

# PLASMONIC EXCITATIONS IN PHOTONICS AND OPTOELECTRONICS

## Polarization Properties of Surface Plasmon Polaritons at the Boundary of Topological Insulators with the Axion Effect

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**Abstract**—Properties of surface plasmon polariton waves are theoretically studied in structures containing topological insulators with the axion effect. The effect of axion properties on dispersion, localization, and polarization of plasmon polaritons is analyzed. A possibility of determining the axion effect from the variation in the plasmon-polariton polarization is shown, and conditions for enhancement of polarization effects are revealed in waveguide structures of the dielectric–metal–dielectric type.

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### 1. INTRODUCTION

Sensitivity of surface plasmon polaritons—electromagnetic waves propagating along the metal–dielectric interface [1]—to optical properties of media in which they are excited allows them to be successfully used as biological and chemical sensors [2, 3]. In a number of works it is also pointed out that this sensitivity favors enhancement of some optical effects related to the effect of magneto-optical [4–6], optically active [7, 8], chiral [9], and other properties of a material on its permittivity tensor. In this work we propose using surface plasmon polaritons to observe the axion effect in topological insulators by optical methods.

Topological insulators are band insulators (dielectrics) featuring surface conducting states [10]. Recently these properties have been found in a wide range of materials, e.g.,  $\text{Bi}_{1-x}\text{Sb}_x$ ,  $\alpha\text{-Sn}$ ,  $\text{Bi}_2\text{Se}_3$ ,  $\text{Bi}_2\text{Te}_3$ , etc. If time inversion excited, for example, by placing the sample in a magnetic field, introducing impurities in it, or applying a ferromagnetic or anti-ferromagnetic layer on its surface, a magnetoelectric effect, the so-called axion effect, arises in topological insulators [10–12]. This effect manifests itself in the coupling of the electric ( $\mathbf{E}$ ) and magnetic ( $\mathbf{B}$ ) components of the electric field. The coupling is defined by a Lagrangian  $L = (\alpha\Theta/4\pi^2)\mathbf{E}\mathbf{B}$ , where

$\alpha = e^2/\hbar c$  is the fine structure constant and  $\Theta$  is the quantized axion angle that is multiple of the odd number of  $\pi$ . The coupling of the magnetic and electric field components resulting from the axion effect can be expressed by constitutive equations

$$\mathbf{D} = \varepsilon_{\text{TI}}\mathbf{E} + \bar{\alpha}\mathbf{B}, \quad (1)$$

$$\mathbf{H} = \mathbf{B} - \bar{\alpha}\mathbf{E}, \quad (2)$$

where  $\varepsilon_{\text{TI}}$  is the permittivity of the topological insulator,  $\bar{\alpha} = \alpha\Theta/\pi$ .

From the Maxwell equations we can obtain a wave equation for propagation of an electromagnetic wave inside a topological insulator with the axion effect:

$$\Delta\mathbf{E} - \frac{\varepsilon_{\text{TI}}}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \quad (3)$$

which does not involve any terms related to the axion effect. This means that though constitutive equations differ from those which describe radiation propagation in an ordinary isotropic dielectric, the axion effect does not affect electrodynamics in the bulk of the medium, namely, it does not make the bulk wave change its polarization and propagation constant at any propagation direction and initial polarization. The only change caused by the axion effect is that vectors  $\mathbf{E}$  and  $\mathbf{B}$  as well as  $\mathbf{D}$  and  $\mathbf{H}$  are pairwise orthogonal while vectors  $\mathbf{E}$  and  $\mathbf{D}$  and  $\mathbf{H}$  and  $\mathbf{B}$  are not collinear, according to Eqs. (1) and (2).

At the same time, as follows from the boundary conditions, on the surface of the topological insulators with the axion effect there arise the effective charge surface density  $\rho_{\text{ef}} = (\bar{\alpha}/4\pi)B_n$  and the effec-

