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Preliminary determination of Newtonian gravitational constant with angular acceleration feedback method

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This paper describes the preliminary measurement of the Newtonian gravitational constant G with the angular acceleration feedback method at HUST. The apparatus has been built, and preliminary measurement performed, to test all aspects of the experimental design, particularly the feedback function, which was recently discussed in detail by Quan *et al*. The experimental results show that the residual twist angle of the torsion pendulum at the signal frequency introduces 0.4 ppm to the value of G. The relative uncertainty of the angular acceleration of the turntable is approximately 100 ppm, which is mainly limited by the stability of the apparatus. Therefore, the experiment has been modified with three features: (i) the height of the apparatus is reduced almost by half, (ii) the aluminium shelves were replaced with shelves made from ultra-low expansion material and (iii) a perfect compensation of the laboratory-fixed gravitational background will be carried out. With these improvements, the angular acceleration is expected to be determined with an uncertainty of better than 10 ppm, and a reliable value of G with 20 ppm or below will be obtained in the near future.

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1. Introduction

The absolute value of the Newtonian gravitational constant G is one of the fundamental constants of nature. Until the CODATA Task Group announced the 2010 recommended value in 2012 [1], there had been six values of G with relative uncertainties of less than 50 ppm [2–8]. Even though the claimed uncertainties in these values are very small, unfortunately, the agreement is very poor. The difference between the smallest and the biggest value of G is about 480 ppm. In 2013, the latest value of G was given by BIPM (Bureau International des Poids et Mesures) with a relative uncertainty of 27 ppm [9], which is only 21 ppm lower than their 2001 result [3].

The current status of G measurements shows that the value of G may be dependent on the measuring method and the experimental site [2–14]. Therefore, we plan to use three different methods to measure the value of G simultaneously at the same laboratory. (i) The time-of-swing method. Our group has obtained three results with this method [6,7,14,15], and some improvements are made in the ongoing experiment. (ii) The atom interferometry method. The atom interferometry gravimeter was built in 2006 and achieved a high sensitivity [16–19]. (iii) The angular acceleration feedback method. This method was first used to measure G by Rose *et al.* [20] at the University of Virginia in 1969, and the value determined was $G = (6.674 \pm 0.012) \times 10^{-11} \, \mathrm{m^3 \, kg^{-1} \, s^{-2}}$ with a relative uncertainty of 1800 ppm. In 2000, Gundlach and Merkowitz at the University of Washington in Seattle measured the value of G with the same method, but made some improvements to overcome the large systematic errors in previous measurements [2,21–23]. They obtained the value of G as $(6.674255 \pm 0.000092) \times 10^{-11} \, \mathrm{m^3 \, kg^{-1} \, s^{-2}}$ with a relative uncertainty of 14 ppm, which is the most precise value to date. In Gundlach and Merkowitz's experiment, the aluminium shelves that support the source masses can easily be influenced by temperature fluctuation. This may affect the uncertainty of source mass distance measurement, which is the largest error during the experiment. Therefore, this method has been used to measure the value of G in our laboratory since 2008. The preliminary results and future work will be described in this paper in detail.

2. Principle of the angular acceleration feedback method

(a) Basic principle

A sketch of the angular acceleration feedback method is shown in figure 1. The torsion pendulum of the two-stage pendulum system that hangs from a magnetic damper is placed at the centre of a vacuum chamber, while the magnetic damper is used to reduce the tilt–twist coupling to the pendulum [24–26]. The chamber is installed on a continuously rotating inner turntable between a set of source masses. The equation of motion of the torsion pendulum in the rotating frame is

$$I\ddot{\theta} + \gamma\dot{\theta} + k\theta = \tau_\mathrm{g} - \tau_\mathrm{t}, \tag{2.1}$$

where θ is the torsion angle, I is the moment of inertia of the pendulum, γ is the damping coefficient, k is the spring constant of the torsion fibre, τ_g is the gravitational torque due to the source masses and τ_t is the inertia torque due to the rotating inner turntable.

