-0.011246485

Now consider the relativistic equation with locking potential

$$(p^2-\Pi_E)\psi(p)=\beta/\pi\int_0^\infty((p-k)^{-2}+(p+k)^{-2})\psi(k)dk, \quad (2.3)$$

which we get from Eq.(1.6) when $\alpha=0$ and the corresponding change of the left-hand side has been made. Here $\Pi_E=(2E-2m)m/4$, where m is a mass of particles forming the connected system, and M=2m is a mass of the connected system. There is an analytic solution of Eq.(2.3)

$$M_n=2m(1+\Lambda\xi_n)\ , \quad (2.4)$$

here ξ_n-function Airi zeros, and $\Lambda=2(\beta/m^3)^{(2/3)}$.

Let us apply Eq.(2.3) to a description of mass spectrum of J/Ψ- and Y-particles. Values of parameters β and m are determinated by masses of two low level /5/. Using the β and m values obtained Eq.(2.3) allows to numerically compute the masses of the rest of excited states (see Table 2). The numerical results of calculating the mass spectrum of J/Ψ- and Y-particles by formula (2.4) with the help of values of β and m obtained are listed in Table 2.

Table 2.

$M_{J/\Psi}$(Mev)			M_{Υ}(Mev)		
Mexp.	M(2.3)	M(2.4)	Mexp.	M(2.3)	M(2.4)
3096.9±0.1	3096.9	3096.93	9460.±0.2	9460	9462
3686.0±0.1	3686	3686.11	10023.4±0.3	10023.4	10027
4159 ± 20	4168	4168.48	10355.5±0.5	10482.8	10489.95
4415 ± 6	4589.5	4594.8	10577±7	10885	10898.9
$\Lambda=0.14578\pm5.16*10^{-5}$ $m=1.1548\pm1.3*10^{-4}$			$\Lambda=0.037097\pm6.5*10^{-6}$ $m=4.3534\pm8.2*10^{-5}$		

Table 2 shows that the numerical results of calculations of the masses spectrum by (1.6) and the computations by the formula (2.4) coincide in three first figures.

3.The use of the potential which consists of both the Coulomb potential and the locking part, allows to describe simultaneously both mass spectrum and lepton decay widths of vector mesons .To determine the vector mesons lepton decay widths we use the Van Royen-Weisskopf formula[6], that can be written in momentum space taking in account quark colour, in a following manner:

$$\Gamma_{V\to e^+e^-}=16\pi\alpha^2 e_q^2 M_{V\alpha}^{-2}\left|\int_0^\infty \varphi(p)dp\right|^2 , \qquad (3.1)$$

where α=1/137,M_V is a meson mass,e_q is a quark charge,and wave function $\varphi(p)$ is normalised by condition (1.7). Computations of meson masses were performed by using the formula:

$$M_V=(4.+E)m^* , \qquad (3.2)$$

where E-solution of Eq.(1.6),and $m^*=m_q/2$-reduced quark mass.

Table 3 results the computations of mass spectrum and lepton decay widths of J/Ψ-particle. Parameter values $m^*=0.71552\pm1.34*10^{-5}$, $\alpha=0.19225\pm1.2*10^{-4}$,$\beta=0.41482\pm3.9*10^{-5}$and $V_0=0.54511\pm8.6*10^{-5}$ have been fitted in masses of the first three states and lepton widths of the basic state[5].

Table 3

$M_{J/\Psi}$(Mev)		$\Gamma_{J/\Psi}$ (Kev)	
Mexp.	Mtheor.	Γexp.	Γtheor.
3096.9±0.1	3096.9	4.8±0.6	4.77
3686.0±0.1	3686.0	2.1±0.3	2.97
4159 ± 20	4153	0.75±0.1	2.24
4415 ± 6	4563.5	0.44±0.14	1.8

As shown in Table 3, Eq.(1.6) describes the mass spectrum quite right, but gives too high values of lepton decay widths for excited states of the $c\bar{c}$ system.

4.Let us turn our attention to the discussion of the relativistic quasipotential equation[2]:

$$\sqrt{1+p^2}(\sqrt{1+p^2}-E)\varphi(p)=\alpha/\pi\int_0^\infty \ln\left|\frac{p-k}{p+k}\right|\varphi(k)dk-\beta/\pi\int_0^\infty((p-k)^{-2}+(p+k)^{-2})\varphi(k)dk. \quad (4.1)$$

For the meson mass to be calculated, we use the formula

$$M_V=2Em, \quad (4.2)$$

where m is a quark mass, and in order to calculate the width $\Gamma_{V\to e^+e^-}$ we utilize expression (3.1), where the wave function $\varphi(p)$ is the solution of Eq.(4.1).Now we fix free parameters $m=1.6928\pm0.00003$, $\alpha=0.56103\pm0.000006$, $\beta=0.016005\pm0.00009$ in the Ψ-particles set the first two states masses and in the J/Ψ-particle basic state lepton width.

Table 4 shows the results of calculations of the J/Ψ mass spectrum with Eq. (4.1).

Table 4

$M_{J/\Psi}$(Mev)		$\Gamma_{J/\Psi}$ (Kev)	
Mexp.	Mtheor.	Γexp.	Γtheor.
3096.9±0.1	3096.9	4.8±0.6	4.5
3686.0±0.1	3686.0	2.1±0.3	1.28
4028 ± 10	3972.9	0.75±0.15	0.81
4159 ± 20	4169.9	0.77±0.23	0.78

As seen from Table 4, the description of the width $\Gamma_{V\to e^+e^-}$ is better than in the framework of the nonrelativistic approach.

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Численное решение интегральных уравнений, описывающих спектр масс векторных мезонов

Описан численный алгоритм для решения квазипотенциальных интегральных уравнений в импульсном пространстве. Приведены результаты численных расчетов спектра масс и ширин лептонных распадов векторных мезонов в сравнении с данными физического эксперимента.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Numerical Solution of Integral Equations, Describing Mass Spectrum of Vector Mesons

The description of the numerical algorithm for solving quazipotential integral equation in impulse space is presented. The results of numerical computations of the vector meson mass spectrum and the lepton decay width are given in comparison with the experimental data.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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