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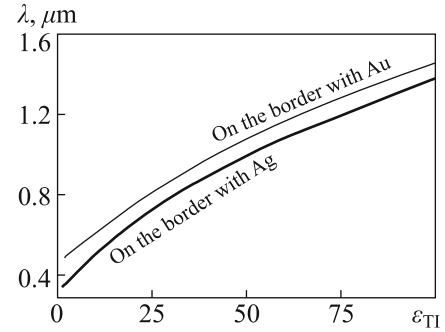
tive current surface density  $\mathbf{j}_{\text{ef}} = (\bar{\alpha}c/4\pi)\mathbf{E}_\tau$ , where the subscripts  $n$  and  $\tau$  denote the normal and tangential components of the corresponding vectors of the electromagnetic field inside the topological insulator. Thus, the axion effect can manifest itself at the interface of two media, one of which has axion properties while the other does not. Specific features of optical radiation reflection from such interfaces are considered in a number of works [13–15]. However, it can be expected that the axion effect will also manifest itself when the electromagnetic wave interacts with the interface of the axion and nonaxion media not once (as in the case of reflection or refraction at the interface) but continuously, having the form of a running surface wave, e.g., a plasmon-polariton wave. In this case, the effective length of the wave–surface interaction increases. The role of the medium without the axion effect is played by the metal needed for existence of surface plasmons. Note that surface plasmons can also be excited on the surface of the topological insulator due to its surface conduction [16–19], but the energy of these plasmons corresponds to the frequencies of the far-infrared and terahertz ranges rather than the optical range. Thus, further analysis will be performed for structures in which the topological insulator (TI) with the axion effect borders a metal, which allows optical response to be observed in the visible and near-infrared ranges. To obtain a structure like this, any sublayer up to 3–7 nm thick, inert with respect to the optical properties of the structure, can be sandwiched between the topological insulator and the metal.

To analyze the axion effect on the properties of the plasmon structure modes, we find eigenmodes in the form of the surface electromagnetic waves propagating along the  $Ox$  axis that is perpendicular to the surface of the TI occupying the region  $z > 0$ . For simplicity, we will describe the system using the linear approximation in the small parameter, fine structure constant  $\alpha = 1/137 \ll 1$ . Then, expressed in terms of the complex wave vector  $\mathbf{k} = \{\beta k_0; 0; i\gamma_{\text{TI}}k_0\}$ , the electromagnetic field of the surface wave inside the TI is

$$\mathbf{E} = \mathbf{E}_0 \exp(ik_0\beta x) \exp(-\gamma_{\text{TI}}k_0 z), \quad (4)$$

where  $k_0 = \omega/c$  is the wavenumber in free space. Note that since bulk modes of the topological insulator are degenerate in polarization (see Eq.(3)), the surface wave is not split into a sum of quasi-TM and quasi-TE components, as is the case, for example, in gyrotropic dielectrics.

Dispersion, polarization, and localization properties of the eigenmodes of the TI–metal plasmon structure can be obtained by solving Maxwell equations with the corresponding boundary conditions, which, in the absence of surface charges and currents,



**Fig. 1.** Dependence of the plasmon resonance wavelength on the topological insulator permittivity (thin curve is for gold, bold curve is for silver).

have an ordinary form: normal components of vectors  $\mathbf{D}$  and  $\mathbf{B}$  and tangential components of vectors  $\mathbf{E}$  and  $\mathbf{H}$  are continuous at the interface. The resulting dispersion relation is free of terms linear in  $\alpha$ , and thus the axion effect does not exert significant influence on the dispersion of plasmon polaritons

$$\beta = \sqrt{\frac{\varepsilon_m \varepsilon_{\text{TI}}}{\varepsilon_m + \varepsilon_{\text{TI}}}}, \quad (5)$$

where  $\varepsilon_m$  is the permittivity of the metal, and on their excitation conditions. At the same time, a distinctive feature of a topological insulator is high permittivity (about a few tens). As a result, the plasmon resonance wavelength corresponding to  $\beta \rightarrow \infty$  shifts to the infrared region. The corresponding shift is shown in Fig. 1.

Solution of the eigenmode problem shows that the axion effect causes a change in polarization of plasmon modes of this structure. A similar effect was earlier revealed for plasmons at the TI–dielectric interface [20]. Complex amplitudes of the electromagnetic field of the plasmon polariton on the TI surface have the form (up to the constant factor)

$$\mathbf{E}_0 = \begin{pmatrix} \gamma_{\text{TI}} \\ i\bar{\alpha} \frac{\varepsilon_{\text{TI}}}{\varepsilon_{\text{TI}} - \varepsilon_m} \\ i\beta \end{pmatrix}, \quad \mathbf{H}_0 = \begin{pmatrix} \bar{\alpha} \gamma_{\text{TI}} \frac{\varepsilon_m}{\varepsilon_{\text{TI}} - \varepsilon_m} \\ -i\varepsilon_{\text{TI}} \\ i\bar{\alpha} \beta \frac{\varepsilon_m}{\varepsilon_{\text{TI}} - \varepsilon_m} \end{pmatrix}, \quad (6)$$

i.e., the TM mode acquires small-value TE components. This means that the plasmon-polariton TM wave elliptically polarized when there is no axion effect (the phase shift between the components  $E_x$  and  $E_z$  is  $\pi/2$ ) suffers polarization ellipse tilting when there is the axion effect. Since the TE component  $E_y$  is in phase with  $E_z$ , the plasmon polarization ellipse in a medium with the axion effect turns out to be rotated about the  $Ox$  axis through an angle

$$\varphi = \arctan\left(\frac{E_y}{E_z}\right) \approx \bar{\alpha} \frac{\varepsilon_{\text{TI}}}{\beta(\varepsilon_{\text{TI}} - \varepsilon_m)}. \quad (7)$$