At first, the pendulum experiences a periodic torque and the fibre twists due to the gravitational attraction. Subsequently, the feedback is turned on to change the rotation rate of the inner turntable, and the turntable is forced to follow the pendulum to minimize the deflection angle; then the torsion fibre ceases to twist. Therefore, the inertia torque equals the gravitational torque,

$$\tau_\mathrm{t} = \tau_\mathrm{g}, \tag{2.2}$$

where τ_t can be expressed as $I\alpha_\mathrm{t}$, with α_t being the angular acceleration of the turntable. We write τ_g as $GC_\mathrm{g} \sin(\omega_\mathrm{s} t)$, where ω_s is the signal frequency, and C_g is calculated from the geometry, metrology and mass distributions of the pendulum and source masses. Via Fourier transform, the
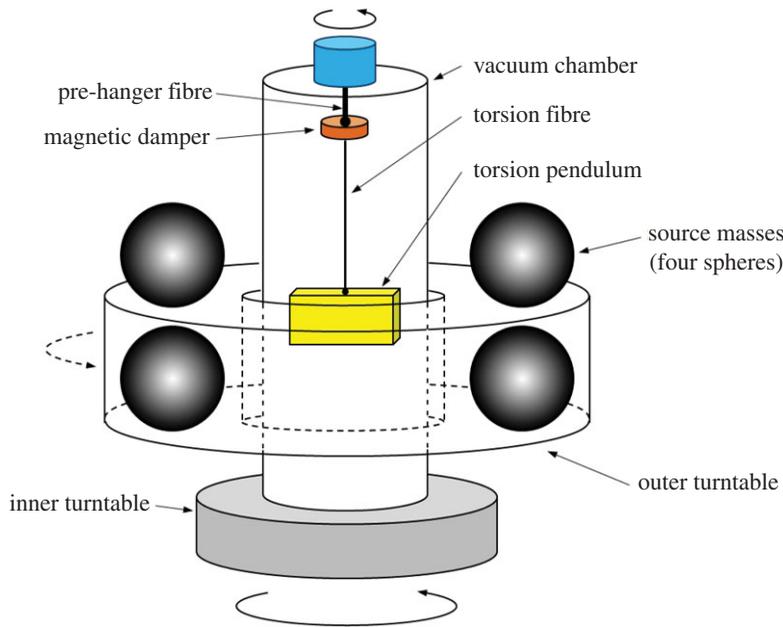
Figure 1. The principle of the angular acceleration feedback method. (Online version in colour.)

expression of G at the signal frequency ω_s is shown as

$$G = \frac{I}{C_g}|\alpha_t(\omega_s)|. \qquad (2.3)$$

By taking into consideration the residual twist angle $\theta(\omega_s)$ due to the finite gain of the feedback loop, the extra acceleration due to the thermal noise $\tau_{th}(\omega_s)$ and the effect of the magnetic damper, the model for G measurement becomes

$$G = \frac{I}{C_g}\left|\left(1 + \frac{k}{k_m}\frac{I_m}{I}\right)\alpha_t(\omega_s) - \frac{\tau_{th}(\omega_s)}{I} + \left[-\omega_s^2 + 2i\beta\omega_s + \omega_0^2\left(1 - \frac{k}{k_m}\right)\right]\theta(\omega_s)\right|, \qquad (2.4)$$

where $\beta = \gamma/2I < 10^{-6}$ in a 10^{-5} Pa vacuum condition [27], and $\omega_0^2 = k/I$ is the natural frequency of the pendulum. Here I_m and k_m represent the moment of inertia of the magnetic damper and the spring constant of the pre-hanger fibre, respectively.

