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surfaces or, more generally, C-complexes), it can be computed by means of the Fox calculus. Combined with the Wirtinger presentation, this gives us a simple algorithm computing the slope in terms of the link diagram.

The original motivation for this work was our formula [1] for the multivariate signature (defined following the approach suggested by Rokhlin and Viro) of the splice $L' \cup L''$ of two colored links $K' \cup L' \subset \mathbb{S}'$ and $K'' \cup L'' \subset \mathbb{S}''$. The signature is almost additive:

$$\sigma_L(\omega', \omega'') = \sigma_{K' \cup L'}(\upsilon'', \omega') + \sigma_{K'' \cup L''}(\upsilon', \omega'') + \delta_{\lambda'}(\omega')\delta_{\lambda''}(\omega''),$$

where $\upsilon^* := \omega^*[K^*]$ and the correction term $\delta_{\lambda'}(\omega')\delta_{\lambda''}(\omega'')$ depends only on the combinatorial characteristics of the links (their *linking vectors* λ', λ''). This formula holds unless $\upsilon' = \upsilon'' = 1$, *i.e.*, unless both characters ω', ω'' are admissible. In the exceptional case, which was left open in [1], the formula takes the form

$$\sigma_L(\omega', \omega'') = \sigma_{L'}(\omega') + \sigma_{L''}(\omega'') + \delta_{\lambda'}(\omega')\delta_{\lambda''}(\omega'') + \Delta\sigma(\kappa', \kappa''),$$

where the extra correction term

$$\Delta\sigma(\kappa', \kappa'') := \operatorname{sg}\kappa' - \operatorname{sg}\Big(\frac{1}{\kappa'} - \kappa''\Big)$$

depends on the slopes $\kappa^* := (K^*/L^*)(\omega^*)$. (For the purpose of this statement, we disambiguate $\infty - \infty$ to 0 and let $\operatorname{sg}\infty = 0$.) Note that this extra term is the only contribution of the knots K', K'' along which the links are spliced. Note also that both slopes are well defined and real as the characters involved are unitary.

Should time permit, I will also discuss further properties of the new invariant. For example, the slope is a concordance invariant away from the so-called *concordance roots*. The concept of slope extends to a special class of tangles; the corresponding signature formula generalizes and refines the skein relations for the signature.

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Ivan Dynnikov. *A method to distinguishing Legendrian and transverse knots*

(The talk is based on recent joint works with Maxim Prasolov and Vladimir Shastin)

A smooth knot (or link) K in the three-space $\mathbb{R}^3$ is called *Legendrian* if the restriction of the 1-form $\alpha = x\,dy + dz$ on K vanishes, where x, y, z are the standard coordinates in $\mathbb{R}^3$. If $\alpha|_K$ is everywhere non-vanishing on K, then K is called *transverse*.

Classification of Legendrian and transverse knots up to respectively Legendrian and transverse isotopy is an important unsolved problem of contact topology. A number of useful invariants have been constructed in the literature, but there are still small complexity examples in which the existing methods do not suffice to decide whether or not the given Legendrain (or transverse) knots are equivalent.

We propose a totally new approach to solving the equivalence problem for Legendrian and transverse knots, which allows to practically distinguish between non-equivalent knots in small complexity cases, and gives rise to a complete algorithm in the general case.

Nikolai Erokhovets. *Combinatorics and hyperbolic geometry of families of 3-dimensional polytopes: fullerenes and Pogorelov polytopes*

By a *polytope* we mean a class of combinatorial equivalence of 3-dimensional convex polytopes. A *k-belt* is a cyclic sequence of k faces such that faces are adjacent if and only if they follow each other, and no three faces have a common vertex. A simple polytope different from the *simplex* Δ^3 is *cyclic edge k-connected (ck-connected)*, if it has no l-belts for $l < k$, and *strongly ck-connected (c*k-connected)*, if in addition any its k-belt surrounds a face. By definition Δ^3 is c^*3-connected but not $c4$-connected. Any simple polytope (family $\mathscr{P}_s$) is $c3$-connected and at most c^*5-connected. We obtain a chain of nested families:

$$\mathscr{P}_s \supset \mathscr{P}_{aflag} \supset \mathscr{P}_{flag} \supset \mathscr{P}_{aPog} \supset \mathscr{P}_{Pog} \supset \mathscr{P}_{Pog*}$$

The family of $c4$-connected polytopes coincides with the family $\mathscr{P}_{flag}$ of *flag* polytopes defined by the property that any set of pairwise adjacent faces has a non-empty intersection. The family of c^*3-connected polytopes we call *almost flag* polytopes and denote $\mathscr{P}_{aflag}$. Results by A.V. Pogorelov (1967) and E.M. Andreev (1970) imply that $c5$-connected polytopes (family $\mathscr{P}_{Pog}$ of Pogorelov polytopes) are exactly polytopes realizable in the Lobachevsky space $\mathbb{L}^3$ as bounded polytopes with right dihedral angles, and the realization is unique up to isometries. Andreev's result implies that flag polytopes are exactly polytopes realizable in $\mathbb{L}^3$ as polytopes with equal non-obtuse dihedral angles. An example of Pogorelov polytopes is given by *k-barrels* B_k, $k \geqslant 5$, see Fig. 1a). Results by T. Dǒslić (1998, 2003) imply that the family $\mathscr{P}_{Pog}$ contains *fullerenes*, that is simple polytopes with only pentagonal and hexagonal faces.

