given mass flow), albeit with a definite scatter, whereas the curves corresponding to different mass flows and also to different spacings of the observation points for injection through a porous zone exhibit a distinct segregation.

The influence of liquid injection on the maximum of the transverse space−time correlation coefficient of the wall pressure fluctuations is illustrated by the results shown in Fig. 7. The correlation coefficient reaches a maximum $R_p$ for zero delay time. It is evident from an analysis of the graphical data that liquid injection ($v_n/U_\infty = 0.014$) produces a certain increase in the correlation at low frequencies; the values of the correlation coefficient in the high-frequency range practically coincide with those in the case of free flow over the surface. A reduction in the transverse correlation is observed over the entire investigated frequency range in the case of large injection ($v_n/U_\infty = 0.0225$).

Thus, large injection of a liquid in the normal direction through a permeable section of a surface in a flow produces a substantial reduction in the spectral power density and correlation of the wall pressure fluctuations.

A decrease in the intensity and correlation of the wall pressure fluctuations in a uniform turbulent boundary layer takes place, despite the increase in the thickness of the boundary layer and the intensity of the velocity fluctuations in the turbulent core of the flow during injection. The principal causes of the observed reductions appear to be the detachment of turbulence from the wall and the formation of a layer with a low velocity-fluctuation level in the wall zone, along with the suppression of correlations in the turbulent flow above the permeable wall.


[1] L. M. Lyamshev, "Acoustics of controlled boundary layer," in: Proc. Seventh Int. Congr. Acoustics, Vol. 2, Budapest (1971), p. 377.
[2] L. M. Lyamshev, Vestn. Akad. Nauk SSSR, No. 7, 22 (1973).
[3] A. G. Shustikov, Akust. Zh. 29, 693 (1983) [Sov. Phys. Acoust. 29, 409 (1983)].
[4] L. M. Lyamshev, B. I. Chelnokov, and A. G. Shustikov, Akust. Zh. 29, 806 (1983) [Sov. Phys. Acoust. 29, 477 (1983)].
[5] B. F. Kur'yanov and L. E. Medvedeva, Harmonic Analysis of Stationary Random Processes [in Russian], ONTI VTs MGU, Moscow (1970).
[6] W. K. Blake, J. Fluid Mech. 44, 637 (1970).


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# Application of the perturbation method for calculating the characteristics of surface waves in anisotropic and isotropic solids with curved boundaries


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A theory of the propagation of surface elastic waves in anisotropic solids with smooth curved boundaries is developed by means of the perturbation method. The general scheme of the method used is described. The propagation of Rayleigh waves in an isotropic medium is discussed as a case with degenerate anisotropy.


In the simplest formulation of the problem, Rayleigh-type surface waves propagate in an anisotropic solid without dispersion. In this formulation the problem is investigated in the linear approximation of the classical theory of elasticity; it is assumed that the surface waves have plane phase fronts, do not interact with other kinds of fields, and propagate in a homogeneous half-space with a plane free boundary in the absence of external disturbances. Departures from the described model can lead to dispersion and additional attenuation (leaky waves have attenuation in the original model). If the variations of the wave characteristics as a result of these departures are small, the additional attenuation and dispersion can be calculated by means of perturbation theory. The perturbation method described in Gurevich's monograph[1] are the most effective. This method has been used previously in the theory of surface elastic waves to account for the influence of perturbations in the volume of the medium and in the boundary conditions.[2] In the present article we apply the indicated method to a new category of problems, in which the perturbation is a variation in the shape of an anisotropic solid. This category of problems has become increasingly relevant in recent years in connection with the widespread application of crystals with curved boundaries in surface-wave acoustoelectronic devices. The curvature of the surface of the crystals makes it possible to design miniature long-delay lines for signal transmission,[3] topographic waveguides,[4] and geodesic lenses.[5] However, the theory of the propagation of Rayleigh surface waves, which are the principal type of surface waves used in acoustoelectronics, has been developed so far for smooth curved surfaces only in the case of isotropic materials[6–19] (an asymptotic theory for the case of the free surface of a solid has been developed in the cited papers). Yet even in this simplest case, the results of different authors are in disagreement, and not enough attention is given to the analysis of the derived expressions. For this reason we also discuss the case of an isotropic medium in the present study.