(b) Experimental design

As described in [2,21,22,28,29], the gravitational torque can be expanded in spherical multipole moments and we obtain

$$\frac{C_g}{I} = -\frac{4\pi}{I}\sum_{l=0}^{\infty}\frac{1}{2l+1}\sum_{m=-l}^{+l}mq_{l,m}Q_{l,m}, \qquad (2.5)$$

where $q_{l,m}$ and $Q_{l,m}$ are the multipole moments of the pendulum and the multipole fields of the source masses, respectively,

$$q_{l,m} = \int \rho(r_t)r_t^l Y_{l,m}^*(\theta_t, \phi_t)\, d^3 r_t, \qquad (2.6)$$

and

$$Q_{l,m} = \int \rho(r_a)r_a^{-l-1}Y_{l,m}(\theta_a, \phi_a)\, d^3 r_a, \qquad (2.7)$$

where $\rho(r_t)$ and $\rho(r_a)$ are determined by the mass distributions of the torsion pendulum and the source masses, respectively.

The pendulum is designed as an up–down symmetric rectangular block and all of the $l =$ odd and $m =$ odd terms are made to vanish. The magnitudes of $l > 2$ higher-order terms in equation (2.5) converge as $(r_t/r_a)^{l-2}$, where r_t and r_a are the typical radius of the torsion pendulum and source masses. Therefore, the $l = m = 2$ term is the dominant one. However, the $l > 2$ and $m = 2$ terms still contribute to the signal of interest, $\sin(\omega_s t)$. So we choose the height h of the pendulum to be $h = \sqrt{3(w^2 + t^2)/10}$, where w and t are the width and the thickness of the pendulum, respectively. Then the $q_{4,2}$ moment can be designed to be zero. Four spheres are used as the source masses surrounding the pendulum, two of which are placed on the upper and lower layers, respectively, on each side, with the vertical distance z of the geometric centre (GC) given by $z = \sqrt{\frac{2}{3}}\lambda$, where λ is the distance between the torsion fibre and the centre of the sphere. So $Q_{4,2}$ is designed to vanish. The rest of the $l > 2$ and $m = 2$ terms, which cannot be eliminated by proper design, are small and easily calculable.

In order to eliminate accelerations caused by the laboratory-fixed objects, the source masses are located on another separate outer turntable that rotates at a different rate and independently from the inner turntable. The angular velocity of the outer turntable $\omega_a(t)$ is expressed as

$$\omega_a(t) = \omega_d + \omega_t(t), \tag{2.8}$$

where ω_d is the difference in angular velocity, which is kept constant, and $\omega_t(t)$ is the angular velocity of the inner turntable. The signal frequency ω_s is set as $2\omega_d$, and can be selected freely.

3. The apparatus

The preliminary apparatus is shown in figure 2, and a detailed description of the apparatus follows.

(a) Two-stage pendulum system

The torsion pendulum, which hangs from the magnetic damper by an $18\,\mu m$ diameter, $880\,mm$ long annealed tungsten fibre, is a gilded rectangular quartz block with mass of $40\,g$ and dimensions of $91.1 \times 50 \times 4\,mm^3$; thus the moment of inertia of the pendulum is calculated to be $I = 2.7 \times 10^{-5}\,kg\,m^2$. The magnetic damper is suspended by an $80\,\mu m$ diameter, $50\,mm$ long pre-hanger annealed tungsten wire for suppressing the unwanted modes of the pendulum. Before use in the formal experiment, the torsion fibre is loaded by a small block to about half of its breaking strength for several months to release the stress. Meanwhile, the fibre is heated for many hours to reduce the drift rate to less than $3\,\mu rad\,h^{-1}$ [7]. According to equation (2.4), the magnetic damper will lower the measured value of the angular acceleration $\alpha_t(\omega_s)$ and contribute an additional correction to the value of G. When substituting the typical parameters of the pendulum and the damper into kI_m/k_mI, this correction is determined to be $130\,ppm$.