The family $\mathscr{P}_{aPog}$ of c^*4-connected polytopes we call *almost Pogorelov* polytopes, and the family $\mathscr{P}_{Pog^*}$ of c^*5-connected polytopes – *strongly Pogorelov*. G. D. Birkhoff (1913) reduced the 4-colour problem to the family $\mathscr{P}_{Pog^*}$.

A simple polytope with all faces except for the n-gon being pentagons and hexagons is called an *n-disk-fullerene*.

Proposition 1 ([1], [2]). *Any 3-disk-fullerene belongs to $\mathscr{P}_{aflag}$, any 4-disk-fullerene – to $\mathscr{P}_{aPog}$, and any 7-disk-fullerene – to $\mathscr{P}_{Pog}$. For each $n \geqslant 8$ there exist an n-disk-fullerene in $\mathscr{P}_{Pog^*}$ and an n-disk fullerene not in $\mathscr{P}_{aflag}$.*

T. E. Panov remarked that Andreev's results should imply that almost Pogorelov polytopes correspond to right-angled polytopes of finite volume in $\mathbb{L}^3$. Such polytopes may have 4-valent vertices on the absolute, while all proper vertices have valency 3.

Theorem 2 ([3]). *Cutting of 4-valent vertices defines a bijection between classes of combinatorial equivalence of right-angled polytopes of finite volume in $\mathbb{L}^3$ and almost Pogorelov polytopes different from the cube I^3 and the pentagonal prism $M_5 \times I$.*

We develop a theory of combinatorial construction of families of polytopes. The main idea is to build a family by a given set of operations from a small set of initial polytopes. A classical result by V. Eberhard (1891) states that any simple polytope can be obtained from the simplex Δ^3 by cuttings off vertices, edges and pairs of adjacent edges.

Proposition 3 ([3]). *A simple polytope belongs to $\mathscr{P}_{aflag}$ if and only if it can be obtained from the simplex with at most two vertices cut by cuttings off vertices, edges and pairs of adjacent edges not equivalent to cutting off a vertex of a triangle, and if and only if it is obtained by simultaneous cutting off a set of vertices of Δ^3 or a flag polytope.*

Results by A. Kotzig (1969) imply that a simple polytope is flag iff it can be obtained from I^3 by cuttings off edges and pairs of adjacent edges of at least hexagonal faces. The family $\mathscr{P}_{aPog}$ contains I^3, $M_5 \times I$, and the 3-dimensional *Stasheff polytope As^3*, which is the cube with three pairwise disjoint orthogonal edges cut. A result by D. Barnette (1974) implies that a simple polytope belongs to $\mathscr{P}_{aPog}\backslash\{I^3, M_5 \times I\}$ iff it can be obtained from As^3 by cuttings off edges not lying in quadrangles and pairs of adjacent edges of at least hexagonal faces. Unlike the case of flag polytopes, not any quadrangle of a polytope in $\mathscr{P}_{aPog}$ is obtained by cutting off an edge of a polytope of the same family. However, results by D. Barnette imply that if a polytope in $\mathscr{P}_{aPog}$ has quadrangles, then at least one quadrangle can be obtained in this way. A *matching* of a polytope is a set of its pairwise disjoint edges. A matching is *perfect*, if it covers all the vertices. Let P_8 be the cube with two disjoint orthogonal edges cut.

Theorem 4 ([3]). *Any almost Pogorelov polytope $P \neq I^3, M_5 \times I$ is obtained by cutting off a matching of a polytope in $\mathscr{P}_{aPog} \sqcup \{P_8\}$ producing all the quadrangles.*

A polytope in $\mathbb{L}^3$ is *ideal*, if all its vertices lie on the absolute. It has a finite volume.

