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Installation diagram of the lattice truss with an arbitrary number of panels

Монтажная схема решетчатой фермы с произвольным числом панелей

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Ключевые слова: ферма; прогиб; индукция; Maple; точная формула; реберная раскраска; гиперграф

Abstract. Hereby the diagram of a flat statically determinate regular lattice truss with the parallel chords is proposed. The task is to obtain an analytical dependence of the truss deflection and forces in the most tension and compressed bars on the number of panels. In order to solve the problem, the Maple computing mathematical system is used. We have considered the case when the lower truss chord is subject to a uniform load. The forces are determined by the Method of Joint. The Maxwell-Mohr formula is used to determine the deflection. The solution obtained for a set of cases with different successively increasing numbers of panels is generalized to a random number of panels by method of induction. The special operators of the Maple system are used to prepare homogeneous linear recurrence relations that are satisfied with the sequences of coefficients in the required formula. It is shown that for the number of panels in the half-span that are divisible to three, the determinant of the equilibrium equation system is becoming zero. The truss is becoming kinematically changeable that is confirmed by the corresponding diagram of possible joint velocity. The algorithm for the truss installation diagram is described, where the cross bars are in different planes and are connected in the nodes so that the truss elements are not subjected to buckling. The solution of this problem is related to the correct edge coloring of graphs and hypergraphs.

Аннотация. Предлагается схема плоской статически определимой регулярной балочной фермы с параллельными поясами. Ставится задача получения аналитической зависимости прогиба фермы и усилий в наиболее растянутых и сжатых стержнях от числа панелей. Для решения задачи привлекается система компьютерной математики Maple. Рассмотрен случай равномерного загружения узлов нижнего пояса фермы. Усилия определяются методом вырезания узлов. Для нахождения прогиба используется формула Максвелла - Мора. Решение, полученное для серии задач с разным последовательно возрастающим числом панелей, обобщается методом индукции на произвольное число панелей. Используются специальные операторы системы Maple для составления однородных линейных рекуррентных уравнений, которым удовлетворяют последовательности коэффициентов в искомой формуле. Показано, что для чисел панелей в половине пролета, кратных трем, определитель системы уравнений равновесия обращается в ноль. Ферма становится кинематически изменяемой, что подтверждается соответствующей схемой возможных скоростей узлов. Описан алгоритм составления монтажной схемы фермы, при которой пересекающиеся стержни лежат в разных плоскостях и соединяются в узлах так, что элементы фермы не подвергаются изгибу. Решение этой задачи связывается с правильной реберной раскраской графов и гиперграфов.

1. Introduction

The lattice trusses are widely used in civil engineering and mechanical engineering. Well-developed numerical algorithms, mainly based on the finite element method, make it possible to easily and accurately calculate a sufficiently wide range of such structures. Development of the modern computing mathematical systems offers new opportunities for solving the problems of the frame structures that is obtaining of accurate solutions. The value of such solutions is determined by the degree of versatility of the design formulas. In particular, it is quite easy to find an analytical expression for the deflection or force in any truss

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bar, depending on the truss dimensions and the load, if a specific number of panels is provided. For this purpose, it is sufficient to solve the system of equilibrium equations and all the design equations in symbolic form. It is much more difficult to obtain the dependence of the solution on the number of panels or bars that make up the structure. In [1–3], the induction method is applied in the similar case that substantially extend the scope of application of the final formulas. The resulting symbolic estimates of deflection, support reactions and forces in the bars are free from errors naturally accumulated by the numerical methods, especially when a large number of panels is taken into account. The inductive method can be used for the trusses that have the regularity property [4, 5]. The same method is used in the present paper for the statically determinate truss (Figure 1). During the research process, a case of kinematic degeneration of the truss has been found and an algorithm for designing the installation diagram has been proposed. The formulas for calculating the deflection of building structures are of practical importance for the design engineers when developing the new diagrams and improving the standard diagrams of frame structures. Earlier deflection values in analytical form using the induction method in the Maple system were found in flat trusses [2,3, 6–9] and in spatial ones [1,10–14].