The twist angle of the pendulum is read out by an autocollimator, which is fixed on the vacuum chamber. The angle is amplified by a factor of four without introducing extra noise due to four reflecting mirrors [22]. All of the pendulum system is located in the stainless-steel vacuum chamber at approximately $10^{-5}\,Pa$ maintained by two $20\,l\,s^{-1}$ ion pumps.

(b) Source masses

Four SS316 stainless-steel spheres are used as the source masses, whose average mass and diameter are $8.5\,kg$ and $127\,mm$, respectively. Each sphere rests on three stainless-steel seats, which are embedded in the double-layer aluminium shelves, and each layer supports two spheres. The horizontal distances and the vertical distances of the geometric centres are $342.3\,mm$ and $139.8\,mm$, respectively. The rotating gauge block method [31] is used to measure the horizontal surface separations of each pair of spheres, whereas the driving gauge method [32] is used to determine the vertical surface separations.
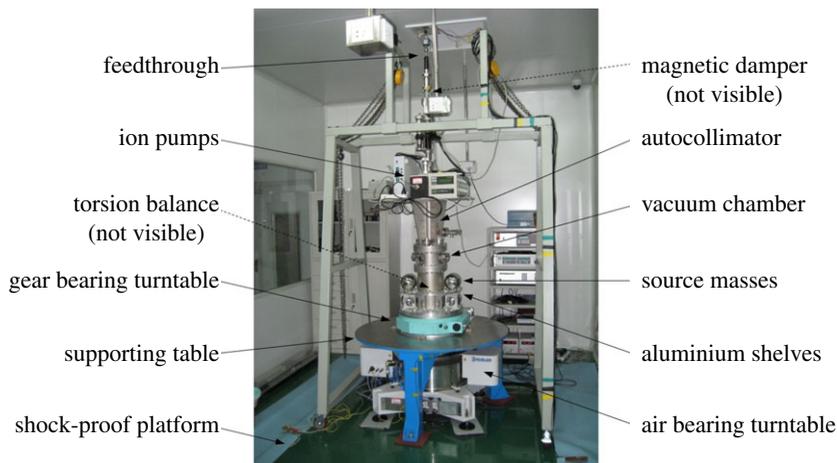
Figure 2. Photograph of the preliminary apparatus. The torsion balance is located in the vacuum chamber, which is installed on the air bearing turntable. The source masses are supported by another separate gear bearing turntable. The two turntables rotate in opposite directions (see [30]). (Online version in colour.)
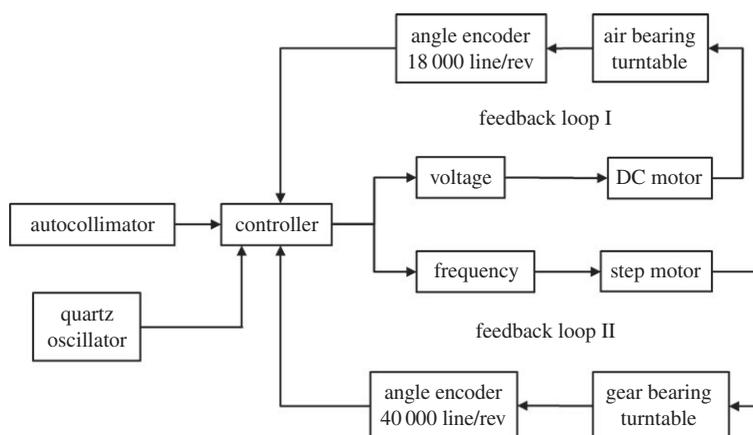
Figure 3. Feedback control system. Feedback loop I is used to control the air bearing turntable to keep the fibre untwisted. Feedback loop II is used to control the gear bearing turntable to hold the difference angular velocity constant (see [30]).