We give the general scheme of the method used here for arbitrary small perturbations in application to the problems of elastic wave propagation in anisotropic solid-state waveguides and unbounded solids. We assume that the perturbed problem is described by the set of equations comprising the equations of motion

$$\rho \dot{V}_i' = \partial_j' T_{ij}' + f_i^F, \tag{1}$$

the equations of state

$$T_{ij}' = c_{ijkl} S_{kl}' + f_{ij}^T, \tag{2}$$

the boundary conditions

$$T_{in}' + Z_{ik} V_k' = f_{ik}^Z V_k', \tag{3}$$

and the stress-strain relation

$$S_{ij}' = (\partial_i' U_j' + \partial_j' U_i')/2 + f_{ij}^S, \tag{4}$$

where $V_i$ is the particle velocity, $U_i$ is the mechanical displacement, $T_{ij}$ is the elastic stress, $S_{ij}'$ is the strain, $\rho$ is the density of the medium, $c_{ijkl}$ is the elasticity tensor (stiffness constants), $Z_{ik}$ is the mechanical impedance, $\partial_i \equiv \partial/\partial x_i$, and $x_i$ denotes the Cartesian coordinates. The indices i, j, k, $l$ take the values 1, 2, 3. The functions f are perturbations. We shall not specify their form at this stage. We assume that they are equal to zero in the unperturbed problem. The prime is used to denote the variables of the perturbed problem that do not coincide with the analogous variables of the unperturbed problem. Allowance for the interaction of elastic waves with other types of fields adds equations for the dynamics of those fields. We also use locally Cartesian coordinates $x_\tau$, $x_n$, $x_t$. The $x_\tau$ axis is in the same direction as the phase velocity of the wave. The $x_n$ axis is directed along the outward normal to the surface, and $x_n = 0$ on the surface. We shall specify the direction of the $x_t$ axis later. We write the wave vector in the form $\exp[-i(\omega t - k x_\tau)]$.

We perform the following operations on the basic equations. We multiply Eq. (1) by $V_i^*$ and multiply the analogous complex-conjugate equation of the unperturbed problem by $V_i'$. Summing these equations with allowance for the relation $\partial/\partial t = \pm i\omega$, we obtain

$$V_i^* \partial_j' T_{ij}' + V_i' \partial_j T_{ij}^* + f_i^F V_i^* = 0. \tag{5}$$

We assume the existence of points $x_i$, $x_i'$ at which $\partial_i = \partial_i'$. We carry out the subsequent transformations at these points. We write Eq. (5) in the form

$$\partial_j\{-T_{ij}^* V_i' - T_{ij}' V_i^*\} + i\omega[T_{ij}' \partial_j U_i^* - T_{ij}^* \partial_j U_i'] - f_i^F V_i^* = 0. \tag{6}$$

We integrate Eq. (6) over the cross-sectional area S of the waveguide and invoke the two-dimensional Gauss divergence theorem $\int_S \partial_j\{\ \}_j ds = \int_S \partial_\tau\{\ \}_\tau ds + \int_L \{\ \}_n dx_t$. Here $\{\ \}_j$ denotes the expression in the braces in Eq. (6), and L is the contour of the cross section S. Integration is not carried out in the one-dimensional case. We also use a relation that follows from Eqs. (2), (4) and the symmetry of $c_{ijkl}$, along with a relation that is valid when the time-average energy flux across the boundary of the unperturbed waveguide is equal to zero:

$$T_{ij}' \partial_j U_i^* - T_{ij}^* \partial_j U_i' = T_{ij}^* f_{ij}^S + S_{ij}^* f_{ij}^T, \quad \int_L \{\ \}_n dx_t = -\int_L f_{ik}^Z V_k' V_i^* dx_t.$$

The indicated transformations lead to the equation

$$\int_S \partial_\tau\{\ \}_\tau ds = -i\omega \int_S [f_{ij}^T S_{ij}^* + f_{ij}^S T_{ij}^* - f_i^F U_i^*] ds + \int_L f_{ik}^Z V_k' V_i^* dx_t = F.$$

We assume that the perturbed and unperturbed solutions are joined in this equation by the relation $\mathscr{A}'(x_\tau, x_n, x_t) \cdot \exp(ik'x_\tau) = a(x_\tau)[\mathscr{A}(x_n, x_t) + \Delta\mathscr{A}(x_n, x_t)]\exp(ikx_\tau)$, where $a(x_\tau)$ is a slowly varying amplitude, $\Delta\mathscr{A}$ is a term characterizing the change in structure of the wave, and $\mathscr{A}$ is the amplitude value of $T_{ij}$, $U_i$, or $V_i$.