2. Methods

2.1. The truss diagram and the equations to find the forces

The truss diagram shows pin-connected bars. The bars are assumed to be elastic with the same elasticity modulus, the cross-sections of the rods are assumed to be the same. The truss containing n number of panels in the half-span has $4n+6$ pin connections and $m = 8n + 12$ bars, including three support bars. The truss is statically determinate.

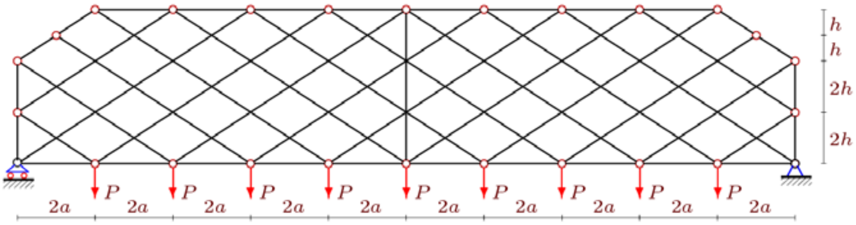

Figure 1. Truss. *n* = 5

The uniform load is applied to the nodes of the lower chord. The Method of Joint is used for determination of forces in the bars with the support of a program compiled in the Maple system [1]. The truss bars and nodes are numbered (Figure 2), the coordinates of the joints and the graph for connection of the bars (edges) and nodes (apex) are provided.

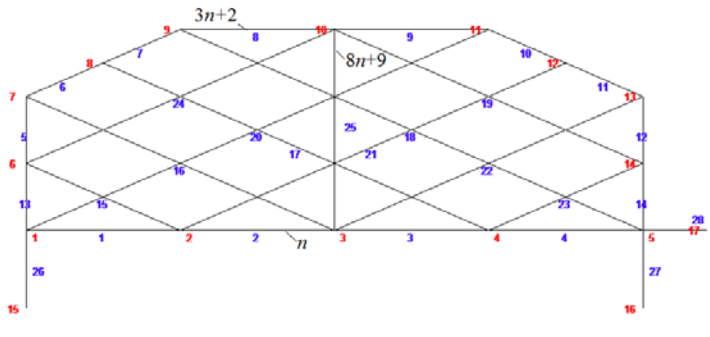

Figure 2. Truss. Numbering of nodes and bars, *n* = 2

The graph of the lattice is represented by conditional vectors $\bar{N}_i$, $i = 1,...,m$ containing the numbers of pin connections at their ends. For example, both chords and side structures are coded with the following vectors:

$$\bar{N}_i = [i, i+1],\ i = 1,..,2n,$$
$$\bar{N}_{i+2n} = [i+2n+1, i+2n+2],\ i = 1,..,2n+4,$$
$$\bar{N}_{4n+5} = [1, 2n+2], \bar{N}_{4n+6} = [2n+1, 4n+6].$$

The matrix of nodes equilibrium equations system $\mathbf{G}$ with the dimensions $m \times m$ consists of the directional cosines of the bars, when projected onto the coordinate axis. The odd matrix rows are projected onto the x-axis, even rows onto the y-axis.

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The directional cosines are calculated using the bars lengths and their vector projections onto the coordinate axes:

$$l_i = \sqrt{l_{1,i}^{\ 2} + l_{2,i}^{\ 2}}, \ l_{1,i} = x_{N_{2,i}} - x_{N_{1,i}}, l_{2,i} = y_{N_{2,i}} - y_{N_{1,i}}, \ \ i = 1,...,m.$$

In the number $N_{i,j}$, the first index i is the bar number, the second one is the number of the vector component $\bar{N}_i$, that takes the value 1 (the beginning of the bar vector) or 2 (the end of the bar). The matrix of the directional cosines $\mathbf{G}$ has the following elements:

$$G_{k,i} = -l_{j,i} / l_i, k = 2N_{i,2} - 2 + j, \ \ k \le m, \ j = 1,2, \ i = 1,...,m,$$
$$G_{k,i} = l_{j,i} / l_i, \ \ k = 2N_{i,1} - 2 + j, \ \ \ k \le m, \ \ j = 1,2, \ i = 1,...,m.$$

The forces in the truss bars are determined based on the solution of the system of linear equations $\mathbf{G}\bar{S} = \bar{B}$, where $\bar{S}$ is the vector of unknown forces in the bars, $\bar{B}$ is the vector of external loads.