It is noteworthy that the direction of the polarization ellipse rotation is uniquely related to the plasmon propagation direction. This is similar to the Rashba effect, when the directions of the electron spin and momentum are uniquely related.

In this case, the ratio of the TE and TM components  $\nu$  is

$$\nu = \left| \frac{E_y}{H_y} \right| = \frac{\bar{\alpha}}{|\varepsilon_{\text{TI}} - \varepsilon_{\text{m}}|}. \quad (8)$$

Numerical estimations show that  $\nu$  is as small as about  $10^{-4}$  because  $\varepsilon_{\text{TI}} \sim |\varepsilon_{\text{m}}| \sim 10$ . Therefore, to observe experimentally changes in plasmon-polariton polarization due to the axion effect, it is important to find conditions for enhancement of this effect.

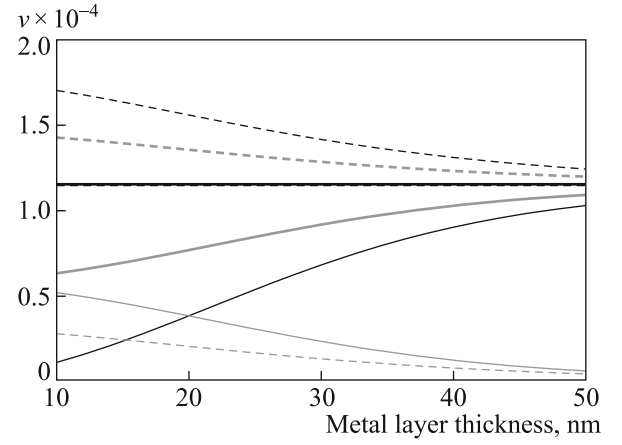
To this end, three-layer structures like TI–metal–TI or TI–metal–dielectric can be used. In these three-layer structures featuring gyrotropic response, enhanced effect of gyrotropy on the dispersion [4, 5] and polarization [6] properties of the plasmon modes was earlier observed. It is these structures with a thin (5 to 20 nm) metal film that are of special interest for plasmonics because long-range modes can be excited in them, which results, first, in a considerably higher  $Q$  factor of plasmon resonances and, second, in increased penetration depth of the electromagnetic field of the plasmon in the dielectric.

In these three-layer structures, modes of two types can be excited, which correspond to two solutions to the dispersion equation for surface plasmon polaritons

$$\left( \frac{\gamma_2^2}{\varepsilon_2^2} + \frac{\gamma_1 \gamma_3}{\varepsilon_1 \varepsilon_3} \right) \tanh(\gamma_2 k_0 w) = -\frac{\gamma_2}{\varepsilon_2} \left( \frac{\gamma_1}{\varepsilon_1} + \frac{\gamma_3}{\varepsilon_3} \right), \quad (9)$$

where subscripts 1 and 3 corresponds to the media surrounding the metal (subscript 2) and  $w$  is the metal layer thickness. The mode with a smaller propagation constant  $\beta$  is the fast mode usually characterized by lower absorption that decreases with decreasing metal layer thickness. However, in nonsymmetric structures it has a cutoff and cannot be excited at metal layer thicknesses below a certain threshold value. When permittivities of media 1 and 3 are significantly different, this mode cannot be excited at all. At the same time, the slow mode corresponding to the upper branch of the dispersion equation solution can be excited at any combination of permittivities and metal film thicknesses.