We ultimately obtain the equation in $a(x_\tau)$

$$\frac{\partial a}{\partial x_\tau} - \frac{F^L a}{4P_W + \Delta p} = \frac{F^{NL}}{4P_W + \Delta p}, \tag{7}$$

$$P_W = (1/2)\,\mathrm{Re}\int_S \mathscr{P}_\tau ds, \quad \mathscr{P}_j = -T_{ij} V_i^*,$$

$$\Delta p = -\int_S (\Delta T_{i\tau} V_i^* + \Delta V_i T_{i\tau}^*) ds,$$

where $\mathrm{Re}\,\mathscr{P}_j$ is the time-average energy flux density, $F^L$ is the linear part of the function F with respect to the amplitude $a$, which is separated from it as a factor: $F^{NL} = F - aF^L$. We have the following expression for the perturbation $\Delta k$ of the wave number from Eq. (7):

$$\Delta k = k' - k^* = -iF^L/(4P_w + \Delta p), \tag{8}$$

$$a(x_\tau) = \exp(i\Delta k x_\tau).$$

In this equation we substitute the solution in the form of a power series in a small parameter characterizing the perturbation. Hereinafter we consider only situations in which the relative variation of the wave structure is of the same order of magnitude as the relative variation of the wave number. We then find that the perturbation $\Delta k$ can be calculated in the (N+1)-st order with respect to the small parameter in terms of the wave fields determined up to the N-th order with respect to this parameter. Equation (8) can therefore be used to calculate the wave number of the perturbed problem by iterations. In the first order with respect to the small parameter

$$\Delta k = -iF_1^L/4P_w, \tag{9}$$

and $F_1^L$ is determined in terms of the solution of the unperturbed problem. The variation of the wave structure is actually neglected in the calculation of $\Delta k$ according to Eq. (9).

We use the above-described algorithm to calculate the perturbation of the surface elastic wave number due to variation of the shape of an anisotropic solid. We assume that the solid has a plane surface in the unperturbed problem and the surface of a circular cylinder in the perturbed problem, and that the ratio of the wavelength to the radius R of the cylinder is a small parameter ($kR \gg 1$). In accordance with the choice of the small parameter we represent the elasticity equations describing the perturbed problem in a cylindrical coordinate system r, z, $\varphi$ in the form of Eqs. (1)-(4). The functions f and the coordinates $x_i'$ have the following form in this case (the indices i, j, k, $l$ take the values r, z, $\varphi$): $f_r^F = \varepsilon\partial_\varphi T_{r\varphi} + (T_{rr} - T_{\varphi\varphi})/r$, $f_\varphi^F = \varepsilon\partial_\varphi T_{\varphi\varphi} + 2T_{r\varphi}/r$, $f_z^F = \varepsilon\partial_\varphi T_{z\varphi} + T_{rz}/r$, $f_{\varphi\varphi}^S = \varepsilon\partial_\varphi U_\varphi + U_r/r$, $\varepsilon \equiv (r^{-1} - R^{-1})$, $2f_{\varphi z}^S = \varepsilon\partial_\varphi U_z$, $2f_{\varphi r}^S = \varepsilon\partial_\varphi U_r - U_\varphi/r$, $x_r' = r - R$, $x_z' = z$, $x_\varphi' = R\varphi$, and all other $f_{ik}^S = 0$. The perturbations $f_{ij}^T$, $f_{ik}^Z$ are absent in this case, and $Z_{ik} = 0$. The perturbation of the wave number can be determined from the known functions f by means of Eq. (9) in the first order with respect to the parameter 1/kR. We first consider two special cases: propagation along the generatrix of the cylinder and propagation perpendicular to the generatrix. We assume that the surface waves have plane phase fronts. Then the integrals over the cross section of the waveguide in the expressions for F and PW in Eq. (9) degenerate into integrals with respect to the depth (i.e., the coordinate $x_n$). For surface waves propagating along a convex cylinder, $x_n = x_r'$, $x_\tau = x_z'$, $x_t = x_\varphi'$, and the $x_\tau$, $x_n$ and $x_t$ axes form a right trihedral. The expression for the perturbation $\Delta k$ in this case has the form

$$(\Delta k/k)_t = (4P_w kR_t)^{-1}\int_{-\infty}^{0}(N_t + i\mathscr{P}_n)\,dx_n, \tag{10}$$

where $N_i = 2\omega\,\mathrm{Re}(U_i^* T_{in} - U_n^* T_{ii})$ and summation is not carried out over i. An analogous expression holds for propagation around a convex cylinder:

$$(\Delta k/k)_\tau = (4P_w kR_\tau)^{-1}\left\{\int_{-\infty}^{0}(N_\tau + i\mathscr{P}_n)\,dx_n + \int_{-\infty}^{0}\int_{-\infty}^{n}(2k\,\mathrm{Re}\,\mathscr{P}_\tau\,dx_n)\,dn\right\}. \tag{11}$$