During calculation of the forces it has been found that the diagram under examination has a hidden and sufficiently dangerous defect. It turns out that for the trusses with a number of panels divisible by three, the determinant of the equation system matrix is becoming equal to zero. With such n values the truss it turned into the instantly changeable mechanism. To confirm this fact, we have obtained the scheme of possible joint velocities for n = 3 (Figure3). It is obvious that the bars 2-8 and 6-18 are rotating around the supports, the rods 12-13, 13-14, 2-3 and 5-6 are rotating around the rigid pin connections. The bars 8-12 and 14-18 make a plane movement. Having considered the rotational motion of the bars 2-8 and 6-18, we obtain that the joint velocities are related to each other: $\omega = v/(2a) = u/(2h)$. The same relation is resulted from the analysis of the plane motion of the bars 8-12 and 14-18 around the instantaneous velocity centers M_1 and M_2.

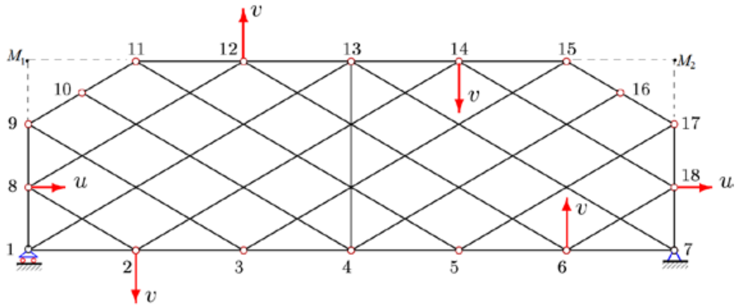

Figure 3. Diagram of possible joint velocities of variable truss at *n* = 3

In order to reveal the regularity of the coefficient formation in the desired formulas for deflection and forces in the typical rods, it is necessary to exclude from the solution sequence the trusses with the number of panels, such as n = 3i, i = 1,2,3, ... To meet these requirements, a sequence with common term is taken

$$n = (6k - 3 - (-1)^k)/4, \ k = 1,2,3,... \tag{1}$$

The elements of this sequence take all natural values except for those divisible by three.

2.2. Deflection

In order to determine the deflection of the central joint of the lower chord (the vertical hinge displacement with the number n + 1) we use the Maxwell-Mohr formula

$$\Delta = \sum_{i=1}^{m-3} \frac{S_i s_i l_i}{EF},$$

where S_i, s_i – the forces in the i truss bar from the applied distributed load and from the single vertical force in the central node of the lower chord with the number n + 1, respectively. Summation is carried out over all deformable rods of the truss. Three support rods are assumed to be rigid and are not included into this sum.

The successive solution of truss deflection equation when k, indicating the number of panels in the half-span, is variable gives every time the same equation regardless of k value

$$\Delta = P(A_k a^3 + C_k c^3 + H_k h^3)/(EFh^2),$$

where $c=\sqrt{a^2+h^2}$. The coefficient values depend on the value of k that determines the number of panels n in the half-span by formula (1). In order to find the common term A_k of the coefficient sequence at a^3, the recurrence equation of the eleventh order, (k > 2), was found using the **rgf_findrecur** operator of the **genfunc** package of rational generating functions of the Maple system.

$$A_k = A_{k-1} + 3A_{k-2} - 3A_{k-3} - 2A_{k-4} + 2A_{k-5} - 2A_{k-6} + 2A_{k-7} + 3A_{k-8} - 3A_{k-9} - A_{k-10} + A_{k-11}.$$

Solution of this equation using the **rsolve** operator provides the general term of the sequence

$$A_k = (30k^4 - 20(\cos 2\varphi + 3)k^3 + 6(5\cos 2\varphi - 29)k^2 + 4(51 - 67\cos 2\varphi)k - \\ -144\cos\varphi + 129\cos 2\varphi - 144\sin\varphi + 79)/64,$$

where $\varphi = \pi k/2$ is given. Similarly, on the basis of the solution of homogeneous equations of the seventh order

$$C_k = C_{k-1} + C_{k-2} - C_{k-3} + C_{k-4} - C_{k-5} - C_{k-6} + C_{k-7},$$

$$H_k = H_{k-1} - H_{k-2} + H_{k-3} + H_{k-4} - H_{k-5} + H_{k-5} - H_{k-7}$$

we can obtain other coefficients of the deflection formula:

$$C_k = (-30k^2 + 6(5 - 9\cos 2\varphi)k + 27\cos 2\varphi - 12\cos\varphi - 12\sin\varphi + 33)/16,$$
$$H_k = (2(5 - 3\sin\varphi - 3\cos\varphi)k - 12\cos\varphi + 18\sin\varphi - 39\cos 2\varphi - 5)/4.$$