Figure 2 shows dependences of the ratio between the TE and TM components of plasmon modes on the metal layer thickness in quasi-symmetric TI–gold–TI structures. Here the TI is characterized by the parameters  $\varepsilon_1 = \varepsilon_3 = 25$ ,  $\bar{\alpha}_1 = 1/137$ , and different values of  $\bar{\alpha}_3$  at the wavelength  $\lambda_0 = 1000$  nm. An impor-

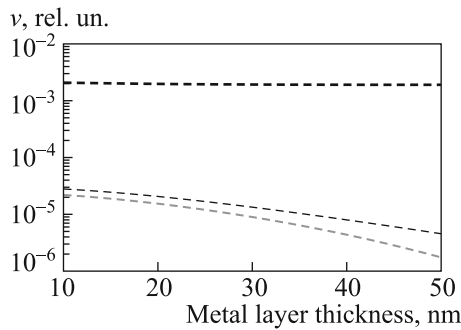


**Fig. 2.** Dependence of the TE/TM ratio at the boundary of medium 1 with  $\bar{\alpha}_1 = 1/137$  on the metal layer thickness in the quasi-symmetric structure with  $\bar{\alpha}_3 = 1/137$  (black bold curve),  $\bar{\alpha}_3 = -1/137$  (black thin curve), and at the boundary of medium 3 with  $\bar{\alpha}_3 = 0$  (gray bold curve) and at the boundary of medium 3 with  $\bar{\alpha}_3 = 0$  (gray thin curve). Solid and dashed curves correspond to the fast and slow modes, respectively.

tant point is the difference between the axion change in the polarization of the surface plasmon polariton shown in this work and the analogous change caused by the gyrotropic properties of the dielectric [6]. While in the latter case the tilting angle of the polarization ellipse substantially depended on the electromagnetic field distribution in the surface wave and increased with fraction of the electromagnetic field energy for the gyrotropic dielectric, in the case of the axion effect the situation is reverse. In symmetric structures with the axion effect parameter  $\nu$  does not change with decreasing metal layer thickness either for the fast or for the slow mode (solid and dashed black bold lines, respectively).

The character of the dependence of the plasmon mode polarization on  $\bar{\alpha}$  in three-layer structures can be explained as follows. At large metal layer thicknesses, the TE/TM component ratio corresponds to the value achieved at the corresponding interface of two semi-infinite media. For the completely symmetric structure (black bold curve in Fig. 2) both metal–TI interfaces turn out to be equivalent and make an identical contribution to the rotation of the polarization plane, which does not depend on the thickness of the metal layer because the TE/TM component ratio does not change inside the metal. In other cases, where axion angles for two interfaces are different, polarization of plasmons at the solitary interfaces of the metal and both media are also different, and thus the effect of the second interface becomes noticeable as the metal layer thickness decreases.

Note that dispersion curves for different  $\bar{\alpha}_3$  are equidistant. This is, as pointed out above, because



**Fig. 3.** Dependence of the TE/TM ratio on the metal layer thickness at the boundary of medium 1 with  $\varepsilon_1 = 1$  (black bold curve),  $\varepsilon_1 = 25$  (black thin curve), and  $\varepsilon_1 = 27$  (gray curve) in the structure comprising the TI with  $\varepsilon_3 = 25$  and  $\bar{\alpha}_3 = 1/137$ .

the contribution to the polarization rotation from each interface of the media both with and without the axion effect are summed.

In structures in which  $\bar{\alpha}_3 \neq \bar{\alpha}_1$  (black thin and grey bold curves), enhancement of polarization effects with decreasing metal thickness is observed for slow modes. In addition, when the metal layer thickness is appreciably large, the tilt of the polarization ellipse is identical for both the fast and the slow mode despite the asymmetry of the structure and the resulting asymmetric redistribution of the electromagnetic field of the plasmon. This is because the tilt of the polarization ellipse is directly caused by the interface of the axion and nonaxion media, and the nonzero tilt of the plasmon polarization is therefore observed near the interface of media 1 and 2 with a different axion angle even at large thickness of the metal layer.

In nonaxion dielectric 3 (gray thin curve), in which TE components are only due to the TI on the other side of the metal, these components decrease in value with increasing metal layer thickness.

Enhancement of polarization effects for slow modes (black thin and gray bold curves in Fig. 2) allows polarization effects to be observed in nonsymmetric structures. Figure 3 shows  $\nu$  at the boundary of medium 1 in the following structures: air/gold/TI ( $\varepsilon_3 = 25$ ,  $\bar{\alpha}_1 = 1/137$ ), gray curve; dielectric ( $\varepsilon_1 = 25$ )/gold/TI, black thin curve; and dielectric ( $\varepsilon_1 = 27$ )/gold/TI, gray curve. The appreciably nonsymmetric structure with the air allows two orders of magnitude higher polarization effect to be obtained, and, in addition, it is simpler to be experimentally implemented.

