

Critical Current in SFIFS Junctions¹

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A quantitative theory of the Josephson effect in SFIFS junctions (S denotes bulk superconductor, F is metallic ferromagnet, and I is insulating barrier) is presented in the dirty limit. A fully self-consistent numerical procedure is employed to solve the Usadel equations for arbitrary values of the F-layer thicknesses, magnetizations, and interface parameters. In the case of antiparallel ferromagnet magnetizations, the effect of critical current I_c enhancement by the exchange energy H is observed, while in the case of parallel magnetizations the junction exhibits a transition to the π state. In the limit of thin F layers, we study these peculiarities of the critical current analytically and explain them qualitatively; the scenario of the $0-\pi$ transition in our case differs from those studied before. The effect of switching between 0 and π states by changing the mutual orientation of F layers is demonstrated. © 2002 MAIK "Nauka/Interperiodica".

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Josephson structures involving ferromagnets as weak link material are presently a subject of intensive study. The possibility of the so-called " π state" (characterized by the negative sign of the critical current I_c) in SFS Josephson junctions was predicted theoretically [1–8]. The first experimental observation of the crossover from the 0 to the I_c state was reported by Ryazanov *et al.* [9] and explained in terms of temperature dependent spatial oscillations of induced superconducting ordering in the diffusive F layer.

More recently, a number of new phenomena were predicted in junctions with more than one magnetically ordered layer. First, the possibility of critical current enhancement by the exchange field in SFIFS Josephson junctions with thin F layers and antiparallel magnetization directions was discussed in the regimes of small S-layer thicknesses [10] and bulk S electrodes [11, 12]. Second, the crossover to the π state was predicted in [11] for the parallel case even in the absence of the order parameter oscillations in thin F layers. Still, the physical explanation of these effects and accurate calculation of their magnitude have not been given so far. To make such estimates in the model with thin S electrodes, one must consider KO-1 type solutions [13] and take into account spatial variation of the superconducting state in the SF bilayers; at the same time, in the bulk S case an approximate method was used in [11] beyond its applicability range [12]. This problem is of a rather general nature, since one may expect from previous

knowledge (see, e.g., review [14]) that the supercurrent in a short weak link is H independent.

The above intriguing scenario motivated us to attack the problem of the Josephson effect in SFIFS junctions by self-consistent solution of the Usadel equations for arbitrary thicknesses of the F layers, barrier transparencies, and exchange field orientations. Below, we show that the $0-\pi$ transition in the case of parallel H orientation or enhancement of I_c by H in the antiparallel case with thin F layers occurs when the effective energy shift in the ferromagnets (due to the exchange field) becomes equal to a local value of the effective energy gap induced into an F layer. Under this condition, a peak in the local density of states (DoS) near the SF interfaces is shifted to zero energy. In the models with DoS of the BCS type, this leads to logarithmic divergence of I_c in the antiparallel case at zero temperature, similarly to the well-known Riedel singularity of ac supercurrent in SIS tunnel junctions at voltage $eV = 2\Delta$. We also describe the general numerical method to solve the problem self-consistently and apply it for quantitative description of the $0-\pi$ transition and I_c enhancement in SFIFS junctions.

The model. We consider the structure of the SFIFS type, where I is an insulating barrier of arbitrary strength. We assume that the S layers are bulk and that the dirty limit conditions are fulfilled in the S and F metals. Although our method is applicable in the general situation of different ferromagnets and superconductors, for simplicity, below we illustrate our results in

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the case where equivalent S and F materials are used on both sides of the structure (although the directions of the exchange field in the two F layers may be different), both F layers have the thickness d_F , and the two SF interfaces have the same transparency. At the same time, we do not put any limitations on d_F and the transparency.

The Usadel functions G, F obey the normalization condition $G_\omega^2 + F_\omega F_\omega^* = 1$, which allows the following parameterization in terms of the new function Φ :

$$G_\omega = \frac{\tilde{\omega}}{\sqrt{\tilde{\omega}^2 + \Phi_\omega \Phi_\omega^*}}, \quad F_\omega = \frac{\Phi_\omega}{\sqrt{\tilde{\omega}^2 + \Phi_\omega \Phi_\omega^*}}. \quad (1)$$

The quantity $\tilde{\omega} = \omega + iH$ corresponds to the general case where the exchange energy H is present. However, in the S layers, $H = 0$ and we have simply $\tilde{\omega} = \omega$.