2.3. Forces in the critical bars

Simultaneously with derivation of the formula for deflection, it is possible to obtain the form of forces in the most compressed and tension bars, depending on the number of panels. These formulas are required to evaluate the structure stiffness and the stability of its bars. The sequences of analyzed solutions, which operator **rgf_findrecur** uses to determine regularity, turns out to be shorter, and the recurrence relations are simpler. Having assumed that the most compressed bar under such a load is located in the middle part of the upper chord (the rod with the number 3n + 2, Figure 2), we obtain the following expression:

$$S_{3n+2} = -P(a/h)(6k^2 + 2(6\cos\varphi - 2\sin\varphi - \cos 2\varphi - 3)k + \cos 2\varphi - 4\sin\varphi + 8\cos\varphi + 7)/16.$$

Similarly, the most tension bar appears to have the value of n in the middle of the lower chord. The force is also determined by the induction method

$$S_n = P(a/h)(6k^2 + 2(2\sin\varphi - 6\cos\varphi - \cos 2\varphi - 3)k + \cos 2\varphi + 4\sin\varphi - 8\cos\varphi - 17)/16.$$

For comparison, we also obtain formulas for the forces in the bars adjacent to the assumed critical bars:

$$S_{n-1} = P(a/h)(6k^2 - 2(2\sin\varphi + 2\cos\varphi + \cos 2\varphi + 3)k + \cos 2\varphi + 12\sin\varphi - 8\cos\varphi - 17)/16,$$

$$S_{3n+1} = -P(a/h)(6k^2 + 2(2\cos\varphi + 2\sin\varphi - \cos 2\varphi - 3)k + \cos 2\varphi - 12\sin\varphi + 8\cos\varphi + 7)/16,$$

and force in the vertical stand in the middle of the span:

$$S_{8n+9} = P(3\sin\varphi - 2\cos\varphi - k(\cos\varphi + \sin\varphi))/2.$$

2.4. Installation diagram

When assembling a structure, it is necessary to ensure that elements are connected is such a way that the bars are placed in parallel to avoid buckling due to the arrangement of the ends in different planes [15–16]. In discrete mathematics (theory of graph coloring) there is the problem of the graph edge coloring when the graph edges correspond to the natural numbers (colors) so that edges of different colors are incident to one graph vertex (truss joint) [17–19]. In the case of the truss assembly, colors imply the conditional plane level in which the rod is mounted.

In order to solve the installation case, it is possible to apply the special operator **EdgeChromaticNumber** from the GraphTheory package of the Maple system. However, such a solution is suitable only in the cases where the graph of the truss lattice is flat, that is, it does not contain intersectional braces, as in the truss under examination. In order to avoid buckling, cross bars must be in different planes. The following algorithm for the installation diagram is proposed. Information about the rods (graph edges) is provided to the algorithm input in the form of a list of edges with the end numbers. For the truss with n = 3 (Figure2), if is the following list R = { R_1, R_2, R_3 , {1,2}, {2,3}, {3,4}, {4,5}, {1,6}, {6, 7}, {7,8}, {8,9}, {9,10}, {10,11}, {11,12}, {12,13}, {13,14}, {14,5 }}. Three separate sets of intersectional rods are determined: ascending edges (upward-directed edges) $R_1 = \{6,10\}$, $\{1,11\}$, $\{2,12\}$, $\{3,13\}$, $\{4,14\}$, descending edges (downward-directed edges) $R_2 = \{2,6\}$, $\{3,7\}$, $\{4,8\}$, $\{5,9\}$, $\{10,14\}$ and a separate vertical middle rod $R_3 = \{3,10\}$ crossing the bars of both first sets. All these edges are placed at the beginning of the general list R. The task of the algorithm is to place the edges in the individual sets (levels) $U_i,\ i = 1,...,n_U$ so that there are no two edges having the same end numbers in every set. Initially, the sets are empty except for the three sets, in which the sets with the pre-reserved places are included $U_i = R_i,\ i = 1,2,3.$. The remaining elements of the list are placed in the sets U_i , based on the condition that the numbers of the edge ends are not repeated on one level. This is done in the cycles by the list elements $R_i,\ i = 4,...,n_R$ and levels $U_i,\ i = 1,...,n_U$. The result of the algorithm operation in relation to the truss n = 3 (Figure2) is as follows:

$$U_1 = \{\{1,\ 11\},\ \{2,\ 12\},\ \{3,\ 13\},\ \{4,\ 14\},\ \{6,\ 10\},\ \{7,\ 8\}\},$$
$$U_2 = \{\{2,\ 6\},\ \{3,\ 7\},\ \{4,\ 8\},\ \{5,\ 9\},\ \{10,\ 14\},\ \{11,\ 12\}\}$$
$$U_3 = \{\{1,\ 2\},\ \{3,\ 10\},\ \{4,\ 5\},\ \{6,\ 7\},\ \{8,\ 9\},\ \{12,\ 13\}\},$$
$$U_4 = \{\{1,\ 6\},\ \{2,\ 3\},\ \{9,\ 10\},\ \{13,\ 14\}\},$$
$$U_5 = \{\{3,\ 4\},\ \{5,\ 14\},\ \{10,\ 11\}\}.$$

Indeed, there are no two identical joints numbers on each level $U_i,\ i = 1,...,5$. The index of the obtained partition is equal to five (Figure4). It can be noted that this number is similar to the graph chromatic index, where the edge intersection is not restricted, and only the incidence of one vertex of edges with the same color is not allowed (the rods of the same level in this paper).

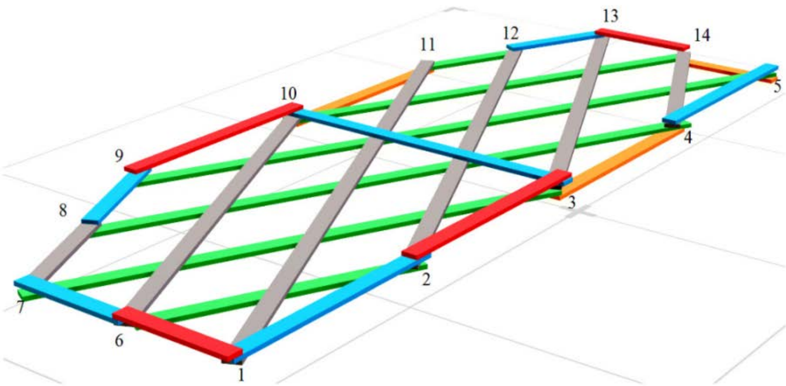

Figure 4. Installation diagram, five levels

Therefore, it is found that in some joints between the bars of different levels there are the gaps in height, requiring additional washers with a thickness equal to the thickness of the bar. It is obvious that the effectiveness of the proposed automatic way to design the installation diagram increases simultaneously with the number of panels, where it is almost impossible to assign the order of the joint assembly manually.

The proposed installation diagram is not the only one. In practice, the short bar elements are not always used in the trusses. For example, the lower chord at n = 2 can consist of one bar. In this case, the truss is relevant to the hypergraph that differs from the usual one by the several available vertices near one edge [20, 21]. The algorithm remains the same with the slight difference that the list of vertices of some edges includes more than two numbers:

$R = \{ R_1, R_2, R_3$, {1,2,3,4,5}, {1,6,7}, {7,8,9 }, {9,10,11}, {11,12,13}, {13,14,5}} .

Four sets of bars of the same level are formed at the program output (Figure 5):

U_1 = {{1, 11}, {2, 12}, {3, 13}, {4, 14}, {6, 10}, {7, 8, 9}},

U_2 = {{2, 6}, {3, 7}, {4, 8}, {5, 9}, {10, 14}, {11, 12, 13}},

U_3 = {{3, 10}, {1, 6, 7}, {5, 13, 14}},

U_4 = {{9, 10, 11}, {1, 2, 3, 4, 5}}.

