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EXTENDED ABSTRACTS

Microscopic model for double-barrier SIS'IS Josephson junctions

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Abstract — As is shown in [1], double barrier SIS'IS structures (I is the tunnel barrier, S' is a thin film with $T_{cS'}<T_{cS}$) combine advantages of weak links and tunnel junctions. Namely, they are intrinsically shunted and have therefore non-hysteretic I-V characteristics, while their resistance is controlled by the tunnel barriers rather than by the interlayer material. Such junctions are thus very promising in RSFQ and programmable voltage standard applications. In the present contribution we develop the microscopic model for stationary supercurrent and I_cR_N product in SIS'IS junctions in the general case of an arbitrary $T_{cS'}/T_{cS}$ ratio and arbitrary barriers. In earlier theoretical papers [1,2] only a few limited cases were studied. The influence of interlayer thickness, critical temperature $T_{cS'}$ and of barrier asymmetry on I_cR_N is quantitatively studied within this model. The current-phase relation in different parameter ranges and an influence of the electronic mean free path in the S' interlayer is also discussed. It is shown that data for *Nb/AlOx/Al/AlOx/Nb* junctions from different groups are well described by the theory.

I. Introduction

Recently it has been demonstrated in a series of experimental studies [3]-[5] that double barrier SS'IS'IS'S structures can be considered as promising candidates for the replacement of SIS tunnel Josephson junctions (JJ) in a large scale integrated circuit technology. Moreover these structures have been used successfully for the fabrication of microcircuits for voltage standards (8192 JJ) and simple RSFQ devices (68 JJ) with an on-chip spread of the junction parameters less than 10% [5]-[7].

Unfortunately, even stationary processes in SIS'IS junctions were studied only in a few limited cases [1,2]. In the present contribution we develop the microscopic model for stationary supercurrent and I_cR_N product in SIS'IS junctions in the general case of an arbitrary $T_{cS'}/T_{cS}$ ratio and arbitrary transparency of the barriers. The transport properties of the interlayer are considered both in dirty and clean limits. The results of calculations are compared with experimental data.

II. Theory

A. Microscopic model

To derive the microscopic model for a double barrier structure we assume the thickness of the interlayer between the barriers to be much smaller then the coherence length ($d \ll \xi_{S'} =(v_F l_{S'}/6\pi T_{cS})^{1/2}$ where v_F is the Fermi velocity for S' and $l_{S'}$ is the electron mean free path in S').

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If the conditions of the dirty limit for the interlayer are fulfilled ($l_{S'} \ll d, \xi_{S'}$), then the supercurrent can be derived within the framework of the Usadel equations [8]. We have found (see also [2]):

$$\frac{eI\,(\varphi)R_N}{2\pi T_{cS}} = \frac{T}{T_{cS}}\sum_{\omega>0}\frac{G_{S'}\Delta_S\sin\varphi}{E_1\Omega\omega}\left[\frac{\Delta_S}{E_1}+\frac{\Delta_{S'}\gamma_{eff}}{\pi T_{cS}}\right], \quad (1)$$

$$\Omega = \frac{\gamma_{eff}\omega}{\pi T_{cS}} + G_S,\; G_{S'} = \frac{\omega\Omega}{\sqrt{\omega^2\Omega^2+\delta^2}},\; \delta = \Delta_S G_S\eta + \frac{\omega\Delta_{S'}\gamma_{eff}}{\pi T_{cS}}$$

Here the summation goes over the Matsubara frequencies $\omega=(2n+1)\pi T$, φ is the phase difference across the junction, $E_1^2= \omega^2+\Delta_S^2$, $G_S^2=\omega^2/(\omega^2+\Delta_S^2)$, $\eta=(\cos^2(\varphi/2)+\gamma_-^2\sin^2(\varphi/2))^{1/2}$, $\gamma_- = (\gamma_{B1} - \gamma_{B2})/(\gamma_{B1} + \gamma_{B2})$. The suppression parameter γ_{eff} is

$$\gamma_{eff} = \frac{d}{\xi_{S'}}\frac{\gamma_{B1}\gamma_{B2}}{\gamma_{B1}+\gamma_{B2}},\quad \gamma_{B1,2} = \frac{2}{3}\frac{l_{S'}}{\xi_{S'}}\left\langle\frac{D_i}{1-D_i}\right\rangle^{-1} \equiv \frac{R_{B1,2}}{\rho_{S'}\xi_{S'}} \quad (2)$$