We choose the x axis perpendicular to the plane of the interfaces with the origin at the barrier I. The Usadel equations [15] in the S and F layers have the form

$$\xi_S^2 \frac{\pi T_c}{\omega G_S} \frac{\partial}{\partial x} \left[G_S^2 \frac{\partial}{\partial x} \Phi_S \right] - \Phi_S = -\Delta, \quad (2)$$

$$\xi_F^2 \frac{\pi T_c}{\tilde{\omega} G_F} \frac{\partial}{\partial x} \left[G_F^2 \frac{\partial}{\partial x} \Phi_F \right] - \Phi_F = 0, \quad (3)$$

where T_c is the critical temperature of the superconductors, Δ is the pair potential (which is nonzero only in the S layers), ω is the Matsubara frequency, and the coherence lengths ξ are related to the diffusion constants D as $\xi_{S(F)} = \sqrt{D_{S(F)}/2\pi T_c}$. The pair potential satisfies the self-consistency equations

$$\Delta \ln \frac{T}{T_c} + \pi T \sum_\omega \frac{\Delta - G_S \Phi_S \operatorname{sgn} \omega}{|\omega|} = 0. \quad (4)$$

In this paper, we restrict ourselves to the cases of parallel and antiparallel orientations of the exchange fields H in the ferromagnets.

The boundary conditions at the SF interfaces ($x = \mp d_F$) have the form [16] (see [17] for detail)

$$\frac{\xi_S G_S^2}{\omega} \frac{\partial}{\partial x} \Phi_S = \gamma \frac{\xi_F G_F^2}{\tilde{\omega}} \frac{\partial}{\partial x} \Phi_F, \quad (5)$$

$$\pm \gamma_B \frac{\xi_F G_F}{\tilde{\omega}} \frac{\partial}{\partial x} = G_S \left(\frac{\Phi_F}{\tilde{\omega}} - \frac{\Phi_S}{\omega} \right), \quad (6)$$

with

$$\gamma_B = R_B \mathcal{A} / \rho_F \xi_F, \quad \gamma = \rho_S \xi_S / \rho_F \xi_F,$$

where R_B and \mathcal{A} are the resistance and the area of the SF interfaces, respectively; and $\rho_{S(F)}$ is the resistivity of the S (F) layer. At the I interface ($x = 0$), the boundary

conditions read

$$\frac{G_{F1}^2}{\tilde{\omega}_1} \frac{\partial}{\partial x} \Phi_{F1} = \frac{G_{F2}^2}{\tilde{\omega}_2} \frac{\partial}{\partial x} \Phi_{F2}, \quad (7)$$

$$\gamma_{B,I} \frac{\xi_F G_{F1}}{\tilde{\omega}_1} \frac{\partial}{\partial x} \Phi_{F1} = G_{F2} \left(\frac{\Phi_{F2}}{\tilde{\omega}_2} - \frac{\Phi_{F1}}{\tilde{\omega}_1} \right), \quad (8)$$

with

$$\gamma_{B,I} = R_{B,I} \mathcal{A} / \rho_F \xi_F,$$

where the indices 1, 2 refer to the left and right side of the I interface, respectively.

In the bulk of the S electrodes, we assume a uniform current-carrying superconducting state

$$\Phi(x = \mp \infty) = \frac{\Delta_0 \exp(i[\mp \varphi/2 + 2m v_s x])}{1 + 2D_S m^2 v_s^2 / \sqrt{\omega^2 + |\Phi|^2}}, \quad (9)$$

where m is the electron mass, v_s is the superfluid velocity, and φ is the phase difference across the junction.

The supercurrent density is constant across the system. In the F part, it is given by the expression

$$J = \frac{i\pi T}{2e\rho} \sum_\omega \frac{G^2(\omega)}{\tilde{\omega}^2} \left[\Phi_\omega \frac{\partial}{\partial x} \Phi_\omega^* - \Phi_\omega^* \frac{\partial}{\partial x} \Phi_\omega \right], \quad (10)$$

while an analogous formula for the S part is obtained if we substitute $\tilde{\omega} \rightarrow \omega$. This expression, together with the boundary condition (8) and the symmetry relation $F(-\omega, H) = F(\omega, -H)$, yields the formula for the supercurrent across the I interface:

$$I = \frac{\pi T}{eR_{B,I}} \sum_\omega \operatorname{Im} [F_{F1}^*(-H_1) F_{F2}(H_2)] \quad (11)$$

(the functions F are related to Φ via Eq. (1)).

