CALIBRATION OF INERTIAL MEASUREMENT UNIT ON A LOW-GRADE TURNTABLE: ESTIMATION OF TEMPERATURE TIME DERIVATIVE COEFFICIENTS

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Abstract

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The aim of the study is to modify previously developed inertial measurement unit calibration method in order to take sensor error variations over temperature time derivative into consideration. The method itself is designed for calibration of inertial measurement units of navigation, tactical and low grade on a turntable with horizontal axis of rotation. Conventional parameters of inertial sensor error model – null biases, errors of scaling factors, sensor misalignments, dynamic drift coefficients and others – are estimated, if necessary. Since the motion of an inertial unit in calibration is not arbitrary (only rotation and no linear translation), sensor output becomes redundant, and not only the motion itself can be determined, but also sensor error parameters, under appropriate conditions of observability. Neither angular nor rate measurement or precise positioning is required from the turntable. This allows us to get estimates for all desired parameters in a simple procedure with no need in precise equipment and without strict compliance with any predefined plan of operations.

Introducing linear sensor error variations over temperature time derivative into the model, we demonstrate that its coefficients are observable in an experiment similar to that in previously developed calibration method, but with temperature changing over time. Using covariance analysis we get an estimation of calibration accuracy in experiments with two different temperature modes – inertial unit self-heating and controlled linear temperature change in a thermal chamber.

1. Introduction

Calibration of inertial sensors is intended to determine parameters of sensor systematic error model via processing measurement data recorded in a special experiment. Knowing estimates of these parameters one can compensate sensor output for systematic error in initial alignment and navigation modes. Conventional models of sensor systematic error incorporate the following components: null biases of accelerometers and angular rate sensors (gyros), errors of their scaling factors, small angles of sensor misalignments, and, for some types of gyros, coefficients of dynamic drift. There is a method of determining these parameters for an assembled 6-axis (3 accelerometers and 3 gyros) inertial measurement unit by processing measurements recorded in a simple experiment conducted on a turntable with horizontal axis of rotation with no need to either follow a strict sequence of operations, or sustain or measure specific angular positions or angular rates [1–3]. It is also known that spatial displacements of inertial sensors from the axis of rotation can also be estimated in calibration experiment (this allows us to calibrate several inertial units simultaneously, since placing the unit close to the axis of rotation is not required in this case) [2], and the same is true for temperature coefficients of null biases and of scaling factors (in an experiment with temperature changing) [3].

At the same time it is known that some types of angular rate sensors, e.g. fiber-optic gyros, have a relation between null biases (drifts) and the temperature rate of change, i.e. time derivative of the temperature, not only temperature itself [4–5]. According to cited literature, this relation is close to linear and is stable in the whole operational temperature range. The present work examines the possibility of determining temperature time derivative coefficients of gyro drifts with minimum changes to the calibration experiment. Two types of temperature change are considered below – inertial unit self-heating and controlled temperature change in a thermal chamber.

We refer to [1–3,6] for some specific notation where it appears.

2. Mathematical models of calibration problem without temperature variations

In [1–3] we state the problem of determining sensor systematic error parameters as an optimal estimation problem for linear dynamic system with measurements, with Kalman filter used to obtain the solution. For accelerometer error we have

$$f'_z - f_z = \Delta f_z^0 + \Gamma f_z + \Delta f_z^s, \qquad f'_z = \begin{bmatrix} f'_1 \\ f'_2 \\ f'_3 \end{bmatrix}, \qquad f_z = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \qquad \Delta f_z^0 = \begin{bmatrix} \Delta f_1^0 \\ \Delta f_2^0 \\ \Delta f_3^0 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \Gamma_{11} & 0 & 0 \\ \Gamma_{21} & \Gamma_{22} & 0 \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} \end{bmatrix},$$

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where f_z' is a column vector of three accelerometer outputs, f_z is a vector of the measured force (per unit mass) as projected onto the instrumental frame of reference Mz, Δf_z^0 is a vector of accelerometer null biases, Γ is a matrix with small errors of accelerometer scaling factors on its diagonal and small angles of accelerometer misalignments off the diagonal (Γ_{21}, Γ_{31} and Γ_{32}). We choose the instrumental reference frame in such a way that elements of Γ matrix above the main diagonal are zeros. For gyros (angular rate sensors) we have:

$$\omega_z' - \omega_z = -\nu_z^0 - \Theta\omega_z - \nu_z^s, \qquad \omega_z' = \begin{bmatrix} \omega_1' \\ \omega_2' \\ \omega_3' \end{bmatrix}, \qquad \omega_z = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}, \qquad \nu_z^0 = \begin{bmatrix} \nu_1^0 \\ \nu_2^0 \\ \nu_3^0 \end{bmatrix}, \qquad \Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ \Theta_{21} & \Theta_{22} & \Theta_{23} \\ \Theta_{31} & \Theta_{32} & \Theta_{33} \end{bmatrix}.$$

Similar to the previous, we have ω_z' for sensor output, ω_z for true absolute angular rate vector as projected onto the instrumental frame, ν_z^0 for null biases, and Θ containing scale factor errors and misalignments. Minus sign on the right side is just a convention to match the earlier models for gimbal systems.

