Any three-dimensional bifurcation of an integrable Hamiltonian system is a neighborhood of the critical level of the certain billiard's book

Irina Kharcheva[1]

[1] *Lomonosov Moscow State University, Moscow, Russia*

Let us consider a standard billiard problem in some fixed domain. A particle moves in a straight line within the domain. When the particle hits the boundary it reflects from it without velocity loss. This dynamic system contains the first integral – the scalar square of the velocity vector. In some special cases such a system has an another integral.

One of these cases is a billiard in the domain bounded by confocal quadrics. The second special integral can be described by the following feature of trajectories: the straight lines containing the segments of the polygonal billiard trajectory are tangents to a certain quadric (ellipse or hyperbola).

This dynamical system has 4-dimensional phase space and two integrals. One of them is a Hamiltonian. Integrable Hamiltonian systems with 2 degrees of freedom have Fomenko-Zieschang invariants[1]. Such invariants allow us to speak about the equivalence between closures of trajectories – Liouville equivalence. Up to Liouville equivalence, such systems have been studied in details in [2, 3] by V.Dragovich, M.Radnovich, and in [4, 5] by V.V.Fokicheva.

A billiard's book is a generalization of these billiards. Such type of billiards formed by gluing a few classical billiard domains along pieces of their boundaries. The special case where we glue two domains called a topological billiard and was researched by V.V. Fokicheva[6].

Researching billiard's books we try model famous integrable systems in terms of Fomenko-Zieschang invariants. The Fomenko conjecture about modeling Fomenko-Zieschang invariants using billiard's books and new results that confirm the part of the conjecture will be presented.

The author was supported by the Russian Foundation for Basic Research (grant No. 16-01-00378-a) and the program "Leading Scientific Schools" (grant no. NSh-6399.2018.1).

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