The limit of small F-layer thickness: $d_F \ll \min(\xi_F, \sqrt{D_F/2H})$. Under the condition $\gamma_B/\gamma \gg 1$, we can neglect the suppression of superconductivity in the superconductors. We further assume that the transparency of the I barrier is small, $\gamma_{B,I} \gg \max(1, \gamma_B)$, and the SF bilayers are decoupled (the exact criterion will be given below). In this case, we can set $v_s = 0$ and expand the solution of Eq. (3) in the F layers up to the second order in small spatial gradients. Applying the boundary condition (6), we obtain the solution in a form similar to that in the SN bilayer [18, 17]:

$$\Phi_{F1, F2} = \frac{\tilde{\omega}_{1,2}/\omega}{1 + \gamma_{BM} \tilde{\omega}_{1,2}/\pi T_c G_S} \Delta_0 \exp(\mp i\varphi/2), \quad (12)$$

with

$$\gamma_{BM} = \gamma_B d_F / \xi_F, \quad G_S = \omega / \sqrt{\omega^2 + \Delta_0^2}.$$

Substituting Eq. (12) into the expression for the supercurrent (11), we obtain $I(\varphi) = I_c \sin \varphi$.

For the parallel orientation of the exchange fields, $H_1 = H_2 = H$, the critical current is

$$I_c^{(p)} = \frac{2\pi T}{eR_{B,I}} \sum_{\Omega>0} \frac{\delta^2 G_S^2}{\Omega^2} \frac{1 - \alpha + \Omega\gamma_{BM}g_1}{(1 - \alpha + \Omega\gamma_{BM}g_1)^2 + 4\alpha g_2}, \quad (13)$$

where $\Omega = \omega/\pi T_c$, $\delta = \Delta_0/\pi T_c$, $\alpha = (h\gamma_{BM})^2$, $h = H/\pi T_c$, $g_1 = 2G_S + \gamma_{BM}\Omega$, and $g_2 = (G_S + \gamma_{BM}\Omega)^2$.

For the antiparallel orientation, $H_1 = -H_2 = H$, the critical current is given by

$$I_c^{(a)} = \frac{2\pi T}{eR_{B,I}} \times \sum_{\Omega>0} \frac{\delta^2 G_S^2}{\Omega^2} \frac{1}{\sqrt{(1 - \alpha + \Omega\gamma_{BM}g_1)^2 + 4\alpha g_2}}. \quad (14)$$

At $h = 1/\gamma_{BM}$ and small Ω , the expression in the sum in Eq. (14) behaves as $1/\Omega$; thus, at low T , the critical current diverges logarithmically: $I_c^{(a)} \propto \ln(T_c/T)$. This effect was pointed out earlier in [10, 11].

The above results become physically transparent in the real energy ε representation. Making an analytical continuation in Eqs. (1) and (12) by the replacement $\omega \rightarrow -i\varepsilon$, we obtain the expression for the DoS per one spin projection (spin ‘‘up’’) $N_F(\varepsilon) = \text{Re} G_F(\varepsilon)$ in the F layers

$$N_F(\varepsilon) = \left| \text{Re} \frac{\tilde{\varepsilon}}{\sqrt{\tilde{\varepsilon}^2 - \Delta_0^2}} \right|, \quad (15)$$

$$\tilde{\varepsilon} = \varepsilon + \gamma_{BM}(\varepsilon - H)\sqrt{\Delta_0^2 - \varepsilon^2}/\pi T_c,$$

which demonstrates the energy renormalization due to the exchange field. Equation (15) yields $N_F(0) = \text{Re}(\gamma_{BM}h/\sqrt{(\gamma_{BM}h)^2 - 1})$, which shows that at $h = 1/\gamma_{BM}$ the singularity in the DoS is shifted to the Fermi level.