In this case, $x_n = x_r'$, $x_\tau = x_\varphi'$, $x_t = x_z'$, and the $x_\tau$, $x_n$, and $x_t$ axes form a left trihedral. The double integral in Eq. (11) occurs in the computation of an integral of the form $\int x_n \mathscr{P}_\tau\,dx_n$ by parts. The indices $\tau$ and t attached to R and $\Delta k/k$ signify that the normal curvature of the surface with respect to the wave vector is longitudinal or transverse (from now on we refer simply to the longitudinal or transverse curvature). The quantities $T_{ij}$ and $U_i$, $V_i$ are determined from the solution of the problem with a plane boundary. The anisotropy of the solid and the orientation of its principal axes in Eqs. (10) and (11) are arbitrary under the condition that the curvature of the surface changes the wave structure only slightly. It is necessary to set $x_n = -x_r'$ for concave cylindrical surfaces, so that $\Delta k$ changes sign. In this case we can also make use of Eqs. (10) and (11) if the radii of curvature are considered to be negative for concave surfaces. The derived expressions are also valid for other types of surface waves with a change in the limits of integration. The results are readily generalized to the case of wave propagation over a smooth surface of arbitrary configuration as follows. The curvature of the smooth surface at any point is completely determined by the radii of normal curvature in two orthogonal directions. In the given problem one of them is naturally associated with the direction of the wave vector. The separate influence of the curvatures of the surface in each of these directions is determined by Eqs. (10) and (11). Their joint influence is determined simply by the sum of Eqs. (10) and (11) owing to the independence of the action of the small perturbations:

$$\Delta k/k = (\Delta k/k)_t + (\Delta k/k)_\tau = A_t/(kR_t) + A_\tau/(kR_\tau), \tag{12}$$

where $R_t$ and $R_\tau$ represent the local radii of curvature. Using this equation to calculate the local velocity of the surface waves, we can then determine their path by applying Fermat's principle. The validity of Eq. (12) for surface waves on a sphere has been confirmed in the derivation of the perturbation $\Delta k$ by the above-described method from the elasticity equations in a spherical coordinate system. The perturbation $\Delta k$ can also be determined in terms of the principal radii of curvature $R_1$, $R_2$ of the surface with the application of the Euler equation:

$$\Delta k = (A_\tau/R_1 + A_t/R_2)\cos^2\theta + (A_\tau/R_2 + A_t/R_1)\sin^2\theta, \tag{13}$$

where $\theta$ is the angle between the wave vector and the principal direction of the surface with the radius of curvature $R_1$. For a circular cylinder of radius R we have $R_1 = 0$, $R_2 = R$, and $\theta$ is the angle between the wave vector and the generatrix of the cylinder.

It is advisable to begin the analysis of the solutions with the simplest case of degenerate anisotropy, viz.: the case of an isotropic medium. The results for Rayleigh waves can be reduced to the following form after transformations using the dispersion relation:

$$A_t^i = \frac{(k^2 - qs)}{2k^3 qK}, \quad A_\tau^i = \frac{(k^2 - 4q^2)(q^2 - s^2)}{2ks^2q^3K}, \tag{14}$$

where

$$K = 1/s^2 + 1/q^2 + 2/k^2 - 8/(k^2 + s^2),$$

$$q^2 = k^2 - k_l^2, \quad s^2 = k^2 - k_t^2, \quad k_l^2 = \omega^2\rho/(\lambda + 2\mu), \quad k_t^2 = \omega^2\rho/\mu,$$