The claimed roundness of the spheres is about $7\,\mu\text{m}$, which introduces $60\,\text{ppm}$ to the value of G. Therefore, a specially fabricated device is used to grind and polish the spheres by hand with great care. Finally, the roundness of the spheres is improved to be $0.8\,\mu\text{m}$, which will only introduce $10\,\text{ppm}$ to the value of G. The offset of the centre of mass from the geometric centre of the spheres is measured with an air bearing [3,8,9], and the measured value is limited principally by the roundness of the spheres.

(c) Feedback control system

We use a feedback function containing proportion, integration and differentiation terms to control the experiment. Two feedback loops in figure 3 are, respectively, used to keep the torsion fibre untwisted and the difference angular velocity constant. Feedback loop I is the air bearing turntable control system. The pendulum in the vacuum chamber is located on the air bearing turntable (Kugler GmbH, RT600). This feedback is turned on due to the small twist angle sensed

Table 1. The designed parameters and expected one σ error budget.

error sources	designed value	uncertainty	$\delta G/G$ (ppm)
pendulum			
mass	40.000 g	10 mg	0.1
width	91.100 mm	3 μm	0.3
thickness	4.000 mm	1 μm	1.9
height	50.000 mm	10 μm	0.1
tilt	0	0.3 mrad	0.1
density inhomogeneity $\Delta\rho/\rho$	0	10^{-3}	0.1
magnetic damper			1.2
source masses			
mass ($\times 4$)	8.500000 kg	10 mg	0.6
horizontal distance of GC ($\times 2$)	342.300 mm	2 μm	9.4
vertical distance of GC ($\times 2$)	139.800 mm	2 μm	7.2
offset ($\times 4$)	0	0.8 μm	3.9
alignment			
tilt	0	0.1 mrad	1.2
radial	0	100 μm	1.1
height	0	1.0 mm	0.3
feedback			
time base	0	10^{-7}	0.1
residual twist angle		5.3×10^{-6} rad Hz$^{-1/2}$	2.0
turntable velocity		3.8×10^{-8} rad s^{-1} Hz$^{-1/2}$	2.0
thermal noise (3 days)			0.2
total			13.1

by the autocollimator to drive the DC motor to change the turntable angular velocity. The rotating angle is measured by an 18 000 line/rev angle encoder in the process. The turntable will follow the pendulum in order to keep the fibre untwisted. Feedback loop II is the gear bearing turntable control system. The gear bearing turntable (Huber Diffraktionstechnik GmbH & Co. KG, Goniometer 440) supports the sphere frame and is coaxial with the air bearing turntable. A 40 000 line/rev angle encoder is installed at the centre of the bearing to read out the rotating angle. The controller obtains the angles from the turntables and drives the step motor to make the outer turntable follow the inner turntable to keep the difference angular velocity constant. The gain of the feedback loop is defined as the total gravitational torque divided by the residual torque on the torsion fibre. For a 2 ppm target of the feedback control, the gain should be larger than 5×10^5, and the feedback control system in this experiment begins to become unstable when the value of the gain is larger than 1×10^8. In addition, a temperature-controlled quartz oscillator is used for timing in the feedback loop and the data are averaged for 1 s and uploaded to a computer. The detailed performance of the feedback control system has been described in Quan *et al.* [30].