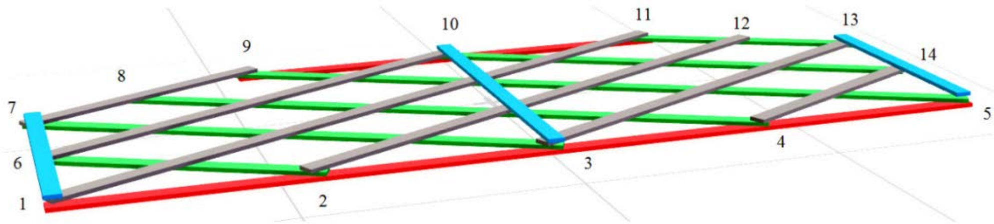

Figure 5. Installation diagram 2, four levels. Hypergraph of truss

3. Results and Discussion

Let us consider some concrete examples, from which the nature of the solution obtained and its features will be clearer.

The curves of the obtained dependence for the non-dimensional deflection $\Delta' = \Delta EF / (P_\Sigma L)$ at a given span length L = 4na = 100m and a fixed total load $P_\Sigma = P(2n-1)$ are plotted in Figure 6. The sharp jumps in the deflection value are typical, especially when a number of panels is small. Moreover, for k = 4 (or for n = 5, that is the same), the node under the load even rises. It means that it is impossible to evaluate the deflection of such structure based on displacement of only one middle node. The adjacent nodes can move in different directions. It is confirmed by the distribution of vertical displacements of the nodes in the lower truss chord at n = 5 (Figure 7).

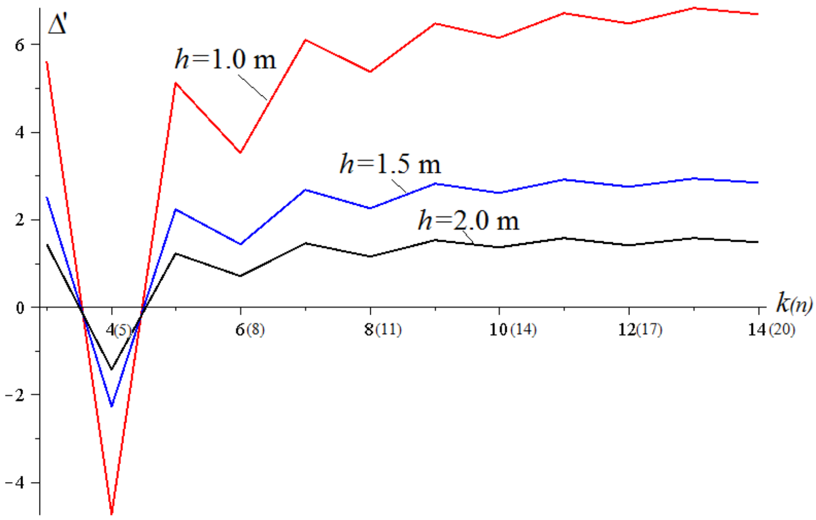

Figure 6. Displacement as a function of number of panels

The formulas for deflections Δ_j of joints j = 1,2,... 11 with due regard to the symmetry have the following form:

$$\Delta_2 = \Delta_{10} = P(57c^3 + 83a^3 + 22h^3)/(EFh^2),$$
$$\Delta_3 = \Delta_9 = -2P(12c^3 + 3a^3 - 4h^3)/(EFh^2),$$
$$\Delta_4 = \Delta_8 = P(69c^3 + 129a^3 + 22h^3)/(EFh^2),$$
$$\Delta_5 = \Delta_7 = 2P(9c^3 + 38a^3)/(EFh^2),$$
$$\Delta_6 = P(a^3 - 33c^3 - 10h^3)/(EFh^2).$$