Exactly at this value of h the maximum of $I_c^{(a)}$ is achieved due to overlap at two $\varepsilon^{-1/2}$ singularities. This leads to logarithmic divergency of the critical current (14) in the limit $T \rightarrow 0$, similarly to the well-known Riedel singularity of a nonstationary supercurrent in SIS tunnel junctions at voltage $eV = 2\Delta_0$, where the energy shift is due to the electric potential. At the same value of the exchange field $h = 1/\gamma_{BM}$, the critical current changes its sign (i.e., the crossover from the 0 to the π contact occurs) for parallel magnetizations in the F layers [see Eq. (13)]. We emphasize that the scenario of the 0– π transition in our case differs from those studied before, where the π shift of the phase was either due to spatial oscillations of the order parameter in F layers or due to the proximity-induced phase rotation in S layers. In our case, the phase does not change in either layer; instead, it jumps at the SF interfaces. This scenario is most clearly illustrated in the limit of large H where

Eqs. (1) and (12) yield $F_F \propto -i\Delta \text{sgn} H$, whereas $F_S \propto \Delta$; thus the phase jumps by $\pi/2$ at each of the SF interfaces, providing the total π shift between $F_{F1}(-H)$ and $F_{F2}(H)$ [it is the phase difference between these two functions that determines the supercurrent according to Eq. (11)].

The considered effects take place only for sufficiently low I-barrier transparency. Indeed, it follows from Eq. (12) that $G_F(\Omega) \propto 1/\sqrt{\Omega}$ for small Ω under the condition $h = 1/\gamma_{BM}$. As a result, the boundary condition (8) results in that, at

$$\Omega \leq \min\left(\frac{\xi_F}{d_F\gamma_{B,I}}, \frac{\gamma_B}{\gamma_{B,I}}\right), \quad (16)$$

the solutions (12) are not valid, since in this frequency range the effective transparency of the I interface (the parameter $G_{F1}G_{F2}/\gamma_{B,I}$ [19]) increases and the spatial gradients in the F layers become large (the limit of large gradients is called ‘‘the KO-1 case’’ [13, 14]). In this case, the nongradient term in Eq. (3) can be neglected and the general solution of the Usadel equation in the F layers has the KO-1 form [13]:

$$\frac{\Phi}{\tilde{\omega}} = \frac{C - iM \arctan[M(Bx + Q)]}{1 - \eta}, \quad (17)$$

where $M = \sqrt{(\eta^2 - 1) - C^2}$, while C , B , Q , and η are integration constants. From Eqs. (1) and (17), it follows that the Green’s functions G , F and hence the contribution to the critical current from these frequencies are H independent. As a result, the barrier transparency parameter $\gamma_{B,I}$ provides the cutoff of the low-temperature logarithmic singularity of $I_c^{(a)}$ at $h = 1/\gamma_{BM}$ [see Eq. (14)]. According to Eq. (16), the critical current saturates at low temperature $T^* = T_c \min(\xi_F/d_F\gamma_{B,I}, \gamma_B/\gamma_{B,I})$. We note that any asymmetry in the SFIFS junction will also lead to the cutoff of $I_c^{(a)}$ divergency [19]. The above estimates are made for the case of low barrier transparency, $\xi_F/d_F\gamma_{B,I} \ll 1$ and $\gamma_B/\gamma_{B,I} \ll 1$. The opposite regime of high transparency deserves separate study.

The general case. For arbitrary F-layer thicknesses and interface parameters, the boundary problem (1)–(9) was solved numerically using the iterative procedure. Starting from trial values of the complex pair potentials Δ and the Green’s functions $G_{S,F}$ we solve the resulting linear equations and boundary conditions for functions $\Phi_{S,F}$. After this, we recalculate $G_{S,F}$ and Δ . Then, we repeat the iterations until convergency is reached. The self-consistency of calculations is checked by the condition of conservation of the supercurrent (10) across the junction. We emphasize that our method is *fully* self-consistent; in particular, it includes the self-consistency over the superfluid velocity v_s , which is essential (contrary to the constriction case) in the quasi-one-