Using logged gyro data we compute the attitude matrix L_y, i.e. the calculated value of transition (orientation) matrix between geodetic and instrumental reference frames [7]. The difference between calculated orientation and what we have in fact is small, and thus, ignoring smaller quadratic and higher order terms, can be represented as a three-dimentional vector β_x, that consists of three small angles of rotation. Henceforth we introduce error equations, which form a system with state vector containing desired sensor error parameters Δf_z^0, Γ, ν_z^0, Θ and orientation errors β_x. Using notation from [1–3,6] we write linear approximation for the system mechanization

$$\frac{d}{dt}\beta_x = \hat{u}_x\beta_x + L^T\left(\nu_z^0 + \Theta\omega_z\right) + L^T\nu_z^s, \qquad \beta_x(0) = \beta_0,$$
$$\frac{d}{dt}\nu_z^0 = 0, \qquad \frac{d}{dt}\Theta = 0, \qquad \frac{d}{dt}\Delta f_z^0 = 0, \qquad \frac{d}{dt}\Gamma = 0. \tag{1}$$

The vector of measurements in the system is a difference between true and measured force given by accelerometers. As the inertial unit is rotating, but not moving linearly, the only force acting on its reference center is a reaction force opposite to gravity vector. The difference between true and measured vectors has a linear connection with the system state vector, which can be represented as follows (smaller terms of higher order are omitted):

$$z^{acc} = \hat{\beta}_x\begin{bmatrix}0 & 0 & g\end{bmatrix}^T + L^T\left(\Delta f_z^0 + \Gamma f_z\right) + L^T\Delta f_z^s. \tag{2}$$

The system state vector in (1) is then estimated using measurements (2) with conventional Kalman filter. Observability of components of the system state vector depends primarily on the rotation of the inertial unit in calibration procedure, as coefficients near estimated values in equations (1), (2) are different combinations of elements of orientation matrix, angular rate vector components and measured force projections. The estimation problem is best conditioned in three rotations, each around one of roughly horizontal instrumental axes [1–2].

In the models, introduced above, no temperature variations of sensor errors are taken into consideration. A conventional approach to determine them is to conduct calibration experiments under approximately constant temperature in different so called "temperature points". After estimating parameters in each temperature point, one can approximate it using piecewise-linear interpolation or smooth functions like polynoms or splines.

To attain the steady temperature conditions (both in the thermal chamber and in the inertial unit) for the experiment several hours are required, before the experiment can be started. Moreover, temperature points should cover the whole operational temperature range of the inertial unit. For example, if the range is from −70 to +60 degrees Celsius, there should be ten or more temperature points, so the whole set of experiments takes days to be completed.

3. Temperature variations

To take temperature variations of sensor errors into consideration, let us modify the models above. First, we introduce temperature sensor outputs for each inertial sensor: T_{f1}, T_{f2}, T_{f3}, for accelerometers and $T_{\omega 1}$, $T_{\omega 2}$, $T_{\omega 3}$ for gyros. These quantities are arranged into matrices:

$$T_f = \begin{bmatrix} T_{f1} & 0 & 0 \\ 0 & T_{f2} & 0 \\ 0 & 0 & T_{f3} \end{bmatrix}, \qquad T_\omega = \begin{bmatrix} T_{\omega 1} & 0 & 0 \\ 0 & T_{\omega 2} & 0 \\ 0 & 0 & T_{\omega 3} \end{bmatrix}.$$

Then we state the following:

- temperature is measured with an accuracy appropriate to compensate temperature variations of inertial sensor errors;
- temperature stays inside the range where sensor errors have nearly linear temperature variations;
- temperature variations of sensitive axes misalignments can be neglected.