Thus, we investigated the axion effect in topological insulators on the properties of the surface plasmon polaritons excited at the boundary with metals in two- and three-layer structures. A change in the polarization of the plasmon polaritons through appearance of

TE components proportional to the axion angle is revealed. Various structure versions are considered, and enhancement of polarization effects by several orders of magnitude is shown to be possible in nonsymmetric plasmon structures in which the second dielectric is a material with a low refractive index, e.g., air.

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## REFERENCES

1. S.A. Maier, *Plasmonics: Fundamentals and Applications* (Springer Science + Business Media LLC, 2007).
2. J. Homola, "Surface Plasmon Resonance Sensors for Detection of Chemical and Biological Species," *Chem. Rev.* **108**, 462 (2008).
3. K.M. Mayer and J.H. Hafner, "Localized Surface Plasmon Resonance Sensors," *Chem. Rev.* **111**, 3828 (2011).
4. V.V. Temnov, G. Armelles, U. Woggon, D. Guzatov, A. Cebollada, A. Garcia-Martin, J.-M. Garcia-Martin, T. Thomay, A. Leitenstorfer, and R. Bratschkitsch, "Active Magneto-Plasmonics in Hybrid Metal-Ferromagnet Structures," *Nature Photon.* **4**, 107 (2010).
5. V. Belotelov, D. Bykov, L. Doskolovich, A. Kalish, and A. Zvezdin, "Giant Transversal Kerr Effect in Magneto-Plasmonic Heterostructures: The Scattering-Matrix Method," *JETP.* **137**, 932 (2010).
6. A. Kalish, D. Ignatyeva, M. Bayer, V. Belotelov, L. Kreilkamp, and A. Sukhorukov, "Transformation of Mode Polarization in Gyrotropic Plasmonic Waveguides," *Laser Phys.* **24**, 094006 (2014).
7. A.P. Sukhorukov, D.O. Ignatyeva, and A.N. Kalish, "Terahertz and Infrared Surface Wave Beams and Pulses on Gyrotropic, Nonlinear and Metamaterial Interfaces," *J. Infrared, Millimeter, and Terahertz Waves.* **32**, 1223 (2011).
8. D.O. Ignatyeva, A.N. Kalish, G.Y. Levkina, and A.P. Sukhorukov, "Surface Plasmon Polaritons at Gyrotropic Interfaces," *Phys. Rev. A.* **85**, 043804 (2012).
9. G. Mi and V. Van, "Characteristics of Surface Plasmon Polaritons at a Chiral-Metal Interface," *Opt. Lett.* **39**, 2028 (2014).
10. M.Z. Hasan and C.L. Kane, "Colloquium: Topological Insulators," *Rev. Mod. Phys.* **82**, 3045 (2010).
11. F. Wilczek, "Two Applications of Axion Electrodynamics," *Phys. Rev. Lett.* **58**, 1799 (1987).
12. R. Li, J. Wang, X.-L. Qi, and S.-C. Zhang, "Dynamical Axion Field in Topological Magnetic Insulators," *Nature Phys.* **6**, 284 (2010).

13. W.-K. Tse and A. MacDonald, "Giant Magneto-Optical Kerr Effect and Universal Faraday Effect in Thin-Film Topological Insulators," *Phys. Rev. Lett.* **105**, 057401 (2010).
14. M.-C. Chang and M.-F. Yang, "Optical Signature of Topological Insulators," *Phys. Rev. B.* **80**, 113304 (2009).
15. F. Liu, J. Xu, and Y. Yang, "Polarization Conversion of Reflected Electromagnetic Wave from Topological Insulator," *JOSA B.* **31**, 735 (2014).
16. J. Qi, H. Liu, and X. Xie, "Surface Plasmon Polaritons in Topological Insulators," *Phys. Rev. B.* **89**, 155420 (2014).
17. A. Karch, "Surface Plasmons and Topological Insulators," *Phys. Rev. B.* **83**, 245432 (2011).
18. P. Di Pietro, M. Ortolani, O. Limaj, A. Di Gaspare, V. Giliberti, F. Giorgianni, M. Brahlek, N. Bansal, N. Koirala, S. Oh, P. Calvani and S. Lupi, "Observation of Dirac Plasmons in a Topological Insulator," *Nature Nanotech.* **8**, 556 (2013).
19. D.K. Efimkin, Y.E. Lozovik, and A.A. Sokolik, "Collective Excitations on a Surface of Topological Insulator," *Nanoscale Res. Lett.* **7**, 1 (2012).
20. Y.-P. Lai, I.-T. Lin, K.-H. Wu, and J.-M. Liu, "Plasmonics in Topological Insulators," *Nanomater. Nanotech.* **4**(13), 1 (2014).