Journal of Physics: Conference Series

PAPER • OPEN ACCESS

On two experiments with falling chains

To cite this article: A A Domnyshev *et al* 2019 *J. Phys.: Conf. Ser.* **1301** 012012

IOP Conf. Series: Journal of Physics: Conf. Series **1301** (2019) 012012 doi:10.1088/1742-6596/1301/1/012012

On two experiments with falling chains

A A Domnyshev[1], V A Kalinichenko[1,2] and P M Shkapov[1]

[1]Bauman Moscow State Technical University, Moscow, 105005, Russia
[2]A. Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, 119526, Russia

E-mail: domnyyshevaa@student.bmstu.ru; kalin@ipmnet.ru; spm@bmstu.ru

Abstract. The article considers paradoxical results of metal chain movement that do not agree with classical theoretical models. We present the results of experiments with the falling U-folded chain and with chain fountain. The velocity and acceleration of chain motion were measured and estimated. The trajectory of the fountaining chain links has been investigated experimentally. A comparison of the experimental data with existing theoretical models is made.

1. Introduction

Modern courses and textbooks on Dynamics usually include the section of "Dynamics of systems of variable mass" in which the problems of the motion of variable-mass rockets are mainly analyzed as examples. Currently, many existing or developed engineering systems include cables, ropes or chains – e.g., tethered satellite systems [1]. For this reason, the department of theoretical mechanics of MSTU planned the possible inclusion of a section on chains motion in the educational program on Dynamics with the subsequent development of laboratory student work. Two classic problems were chosen. The first is the problem of free falling folded chain [2, 3], and the second is Cayley's task of chain falling from a pile on the table to the floor [4].

A review of the existing literature on these problems has shown that experiments conducted in the last 20-30 years give paradoxical results that do not agree with classical theoretical models.

First, in experiments with a chain that is suspended at both ends at the same level and then one of the ends is released (figure 1, a), the acceleration of the free end is greater than g the acceleration of gravity [5-7].

The second paradoxical behavior is a chain fountain (figure 1, b). The chain falling under the action of gravity rises above the plate, forming a stable arch (fountain). Since the first video posted on the Internet [8], a huge amount of models has been published on the possible mechanism for the fountain formation [9-11]. However, a single point of view has not yet been achieved.

2. Experiment 1: Dynamics of U-folded chain (figure 1, a)

The effect in our experiments we used two metal ball chains of length l =80 cm and diameter of d = 1.5 and 4.5 mm – Figure 2, a. The each ball chain consists of identical segments made from rods (0.6 and 2.9 mm) and hollow spheres attached to each other. The mass of each chain is m =5 and 30 g, and the number of balls in the chain is N = 426 and 133, respectively.

For the U-chain experiment, one of the ends of the chain was fixed and the other end was released from the same height. The system was in vertical position, and to determine the position of the falling chain end we used high-speed video (500 fps; VS-Fast). Further video processing was performed using the ImageJ 1.46r and Wolfram Mathematica softwares. The purpose of processing was to determine the time dependence of the vertical coordinate $y(t)$ of the falling end of the chain – see figure 1, a. The obtained dependence was compared with data on the free fall of a steel ball with a diameter of 8 mm.