The difference angular velocity ω_d is set to 6.28318 mrad s^{-1}, and the signal frequency $\omega_s = 2\omega_d$ is 2 mHz, which occurred at a frequency range with low $1/f$ noise. For most of the experiments, the inner turntable angular velocity $\omega_t(t)$ is set to ≈ 2.79 mrad s^{-1}, and the outer turntable angular velocity $\omega_a(t)$ is ≈ -3.49 mrad s^{-1}, where minus means the opposite direction.
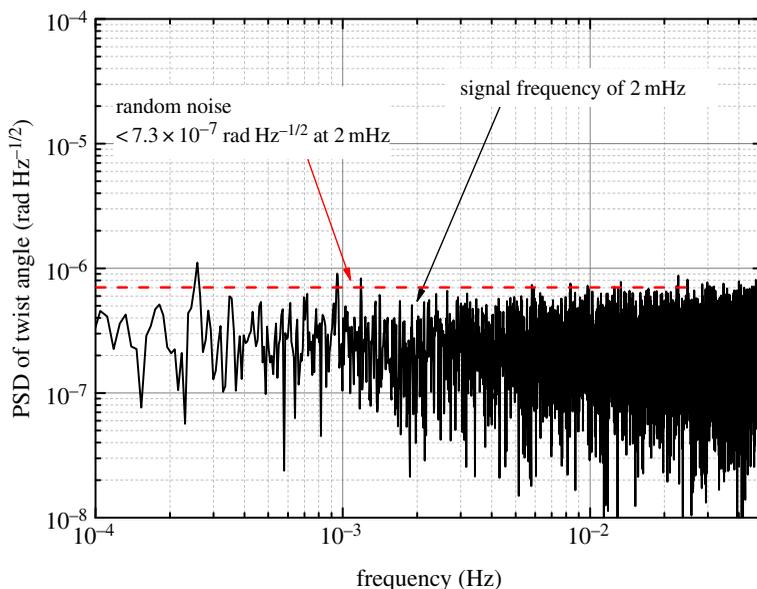
Figure 4. The typical PSD of the pendulum twist angle. The random noise is less than 7.3×10^{-7} rad Hz$^{-1/2}$, which contributes 0.4 ppm to the value of G. (Online version in colour.)

(d) Expected error budget

Table 1 lists the designed parameters and the expected error budget. With the proper design, the pendulum dimensions and mass distribution, and alignment of the two turntables need not be determined exactly. The tilt of the pendulum can break the up–down symmetry to introduce the $l =$ odd and $m =$ odd terms. The roundness of the 127 mm-diameter stainless-steel spheres, which directly introduces uncertainties into the measurement of the distances of the spheres, has reached the limit of manual processing. Nevertheless, the largest error is still the measurement uncertainties of the horizontal and vertical distances. Meanwhile, owing to the finite gain of the feedback loop, a small residual twist angle remains in the torsion fibre.

4. Preliminary experimental results

The preliminary determination for the value of G has been performed to test all aspects of the experimental design and particularly the feedback function. With two feedback loops activated, the two turntables rotate with different angular velocities in opposite directions. The inner turntable is forced to closely follow the torsion pendulum. The typical power spectral density (PSD) of the residual twist angle of the pendulum from approximately 40 h of data is shown in figure 4. We do not find any peaks above the random noise at the signal frequency of 2 mHz. The random noise is less than 7.3×10^{-7} rad Hz$^{-1/2}$. The residual twist angle $\theta(\omega) = 2$ nrad at the signal frequency, and it contributes 0.4 ppm relative uncertainty to the experiment. In equation (2.4), the $[-\omega^2 + 2i\beta\omega + \omega_0^2(1 - k/k_m)]\theta(\omega)$ term indicates the torsion fibre properties, including anelasticity, which leads to a bias [12,33–35] in previous measurements. Therefore, the influence of the torsion fibre properties must be smaller than 0.4 ppm. Finally, the thermal noise of the torsion balance introduces 0.2 ppm to the value of G [36].

The angular acceleration of the inner turntable is obtained by twice numerically differentiating the rotating angle, which is recorded for about 40 h. In order to improve the angle resolution, the output signal is subdivided into 1600 times. Figure 5 shows a 2 h segment of the angular acceleration. All of the turntable angle data are divided into segments including 10 sinusoidal