Under the above assumptions we modify the models of sensor errors introducing temperature variations:

$$f'_z - f_z = \Delta f_z^0 + T_f k_{\Delta f} + \Gamma f_z + T_f K_\Gamma f_z + \Delta f_z^s, \qquad k_{\Delta f} = \begin{bmatrix} k_{\Delta f 1} \\ k_{\Delta f 2} \\ k_{\Delta f 3} \end{bmatrix}, \qquad K_\Gamma = \begin{bmatrix} K_{\Gamma 11} & 0 & 0 \\ 0 & K_{\Gamma 22} & 0 \\ 0 & 0 & K_{\Gamma 33} \end{bmatrix},$$

$$\omega'_z - \omega_z = -\nu_z^0 - T_\omega k_\nu - \dot{T}_\omega \Lambda_\nu - \Theta\omega_z - T_\omega K_\Theta \omega_z - \nu_z^s,$$

$$k_\nu = \begin{bmatrix} k_{\nu 1} \\ k_{\nu 2} \\ k_{\nu 3} \end{bmatrix}, \qquad K_\Theta = \begin{bmatrix} K_{\Theta 11} & 0 & 0 \\ 0 & K_{\Theta 22} & 0 \\ 0 & 0 & K_{\Theta 33} \end{bmatrix}, \qquad \Lambda_\nu = \begin{bmatrix} \Lambda_{\nu 1} & 0 & 0 \\ 0 & \Lambda_{\nu 2} & 0 \\ 0 & 0 & \Lambda_{\nu 3} \end{bmatrix}.$$

New coefficients are now added as components to the system state vector. Now we have an extended model instead of (1):

$$\frac{d}{dt}\beta_x = \hat{u}_x \beta_x + L_y^T\left(\nu_z^0 + T_\omega k_\nu + \dot{T}_\omega \Lambda_\nu + \Theta\omega_z + T_\omega K_\Theta \omega_z\right) + L_y^T \nu_z^s \qquad \beta_x(0) = \beta_0,$$
$$\frac{d}{dt}\nu_z^0 = \frac{d}{dt}\Delta f_z^0 = \frac{d}{dt}k_{\Delta f} = \frac{d}{dt}k_\nu = 0, \qquad \frac{d}{dt}\Theta = \frac{d}{dt}\Gamma = \frac{d}{dt}K_\Theta = \frac{d}{dt}K_\Gamma = \frac{d}{dt}\Lambda_\nu = 0. \tag{3}$$

The measurement model is also changed:

$$z^{acc} = \hat{\beta}_x \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T + L_y^T\left(\Delta f_z^0 + T_f k_{\Delta f} + \Gamma f_z + T_f K_\Gamma f_z\right) + L_y^T \Delta f_z^s. \tag{4}$$

As before, we use Kalman filter for the system state estimation [1–3]. To provide observability in this new extended model, the coefficients at new parameters (i.e. temperature and temperature time derivative) should change independently of each other and of other coefficients defined by the inertial unit motion. This is easily achieved in real experiments, e.g. in self-heating or under controlled temperature change in the thermal chamber.

Besides the formal presence of observability in the estimation problem, the quantitative level of accuracy achieved is also important. It can be obtained from the error covariance matrix explicitly given by Kalman filtering algorithm. For example, if the rotation about one of instrument axes occurs when temperature changes too slow (e.g. at the end of self-heating cycle), the error covariances of parameters being observable at this stage of the experiment will also decrease too slowly. Thus, the desired accuracy of estimation is not attained. Because of high order of the system (39 in the case shown above) and complexity of its inner correlations, full analytic accuracy determination seems not to be possible. Therefore even before setting up a real calibration experiment we need to predetermine its conditions. Here we use covariance modeling to estimate the level of accuracy of calibration under different modes of rotation and temperature change.

As now there is no need to wait until temperature gets steady in the inertial unit, the calibration experiment with the new sensor error model (3), (4) becomes much shorter. Although since the number of estimated components is higher and therefore more data (longer record) should be processed, the record can be (and even should be) started right after the inertial measurement unit is switched on.

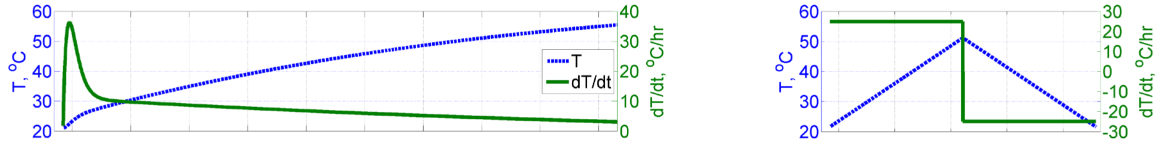
Furthermore, having determined temperature coefficients, but not only parameter value at given temperature, allows extrapolating temperature variation of the sensor error to some extent. Thus, for the given temperature range the required number of experiments becomes smaller.