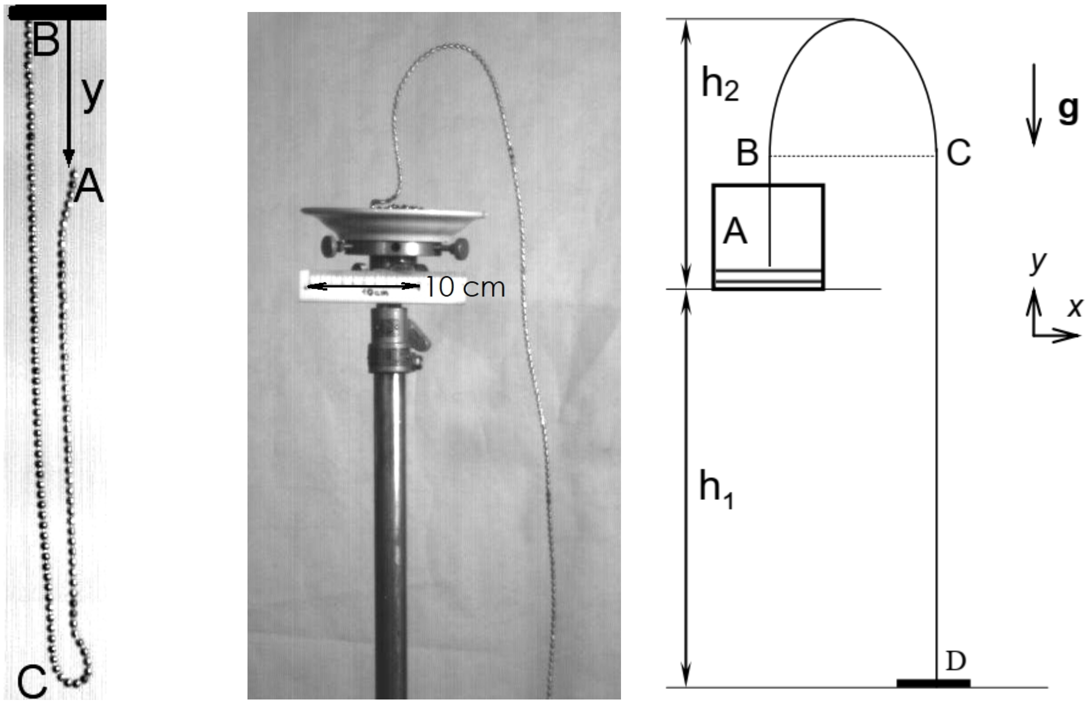


Figure 1. a) The falling U–folded chain. b) The chain fountain

Data on the fall of the U-folded chains (*1*), the free fall of the steel ball (*2*) and the calculated dependence (3) of $y = gt^2/2$ are shown in Figure 2, b. One can see that the data (*1*) lie somewhat above (*2*) and (*3*), which allows us to conclude that the chain has fallen faster.

To interpret our experimental data, we use models [5, 12] that describe the dynamics of an energy-conservative chain.

We use the one-dimensional approximation (figure 1, a), in which the chain is represented by two segments of the AC and the EC. The kinetic energy of the falling segment AC is

$$T = \frac{m\dot{y}^2}{2} = \frac{M}{4l}(l-y)\dot{y}^2 = \frac{\mu}{4}(l-y)\dot{y}^2,$$

and the potential energy relative to point B is

$$\Pi = -\frac{\mu g}{4}(l^2 + 2ly - y^2)$$

Here, $m = \dfrac{M(l-y)}{2l}$ is the mass of the AC segment; $\mu = M/l$ is the linear mass density of the chain; M and l are the mass and length of the chain, respectively.

From the Lagrange equation the acceleration of AC is equal

$$\ddot{y} = g + \frac{\dot{y}^2}{2(l-y)} \qquad (1)$$

Eq. (1) shows that the acceleration of the falling chain exceeds the acceleration of gravity.

Because the system is conservative, then

$$E = T + \Pi = E\big|_{\substack{y=0\\ \dot{y}=0}}$$

or

IOP Conf. Series: Journal of Physics: Conf. Series **1301** (2019) 012012 doi:10.1088/1742-6596/1301/1/012012

$$\frac{\mu}{4}(l-y)\dot{y}^2-\frac{\mu g}{4}(l^2+2ly-y^2)=-\frac{\mu g l^2}{4}$$

For the velocity $v(y)$ we obtain

$$v(y)=\dot{y}=\sqrt{2gy}\sqrt{\frac{1-y/2l}{1-y/l}} \qquad (2)$$

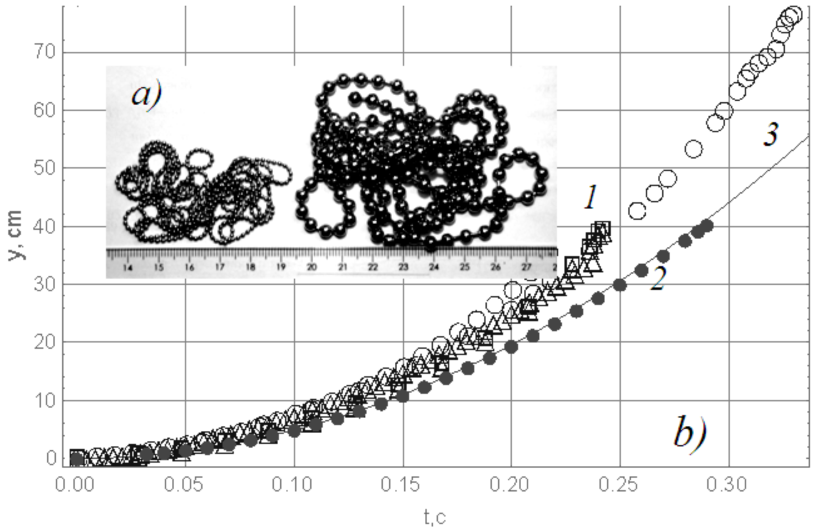


Figure 2. a) Two chains used in experiments. b) The dependence of the vertical coordinate of the chains (*1*) and the ball (*2*) on time; *3* – the graph of $y=gt^2/2$