and $\lambda$, $\mu$ are the Lamé coefficients. The corresponding equations of Refs. 6–14 can be reduced to the same form, confirming the validity of the solutions obtained here. The expressions given in Refs. 15–19 yield results that do not concur with Eqs. (14) and that differ from one another. The functions $A_t^i$, $A_\tau^i$ depend only on the Poisson ratio $\nu$ and vary monotonically with $\nu$ between the following limits for real solids: $\nu = 0$: $v_R^2/v_t^2 = 3 - \sqrt{5}$, $A_t^i = \sqrt{10(\sqrt{5}+1)}/20 \approx 0.284$, $A_\tau^i = -(4 + \sqrt{5})A_t^i \approx 1.774$; for $\nu = 0.5$: $v_R^2/v_t^2 \equiv \eta = (2/3)(4 + \sqrt[3]{3\sqrt{33} - 17} - \sqrt[3]{3\sqrt{33} + 17})$, $A_t^i = (4-\eta)(1-\eta)(2-\eta)/(24\eta - 16) \approx 0.050$, $A_\tau^i = 3(2-\eta)/(6\eta - 4) \approx -2.211$; $v_R$ and $v_t$ are the velocities of Rayleigh and shear waves. It follows from these expressions that $A_\tau^i$, unlike $A_t^i$, depends weakly on the elastic properties of the medium (max $A_\tau^i$/min $A_\tau^i \approx 1.25$). An analysis of the values of $A_t^i$, $A_\tau^i$ and Eq. (12) leads to a number of qualitative conclusions.[6,7,19–21] We give some of them, which have not been noted previously in the literature. First, owing to the curvature of the surface, the nature of the dispersion depends strongly on how the surface is curved in relation to the wave vector. Specifically, whereas longitudinal curvature induces normal dispersion for convex surfaces, transverse curvature induces anomalous dispersion; the situation is reversed for concave surfaces. Second, the influence of longitudinal curvature is much stronger than that of transverse curvature for equal radii of curvature. This means that the influence of transverse curvature can be neglected in approximate calculations of the phase velocity of Rayleigh waves on spherical surfaces. Third, anisotropic curvature of the surface alters the diffraction spreading of surface waves. In particular, the diffraction spreading of Rayleigh waves at the rounded edges of delay lines in the form of plates with such edges is greater than on a flat surface. A number of special conclusions can be formulated for bodies of specific geometries. We note the special directions of propagation of Rayleigh waves over the surface of an isotropic circular cylinder. The phase velocity and diffraction spreading attain extrema in the directions $\theta = 0$, 90°, and the angle $\psi$ of deviation of the group velocity from the phase velocity becomes equal to zero:

$$\psi(\theta) = \mathrm{arctg}(v_R^{-1}\partial v_R/\partial\theta) \approx (A_\tau - A_t)\sin(2\theta)/(kR). \tag{15}$$

The nature of the dispersion is exactly opposite in these directions. The nature of the dispersion changes at $\theta_0 = \mathrm{arctg}\sqrt{-A_t^i/A_\tau^i} \approx 22°–28°$. Despite the fact that dispersion is absent for this direction, the angle $\psi(\theta_0)$ has a finite frequency-dependent value, $\psi(\theta_0) = -2\sqrt{-A_t'A_\tau'}/(kR) \approx (46°\nu - 82°)/(kR)$. The largest value of $\psi$ is attained at 45°, $\psi(45°) \approx (28°\nu - 102°\nu^2 - 118°)/(kR)$.

The analysis of the functions $A_t$ and $A_\tau$ in the anisotropic case presents a large-scale and difficult problem and is therefore not carried out within the scope of the present article. We merely note one difference associated with anisotropy, which is readily discerned without calculating the specific values of $A_t$ and $A_\tau$. According to Eqs. (12) and (15), even isotropic curvature of the boundary (case of a sphere) can alter the angle of deviation of the group velocity from the phase velocity in crystals.