where $D_{1,2}$ are the barriers transparencies, $R_{B1,2}$ are the boundary resistances and $\rho_{S'}$ is the resistivity of the interlayer. The brackets denote angle averaging. The order parameter in the interlayer $\Delta_{S'}$ is the solution of the selfconsistency equation

$$\Delta_{S'}\left\{\ln\frac{T}{T_{cS'}} + 2\pi T\sum_\omega\left[\frac{1}{\omega} - \frac{\gamma_{eff}G_{S'}}{\omega\Omega}\right]\right\} = \pi T\sum_\omega\frac{G_S G_{S'}\Delta_S\pi T_{cS}}{\omega^2}.$$

Expression (1) is valid for any shape of an atomically sharp interface barrier potential, and $\gamma_B \gg d/\xi_{S'}$.

B. Limit of a small suppression parameter

In the case of a small suppression parameter, $d/\xi_{S'} \ll \gamma_{eff} \ll \xi_{S'}/d$, equation (1) can be simplified into

$$\frac{eI_S R_N}{2\pi Tc} = \frac{T}{Tc}\sum_\omega\frac{\Delta_0^2\sin\varphi}{\sqrt{\Delta_0^2\cos^2(\varphi/2)+\omega^2}\sqrt{\Delta_0^2+\omega^2}}. \quad (3)$$

This current-phase relationship is not sinusoidal anymore but has it's maximum at φ=1.86 if $T \ll T_{cS'}$. This limit was first found by [1] for a normal metal interlayer (see Fig.1). Here we have proved that it holds for any kind of interlayer, namely the critical voltage is independent of the material properties or the thickness of the interlayer and the barriers.

C. Limit of a large suppression parameter

In the practically interesting case of a large suppression parameter and relatively high temperatures

$$\gamma_{eff} \gg \frac{T_{cS}}{T},\quad T \ge T_{cS'} + T_{cS'}\left[\frac{\sqrt{T_{cS}(T_{cS}-T_{cS'})}}{\gamma_{eff}T_{cS'}}\right]^{2/3}$$

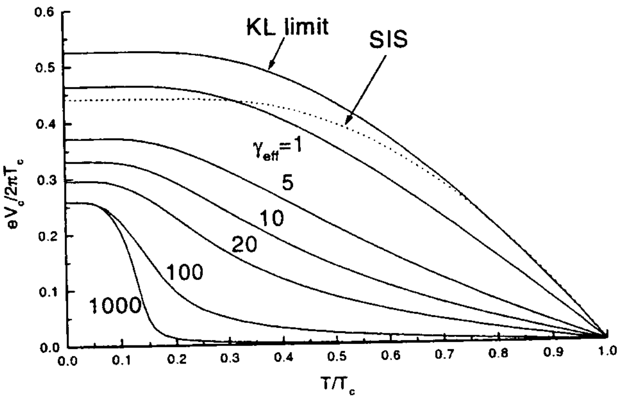

Fig. 1. Temperature dependence of the critical voltage for several values of the suppression parameter for $T_{cS}/T_{cS'} = 9.2 / 1.25$.

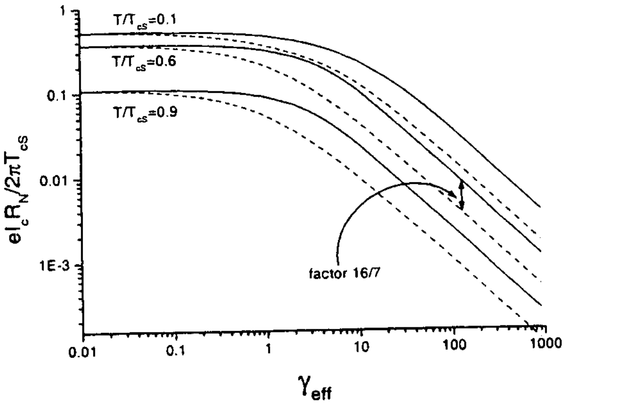

Fig 2. Relation between the normalized critical voltage and the suppression parameter for a double barrier structure with a normal metal interlayer. Solid lines correspond to the clean limit, dotted lines to the dirty limit.