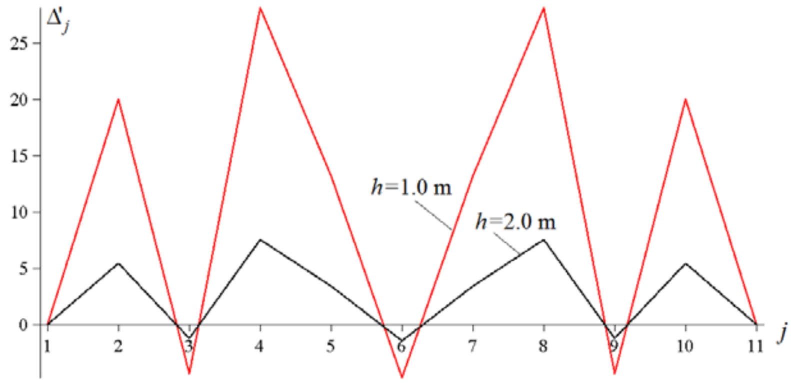

Figure 7. Displacements of the nodes in the lower truss chord. *L*=4*na*=100m, *n*=5, $\Delta'_j = EF\Delta_j/(PL)$

With the increase in the truss height and the value of *n*, the jump rate is decreased. The greatest differences in the values of deflections take place in the middle of the span.

The curves for the non-dimensional values $S'_i = S_i / P_\Sigma$ in Figure 8 show that alternation of the numbers of dangerous bars is possible, depending on the number of panels. Thus, if at k = 5 and k = 6, the bar with the value of *n* in the middle of the span is the most tension rod, then at k = 7 the greatest positive force acts on the adjacent bar *n* – 1. Similarly, the compressed rods in the middle of the upper chord are alternating and are calculated based on the condition of stability loss according to the Euler's formula.

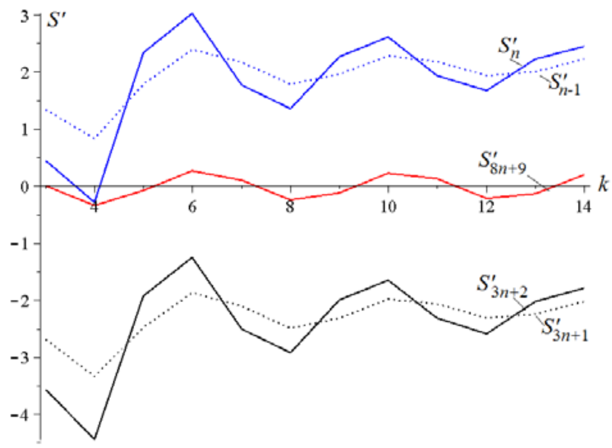

Figure 8. Non-dimensional values of forces in bars

A review of works on the application of the induction method and the computer mathematics system to problems of derivation of exact relations in flat statically determinate trusses is given in [22–24].

4. Conclusions

The obtained formulas for calculation of the deflection and forces in the critical bars when the number of panels are random make it possible to find the values actual for practice in a quite simple way, and what is more important – accuracy, devoid of errors accumulated with a large or very large number of panels and typical for numerical models. A special role here is also played by the induction method. It would seem that if it is necessary to obtain an exact analytical formula with geometric dimensions of the truss as the parameters, then it is simply enough to transfer the calculation program into a symbolic form, and the result is to be ready. However, this is not the case. The peculiarity of symbolic transformations does not allow this to happen even with a very small number of panels (40–50). The period of time for analytical transformations is much longer than for the numerical ones, and, most importantly, the frequently obtained formulas turn out to be unrealistically cumbersome and unsuitable for practice. In addition, the case of kinematic structural variability, revealed in the above example, can not be noticed in calculations (unless the determinant is followed) due to the rounding errors. The problem, solved in this paper, is related to the practice of using the lattice trusses. The truss installation with four panels in the span (Figures 4, 5) is given only as an illustration of the algorithm operation. Such a diagram could be worked out only by listing the possible options without the help of computer methods and concepts of discrete mathematics. However, for the trusses with a large number of panels, the application of the algorithm for automatically preparation of the installation diagram is becoming relevant. All the algorithms considered in this paper can be used in other regular frame structures.

Outside the study of the proposed design, there remained such an important question as the stability of the truss and its elements. The analytical expressions found for the effort make it possible to simply study this question. Regardless of this, the stability of the truss as an element of a spatial construction consisting of individual trusses with horizontal links should also be investigated. Moreover, additional constraints can distort the stress state of a plane model, for which analytical dependencies are obtained. The cases of the found kinematic variability will be valid for the truss and as part of the spatial construction, and the found formulas for the deflection will be an approximate estimate.

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