Corollary 5 ([3]). *Any ideal right-angled polytope P is obtained from some polytope $Q \in \mathscr{P}_{aPog} \sqcup \{P_8\}$ by the contraction of edges of some perfect matching not containing opposite edges of any quadrangle.*

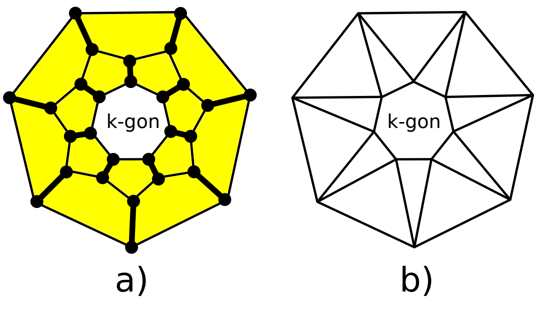

Figure 1. a) canonical perfect matching of the k-barrel; b) k-antiprism.

Example 6. The k-barrel has a canonical perfect matching drawn on Fig. 1a). The corresponding ideal polytope is called a *k-antiprism*, see Fig. 1b).

An operation of an *edge-twist* is drawn on Fig. 2. Two edges on the left lie in the same face and are disjoint. Let us call an edge-twist *restricted*, if both edges are adjacent to an edge of the same face. In the survey (2017) A. Yu. Vesnin combining results by I. Rivin (1996) on ideal polytopes and by G. Brinkmann, S. Greenberg, C. Greenhill, B.D. McKay, R. Thomas, P. Wollan (2005) on quadrangulations of a sphere stated that any ideal right-angled polytope can be obtained from a k-antiprism, $k \geqslant 3$, by edge-twists.

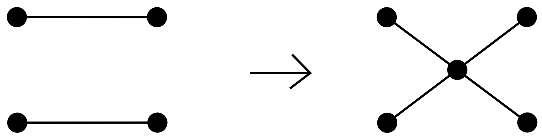

Figure 2. An edge-twist.

Theorem 7 ([3]). *A polytope is realizable as an ideal right-angled polytope iff it either is a k-antiprism, $k \geqslant 3$, or can be obtained from the 4-antiprism by restricted edge-twists.*

Results by I. Rivin (1994) imply that a realization of a polytope as an ideal polytope in $\mathbb{L}^3$ is unique up to isometrices.

Conjecture 8. *An edge-twist increases the volume of a right-angled polytope in $\mathbb{L}^3$.*

All k-barrels, $k \geqslant 5$, belong to $\mathscr{P}_{Pog*}$. Results by D. Barnette (1974,1977), J. W. Butler (1974) and results from [1] imply that a simple polytope different from these barrels belongs to $\mathscr{P}_{Pog}$ iff if it can be obtained from the 5- or the 6-barrel by cuttings off pairs of adjacent edges of at least hexagonal faces and connected sums with the 5-barrel (Fig. 3), and to the family $\mathscr{P}_{Pog*}$ iff it can be obtained from the 6-barrel by cuttings off pairs of adjacent edges of at least hexagonal faces. T. Inoue (2008) showed that both operations increase the hyperbolic volume and enumerated the first 825 bounded right-angled polytopes in the order of the increasing volume (2015).

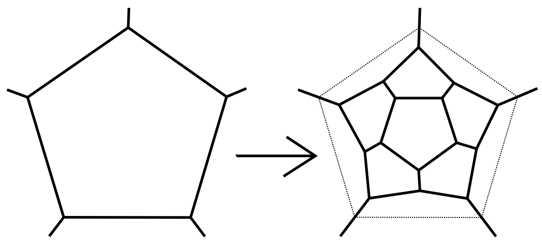

Figure 3. A connected sum with the 5-barrel.

For fullerenes there is a stronger result than for Pogorelov polytopes. There is a 1-parametric series of fullerenes obtained from the 5-barrel by connected sums with the 5-barrel along pentagons surrounded by pentagons. It consists of the 5-barrel and the so-called $(5,0)$-*nanotubes*. Results by F. Kardoš, R. Skrekovski (2008) and, independently, by K. Kutnar, D. Marušič (2008) imply that all the other fullerenes lie in $\mathscr{P}_{Pog*}$.

Theorem 9 ([1]). *Any fullerene different from the* 5*-barrel and the* $(5,0)$*-nanotubes can be obtained from the* 6*-barrel by a sequence of cuttings off pairs of adjacent edges of at least hexagonal faces in such a way that intermediate polytopes are either fullerenes or* 7*-disk-fullerenes with the heptagon adjacent to a pentagon.*

The difficulty is that the construction of the family $\mathscr{P}_{Pog*}$ does not guarantee that intermediate polytopes are close to fullerenes.

Theorem 10 ([2]). *A* 7*-disk-fullerene is not in* $\mathscr{P}_{Pog*}$ *iff it is obtained from a fullerene by a sequence of connected sums with the* 5*-barrel. Any* 7*-disk-fullerene from* $\mathscr{P}_{Pog*}$ *can be obtained from the* 6*-barrel by a sequence of cuttings off pairs of adjacent edges of at least hexagonal faces in such a way that intermediate polytopes have pentagonal, hexagonal and at most two heptagonal faces.*

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