[1]A. G. Gurevich, Ferrites at Microwave Frequencies [in Russian], Fizmatgiz, Moscow (1960), pp. 167-180.
[2]B. A. Auld, Acoustic Fields and Waves in Solids, Vol. 2, Interscience, New York (1973), pp. 271-332.
[3]E. A. Ash, "Fundamentals of signal processing devices," in: Surface Acoustic Waves, A. A. Oliner (ed.), Springer-Verlag, Berlin–New York (1978), p. 97.
[4]L. A. Coldren, Appl. Phys. Lett. 25, 367 (1974).
[5]T. Van Duzer, Proc. IEEE 58, 1230 (1970).
[6]I. A. Viktorov, Surface Acoustic Waves in Solids [in Russian], Nauka, Moscow (1981).
[7]I. A. Viktorov, Akust. Zh. 4, 131 (1958) [Sov. Phys. Acoust. 4, 131 (1958)].
[8]V. M. Babich, Dokl. Akad. Nauk SSSR 137, 1263 (1961).
[9]V. M. Babich and N. Ya. Rusakova, Zh. Vychisl. Mat. Mat. Fiz. 2, 652 (1962).
[10]N. Ya. Kirpichnikova, Zap. Nauchn. Sem. Leningr. Otd. Mat. Inst. Akad. Nauk SSSR 15, 91 (1969).
[11]R. D. Gregory, Proc. Cambridge Philos. Soc. 70, 103 (1971).
[12]L. O. Wilson and J. A. Morrison, J. Math. Phys. 16, 1795 (1975).
[13]J. A. Morrison, J. Math. Phys. 17, 958 (1976).
[14]J. A. Morrison, J. B. Seery, and L. O. Wilson, Bell System Tech. J. 56, 77 (1977).
[15]G. I. Petrashen', Uch. Zap. Leningr. Gos. Univ. Ser. Mat. Nauk 27, 96 (1953).
[16]B. Ya. Gel'chinskii, Uch. Zap. Leningr. Gos. Univ. Ser. Mat. Nauk No. 32, 322 (1958).
[17]S. Yu. Babich and A. N. Guz', Prikl. Mekh. 14, 3 (1978).
[18]V. V. Krylov, Vopr. Radioelektron. Ser. Obshch. Tekh. No. 11, 3 (1976).
[19]V. V. Krylov, Akust. Zh. 25, 754 (1979) [Sov. Phys. Acoust. 25, 425 (1979)].
[20]I. A. Molotkov, Zap. Nauch. Sem. Leningr. Otd. Mat. Inst. Akad. Nauk SSSR 17, 168 (1970).
[21]L. N. Komarova, "Wave propagation in a circular cylindrical shell," in: Proc. Sixth All-Union Acoustics Conf. [in Russian], Vol. 3, Akust. Inst. Akad. Nauk SSSR, Moscow (1968), Sec. L-O, M118.


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# Absorption of sound in transmission through a liquid layer


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The problem of sound transmission through a liquid layer is solved within the framework of the linearized Navier–Stokes and heat-transfer equations. It is shown that a strong absorption effect must exist under definite conditions, when the dissipated energy flux density becomes commensurate in magnitude with the acoustic energy flux density in the incident wave.


The absorption of acoustic energy is usually taken into account in the solution of problems of sound transmission through layered systems on the assumption that the equations of classical acoustics are valid with a complex wave number (for each case), the imaginary part of which corresponds to the ordinary coefficient of sound absorption in an unbounded medium (see Ref. 1, p. 18 Russ. ed.). It is well known, however, that the process cannot be described within the framework of classical acoustics in the acoustic layers near the interfaces of media consisting of immiscible liquids (the present paper is concerned strictly with liquids); in this case it is required to invoke the linearized equations of the mechanics of viscous heat-conducting liquids. Since the temperature and velocity gradients are anomalously high in the acoustic layers, the contribution from these layers to the dissipated energy can be decisive in a number of cases of sound transmission through a liquid body.[2,3] A quantitative criterion for comparison of the energies absorbed per unit time in an acoustic layer and in the remaining volume of the liquid body can be obtained from the theory of mechanical energy dissipation in liquids (see Ref. 4, p. 368). Specifically, if the following condition is satisfied:

$$l/\lambda \ll (c^2/\chi\omega)^{1/2}, \tag{1}$$

the absorption in the volume of the medium can be completely neglected in comparison with the absorption in the acoustic boundary layer. Here $l$ is the path traversed by the sound in the liquid body, $\lambda$ is the wavelength, and $\chi$ is the thermal diffusivity (allowance for viscosity produces a similar expression with the kinematic viscosity coefficient). Inasmuch as the right-hand side of Eq. (1) is almost always considerably greater than unity for ordinary gases and liquids, it can be inferred that the sound absorption even in a liquid body of comparatively large wave dimensions is associated mainly with the "nonclassical" behavior of the liquid in the acoustic layer.

It is clear from the foregoing considerations that the solutions of problems of sound absorption in layered systems[1] must be reexamined. In the present article we solve such a problem for the simplest case of sound transmission through one liquid layer.

Let an infinitely extended plane layer of one liquid (whose parameters are identified by the index 2) of thickness $l$ be situated in an unbounded different liquid (index 1); a plane sinusoidal sound wave is incident on the layer at an arbitrary angle. The behavior of the liquid in any of the three regions is described by the linearized hydro-