equation (1) reduces to:

$$\frac{eJ_S(\varphi)R_N}{2\pi T_{cS}} = \frac{T}{T_{cS}}\frac{1}{\gamma_{eff}}\sum_{\omega>0}\frac{\Delta_S{}^2\sin\varphi}{\omega^2+\Delta_S{}^2}\frac{\pi T_{cS}}{\omega}. \qquad (4)$$

The current-phase relation is now purely sinusoidal. The critical voltage is inversely proportional to the suppression parameter.

Another limiting case in the regime of large suppression parameters is the situation in which $T \ll T_{cS'}$. This situation is described by two tunnel junctions in series, each barrier carrying a phase difference of $\varphi/2$ if the barriers are equal.

Fig.1 shows the calculated normalized critical voltage as a function of the reduced temperature for different values of the suppression parameter. This plot is made for a ratio of critical temperatures of $T_{cS}/T_{cS'} = 9.2/1.25$ as is the case for *Nb/Al*.

D. Electronic mean free path

The derived microscopic model is based on the assumption that the condition of the dirty limit is fulfilled in the interlayer material. But a recent systematic study [9] of the transport parameters of thin aluminum films in *Nb/Al/AlOx/Al/Nb* tunnel structures shows that they are mainly controlled by the scattering at the interfaces. This demonstrates that the dirty limit assumption is difficult to justify. Therefore we have developed the microscopic model for an interlayer material in the clean limit.

Starting from a set of Gor'kov equations [10], we have found an expression for the temperature Green's functions in a double barrier structure. The supercurrent can then be derived after angle-averaging over the wavevector component $k_{//}$ and summing over the Matsubara frequencies:

$$J_S = \frac{e}{\hbar}\int\frac{d^2k_{//}}{(2\pi)^2}T\sum_{\omega>0}\frac{\Delta_S{}^2\sin\varphi+\Delta_S\Delta_{S'}\sqrt{\frac{E_1}{E_2}}\frac{d}{\xi_{S'}}W^2\sin\frac{\varphi}{2}}{2W^4E_1{}^2\left(\cosh\frac{d}{\xi_{S'}}-\cos 2k_Fd\right)+E_3{}^2}. \qquad (5)$$

Here $E_1{}^2=\omega^2+\Delta_S{}^2$, $E_2{}^2=\omega^2+\Delta_{S'}{}^2$, $E_3{}^2=\omega^2+\Delta_S{}^2\cos^2\varphi/2$ and $W=W_{1,2}/hv_F$ is the potential barrier height. The coherence length is now defined as $\xi_{S'}=hv_F/2\omega$ and $\Delta_{S'}$ should be determined from the solution of the appropriate selfconsistency equation.

To compare results with the findings of the model for the dirty limit, we define the suppression parameter as $\gamma_{eff} = (W^2d/\xi)/4$, which coincides with the previous definition of γ_{eff}.

For a small suppression parameter ($\gamma_{eff} \ll 1$) we found (3) again. In this coherent regime the electron mean free path does not play a role in the value of the critical voltage (as does neither one of the material parameters of the interlayer).

In the case of a large suppression parameter and $T \gg T_{cS'}$ the supercurrent is again given by (4) multiplied by a factor of *16/7*. Here we see that the coincidence of the two models breaks down but that we can still use the model for the dirty limit as long as we take this correction into account. The crossover between the two models is shown in Fig.2

E. Proximity effect in the composite electrodes

So far we used rigid boundary conditions in our model. This means that the pair potential is assumed to be constant in each layer. In the case of *Nb/Al/AlOx/Al/AlOx/Al/Nb* junctions however, the electrodes consist of the Nb superconductor and a proximized thin normal layer of *Al*. To implement the spatial variation of the pair potential in our model for the double barrier junction we used the method described in [11].

We found that the influence on the critical voltage in the practical regime of large suppression parameters can be described by a single scaling factor. Even for 10 nm thick *Al* layers we found that the result differed less than 10% from the previous results. For thinner *Al* layers, the influence becomes even less.