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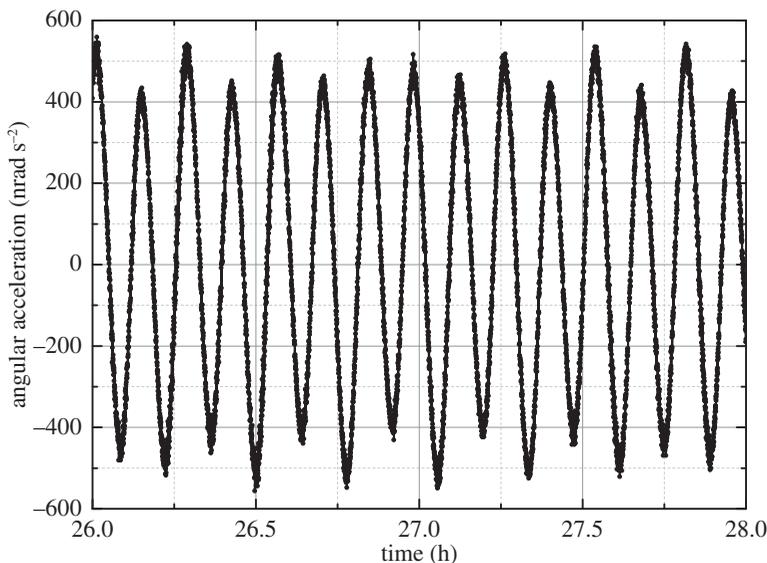
Figure 5. A 2 h segment of the angular acceleration of the inner turntable. The signal frequency is 2 mHz.
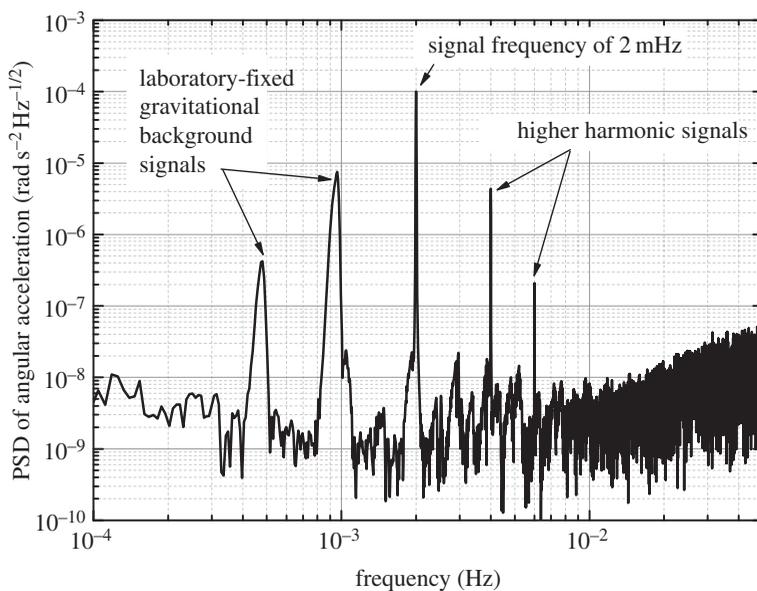
Figure 6. The typical PSD of the turntable angular acceleration. The 2.0 mHz gravitational acceleration signal is nearly four orders of magnitude larger than the random noise. The two peaks at 4.0 and 6.0 mHz are the higher harmonic signals. The primary laboratory-fixed gravitational background is cleanly separated.

cycles and fitted with a least-squares method [2]. The typical PSD of the angular acceleration of the inner turntable is shown in figure 6. In this figure, the 2 mHz gravitational angular acceleration signal is nearly four orders of magnitude larger than the random noise. Therefore, the relative uncertainty of the angular acceleration is approximately 100 ppm, which is mainly limited by the stability of the apparatus. The two additional peaks are higher harmonic signals due to the geometric design. The $l = m = 4$ and $l = m = 6$ terms, which cannot be made to vanish, generate the second harmonic signal and the third harmonic signal, respectively. Moreover, as shown in

the figure, the two primary laboratory-fixed gravitational background signals due to the attitude of the pendulum and the asymmetrical laboratory-fixed masses are cleanly separated, but they are about three orders of magnitude larger than the random noise. Furthermore, the higher-order signals are very close to the signal of interest, which directly introduce the uncertainty to the value of G. Therefore, a perfect compensation to the laboratory-fixed gravitational background needs to be carried out.