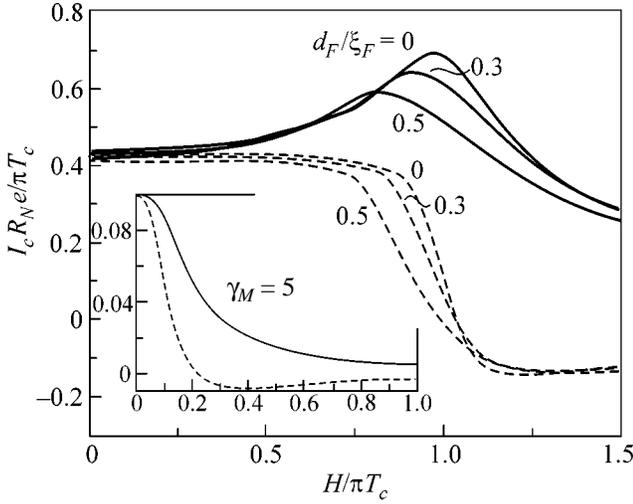


Fig. 1. Enhancement of the critical current (antiparallel magnetizations, solid lines) and the $0-\pi$ transition at which I_c changes its sign (parallel magnetizations, dashed lines) in the SFIFS junction at $T/T_c = 0.05$, $\gamma_{BM} = 1$, and $\gamma_M = 0$. Inset: the same for large values of γ_M (when $d_F \ll \xi_F$, the results depend only on this parameter).

dimensional geometry. The details of our numerical method will be presented elsewhere [19].

Figure 1 shows $I_c(H)$ dependences calculated at $T = 0.05T_c$ from the numerical solution of the boundary problem (1)–(9) for the fixed value of $\gamma_{BM} = 1$ and a set of different F-layer thicknesses and SF interface parameters γ . The normal junction resistance is $R_N = R_{B,I} + 2R_B + 2\rho_F d_F / \mathcal{A}$. The curves $d_F/\xi_F = 0$ are the limits of the vanishing d_F/ξ_F ratio at fixed γ_{BM} and are calculated from Eqs. (13) and (14). For thin F layers, the results depend only on the combination $\gamma_M = \gamma d_F/\xi_F$. The enhancement of I_c and the crossover to the π state are clearly seen for the antiparallel and parallel orientations, respectively. In accordance with the estimates given above, these effects take place for the values of the exchange field H close to πT_c . The enhancement disappears with increasing gradients in the F layers, since the solution to Eq. (12) loses its validity. This is illustrated in Fig. 1 by increasing the thickness d_F or γ_M . In particular, in the case of large γ_M the enhancement is absent, in contrast to the statement in [11] (see [12]).

The influence of temperature and barrier transparency on the critical current anomaly is shown in Fig. 2. One can see that, in accordance with the above estimate, the cutoff of the $I_c^{(a)}$ singularity is provided by finite temperature or barrier transparency; i.e., with the decrease of the barrier strength parameter $\gamma_{B,I}$, the peak magnitude starts to drop when the ratio $d_F \gamma_{B,I} / \xi_F$ becomes comparable to T/T_c . With a further decrease of $d_F \gamma_{B,I} / \xi_F$, the singularity disappears, while the transition to the π state shifts to large values of H .

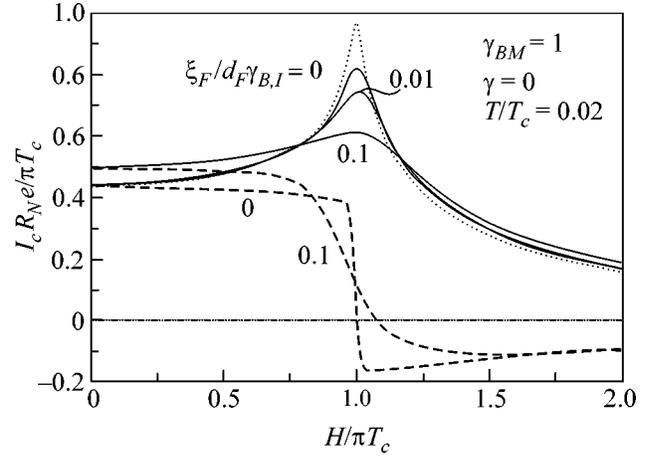


Fig. 2. Enhancement of the critical current (antiparallel magnetizations, solid lines) and the $0-\pi$ transition at which I_c changes its sign (parallel magnetizations, dashed lines) in the SFIFS junction: influence of temperature and barrier transparency. The dotted line corresponds to $T/T_c = 0.01$ and $\xi_F/d_F \gamma_{B,I} = 0$; the parameters for other curves are given in the figure.