III. EXPERIMENTAL RESULTS

Most fabrications of double barrier junctions are done in standard process developed for *Nb/Al/AlOx/Al/AlOx/Al/Nb* technology [12]. This technology is well developed for the case of single barrier type devices and *Al* is very suitable as an interlayer because of it's large coherence length (which reduces the suppression parameter).

An example of a theoretical fit to measured data of I_c vs. T is plotted in Fig. 3.

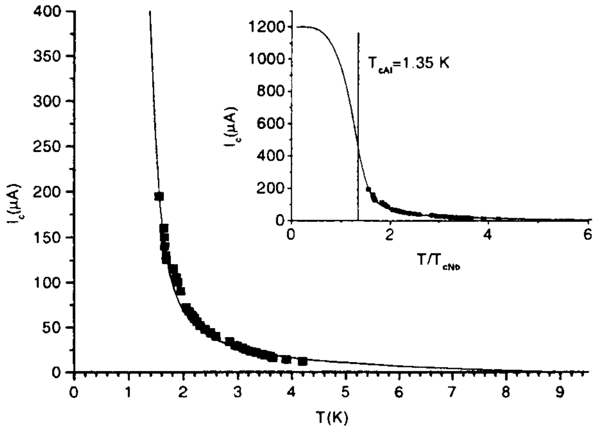

Fig. 3. Experimental data points in an $Ic(T)$ measurement. The solid line is a theoretical fit (γ_{eff}=750, T_{cAl}=1.35 K). The onset shows the entire shape of the theoretical curve.

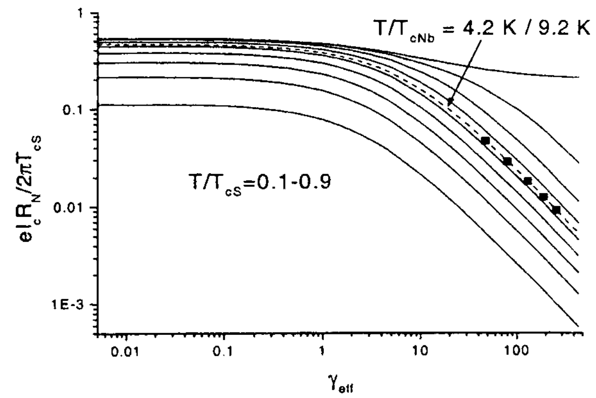

Fig. 4. Dependence of the normalized critical voltage on the suppression parameter for different temperatures, T_{cS}=9.2K and $T_{cS'}$=1.25K, corresponding to the *Nb-Al* case. The squares are experimental data at 4.2 K from [5].

We fabricated this double barrier junction with minor changes to our standard technology for *Nb-Al-AlOx-Al-Nb* junctions [13]. The normal state resistance is extracted from the non-hysteretic current voltage characteristics. In our case of rather thick oxide layers, R_N=1.1 Ω for a 9 μm x 9 μm junction with d_{Al}=7nm. We obtained a value of γ_{eff} =750 for the suppression parameter.

Several double barrier junctions measurements from other groups [5],[14] were found to be well described by the microscopic model. Until now all realized double barrier junctions have suppression parameters much larger than one. Some data which have been extracted from [5] are shown in Fig. 4. This indicates that practical double barrier junctions still only exist in the regime where the supercurrent is inversely proportional to the suppression parameter.

IV. Conclusion

The developed model of double barrier SS'IS'IS'S devices, which takes the proximity effect in the electrodes and the electronic mean free path of the interlayer into account, describes the existing experimental data well.

In the coherent regime of a small suppression parameter the critical voltage becomes independent of the thickness, barrier transparencies and material properties of the interlayer.

In the practical case of a large suppression parameter increasing barrier transparencies or $T_{cS'}$ and decreasing the interlayer thickness will lead to a higher I_cR_N product. Pinholes due to this increase in the barrier transparency do not pose problems if they are smaller than the coherence length of the interlayer material.

The microscopic model predicts that the I_cR_N product can still be very much improved compared to the current status of experimental results (Fig. 4). This implies the possibility for a critical voltage of 1 mV (J_c=10-20 kA/cm^2, γ_{eff}=10) while still having non-hysteretic current voltage characteristics.

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