4. Results

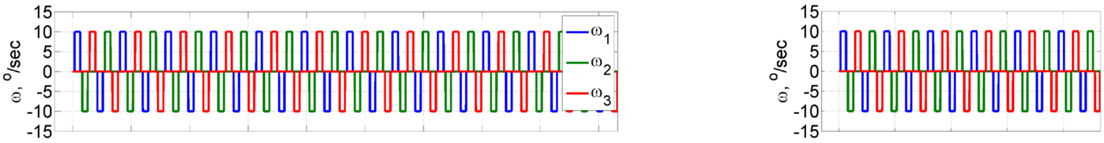
To verify new modifications and make assumptions on what kind of experiments would give the appropriate estimation accuracy, simulation of the navigational grade inertial measurement unit output was processed for two different types of experiments. Both include inertial unit rotation at the rate of approximately 10 degrees per second about each instrumental axis, roughly horizontal. The first one is self-heating, where temperature was recorded from the real inertial unit. The second simulation represents the inertial measurement unit in a thermal chamber, where temperature increases and decreases linearly. Real experiments are expected to have a combination of both temperature modes.

In addition to the conventional set of parameters being calibrated, the new one incorporated time temperature derivative coefficients for gyro drifts, temperature coefficients for sensors null biases and scaling factors. Fig. 1 demonstrates the results (only temperature coefficients are shown).

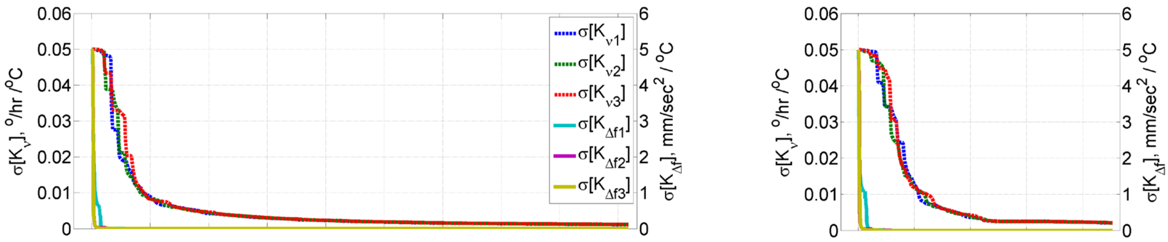
Self-heating **Linear temperature change**



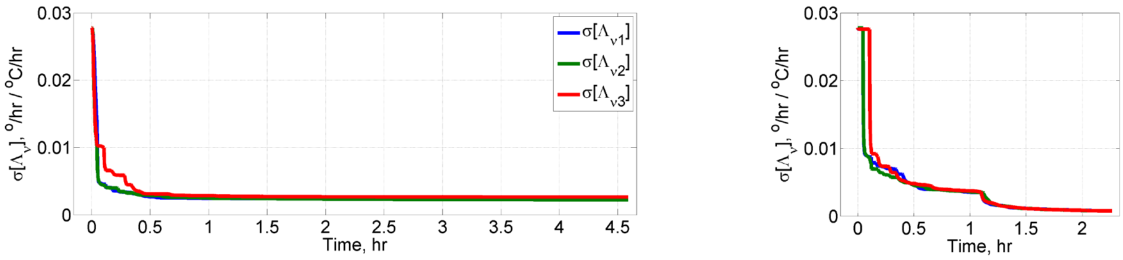
a. Temperature and temperature time derivative



b. Rotation rate



c. Estimation of error covariances (square root) for temperature coefficients of null biases



d. Estimation of error covariances for time temperature derivative coefficients of gyro drifts

Fig. 1. Estimation error covariances in two types of experiment

The plots show that the observability exists in both types of experiment, with final covariances being at least one order of magnitude smaller than its starting values.

5. Conclusions

The modifications of the calibration method for the inertial measurement unit on a single-axis low grade turntable include changes in error equations from (1), (2) to (3), (4). After that temperature time derivative coefficients of gyro null biases can be estimated simultaneously with the rest of conventional parameters of sensor instrumental errors model. The calibration experiment then should involve temperature change along with rotations about every instrumental axis, roughly horizontal. Covariance modeling showed the following.

1. A self-heating mode is expected to ensure all unique rotations over every inertial unit axis while time temperature derivative is changing intensively enough (for the inertial unit used in the experiment it is approximately 10 minutes). In this case two-axis turntable is much preferred as it allows quick switching between axes of rotation and their easy horizontal positioning.

2. In a thermal chamber with temperature decreasing and increasing linearly over time, the variation of temperature and its derivative is greater. As a result, the experiment can be two times shorter having the same or better estimation accuracy.

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