5. Conclusion and future

We have performed a preliminary measurement of the Newtonian gravitational constant G. With the feedback control, the experimental results are independent of the properties of the torsion fibre, as the fibre does not twist. In order to reduce the $1/f$ noise inherent in the torsion fibre and separate the laboratory-fixed gravitational background, the signal can be selected at higher frequency, owing to the rotating source masses. The preliminary results show that the relative uncertainty of the angular acceleration is approximately 100 ppm. However, we cannot give the value of G in the preliminary experiment, because some parameters need to be determined more precisely and further confirmed carefully, such as the distance of the spheres, which can introduce a large uncertainty, and the gravitational background caused by the rotating sphere shelves, which will introduce a large systematic bias to the value of G, and so on. Nevertheless, these will not influence the measurement of the repeatability of the angular acceleration. In order to obtain a reliable value, the experiment has been optimized as follows:

(i) The preliminary apparatus in figure 2 is nearly 3 m high, and the centre of mass is close to the top of the chamber. The inner turntable supports the vacuum chamber at the bottom, so the apparatus can be simplified as an inverted pendulum. The vibration of the turntable can easily affect the stability of the torsion pendulum. With the feedback control activated, the random noise of the torsion pendulum due to the vibration of the turntable will be totally transferred to the random noise of the angular acceleration, which directly influences the relative uncertainty of the value of G. Therefore, a new vacuum chamber 1.5 m high has been built, and an air bearing turntable with less than 100 nm runout is used to support the top section to make the system more stable.

(ii) The uncertainty of the distance of spheres is the largest error. All of the spheres are located on the two aluminium shelves whose thermal expansion coefficient is $23.2 \times 10^{-6} \,^{\circ}\mathrm{C}^{-1}$, and the shelves are supported by the gear bearing turntable, which is driven by a step motor. The temperature fluctuations around the turntable are simultaneously monitored by five temperature sensors with an accuracy of $0.01^{\circ}\mathrm{C}$ when the feedback is turned on to drive the step motor. The results show that the biggest difference of the temperature between the different parts of the turntable is $0.7^{\circ}\mathrm{C}$. Therefore, the variation of the horizontal and vertical distances between spheres caused by the aluminium shelves are $5.6\,\mu$m and $2.3\,\mu$m, which can introduce relative uncertainties of 26.2 ppm and 8.2 ppm, respectively. In order to reduce the effect of the temperature fluctuation on the distances of the spheres, an ultra-low expansion (ULE) material will be selected as the shelf material, which has an ultra-low thermal expansion coefficient of $0.1 \times 10^{-6}\,^{\circ}\mathrm{C}^{-1}$, about two orders of magnitude better than aluminium. With the same temperature fluctuation, the relative uncertainties of the distance variation for the value of G caused by the ULE material shelves can be improved to be less than 0.1 ppm.

(iii) According to the PSD of the angular acceleration in figure 6, the signals of the laboratory-fixed gravitational background introduce an uncertainty to the signal of interest. As a result, the fundamental frequency due to the $l = 2$ and $m = 1$ term caused by the attitude of the torsion pendulum brings in about 9 ppm. In order to reduce the influence of the fundamental frequency to less than 1 ppm, the requirement of the attitude of the pendulum should be better than 0.3 mrad. Meanwhile, the second

harmonic frequency due to the $l = 2$ and $m = 2$ term of the asymmetrical laboratory-fixed masses introduces about 400 ppm. To solve the problem, a perfect compensation of the laboratory-fixed gravitational background that can reduce the amplitude of the second harmonic frequency by over two orders of magnitude needs to be carried out.

With the improvements shown above, we expect to obtain a reliable value of G to about 20 ppm or less in the near future, and compare the values given by the time-of-swing method and the atom interferometry method to help us evaluate the potential systematic errors.

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