Figure 3 demonstrates the DoS in the F layers for a certain spin projection calculated numerically in the limit of small I-barrier transparency. At $H = 0$, we reproduce the well-known minigap existing in an SN bilayer. At finite H , the gap shifts in energy (asymmetrically) and the peak in the DoS reaches zero energy at $h = 1/\gamma_{BM}$. One can see that, even for a small value $\gamma_M = 0.05$, the peaks are rather broad; this is the reason why the singularity in $I_c^{(a)}$ is suppressed by γ_M very rapidly.

In the limit of finite F-layer thickness (see Fig. 4), which is of practical interest, the numerical calculations show monotonic suppression of I_c with an increase of

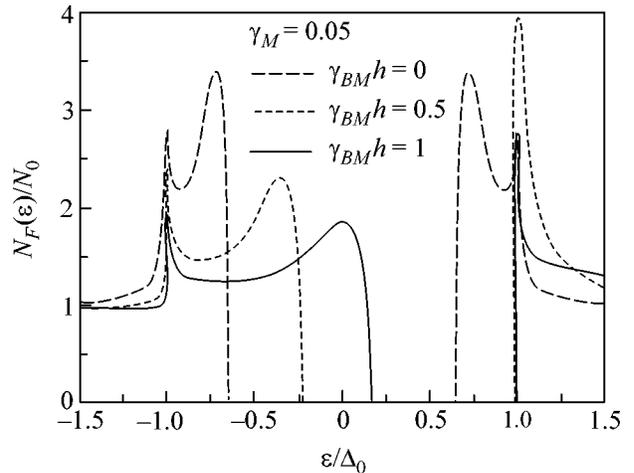


Fig. 3. Normalized density of states for spin “up” in the F layer for various exchange fields.

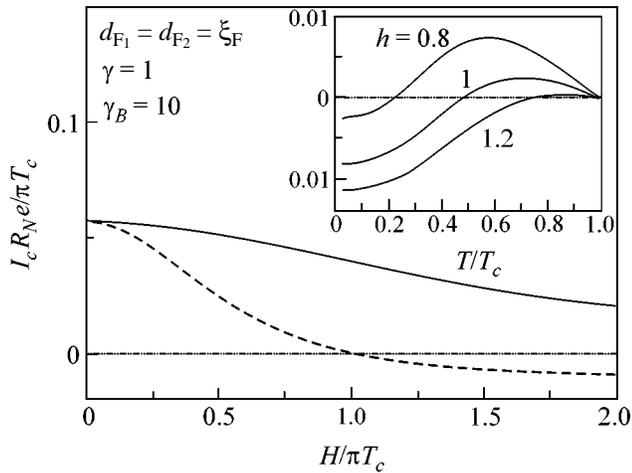


Fig. 4. Critical current in SF₁F₂S junction: switching effect. $T/T_c = 0.5$, the solid and dashed lines correspond to the antiparallel and parallel orientations of magnetizations, respectively. Inset: thermally induced $0-\pi$ crossover in the parallel case.

the exchange field H for antiparallel magnetizations of the F layers and the $0-\pi$ crossover for the parallel case. One can see from Fig. 4 that, for given temperature and thickness of the F layers, it is possible to find the value of the exchange field at which switching between parallel and antiparallel orientations will lead to switching of I_c from near-zero to a finite value (or to switching between 0 and π states). This effect may be used for engineering cryoelectronic devices manipulating spin-polarized electrons.

The case of parallel F-layer magnetizations in the absence of the I barrier corresponds to the standard SFS junction where the $0-\pi$ transition is possible due to spatial oscillations of induced superconducting ordering in the F layer. The thermally induced $0-\pi$ crossover in an SFS junction was observed in [9], where a simple theory based on the linearized Usadel equations was also presented. Here, we show such a crossover (see the inset in Fig. 4) from the fully self-consistent solution in the range of the exchange fields corresponding to that of [9]. Comparison with the experimental data and more detailed results of our model will be given elsewhere [19].

In conclusion, we have presented a general method for solving Usadel equations in SFIFS junctions self-consistently. Using our method, we have theoretically investigated the Josephson current in SFIFS and SFS junctions as a function of relative F-layer magnetizations, thicknesses, and parameters of the S/F and F/F interfaces. We have identified the physical mechanisms of the critical current enhancement and of the $0-\pi$ transition in these junctions.

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