Eq. (2) can be integrated numerically. The total fall time of the free end of the chain is $t_0=0.85\sqrt{2l/g}$, which is 15% less than the time of free fall $\sqrt{2l/g}$. In dimensionless variables of y/l and $\tau=t/\sqrt{2l/g}$, the law of motion of the free chain end $y/l=f(\tau^2)$ is shown in figure 3 (curve 2).

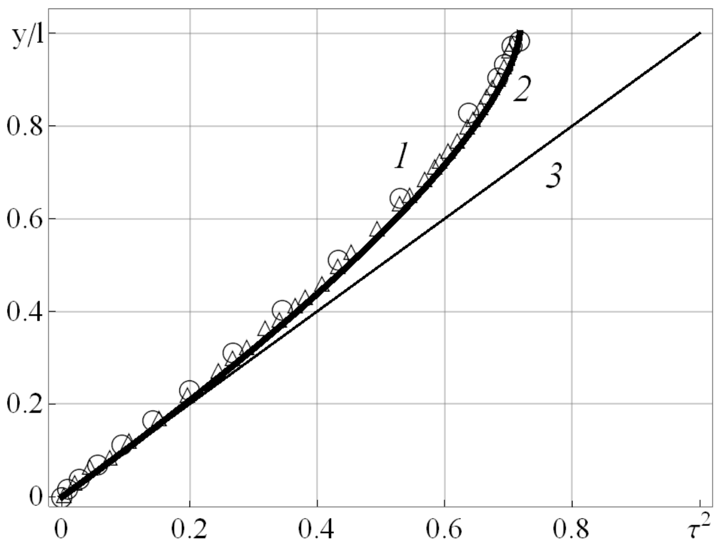


Figure 3. The law of motion of the free chain end: of the chain: *1* – experimental data; *2* – numerical solution of Eq. (2); *3* – free fall, $y/l=\tau^2$

IOP Conf. Series: Journal of Physics: Conf. Series **1301** (2019) 012012 doi:10.1088/1742-6596/1301/1/012012

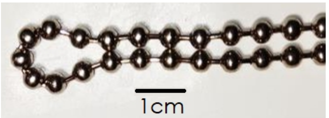
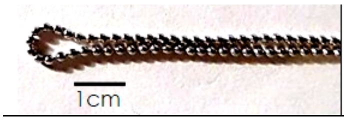
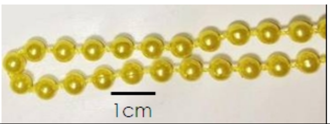
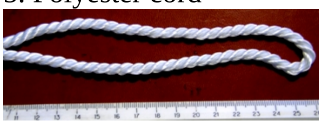
In figure 3, the experimental data (*1*) in a dimensionless form are perfectly described by the calculated curve (*2*), but the conservative model used is not free from shortcomings.

The acceleration of the falling chain is greater than g , and its velocity is greater than the free fall gt . The reason for this is a non-zero reaction $\mu \dot{y}^2 / 4$ in the chain to the free side of the bend C (figure 1, a) – this force pulls the chain down in addition to gravity [5–7, 11, 12]. But for energy-conserving chains, the acceleration (1) and velocity (2) of the falling chain tend to infinity when $y = l$. It can be assumed that energy losses should be taken into account at the stage of joining the last moving link to the fixed part of the chain. This is what will allow avoiding infinite values of velocity and acceleration.

3. Experiment 2: Chain fountain (figure 1, b)

The experiments were conducted as part of a student research project [13]. In the chain fountain experiments discussed below we used five different chains and a cord (Table 1).

Table 1. Characteristics of chains and cord.

Chain	Length, m	Number of link per meter	Diameter, mm	Number of link required for U turn	h_1, cm	h_2, cm
1. Metal balls 	23.45	167	4.58	6	215	19.5
2. Metal balls 	3.56	520	1.20	6	215	12.5
3. Plastic balls 	6.00	170	4.94	3	215	5.1
4. Makarony (maccheroni, pasta with a hole) 	9.00	25	15.00	6	215	11
5. Polyester cord 	15.00	-	6.00	-	215	9

As experiments have shown a necessary condition for the phenomenon of chain fountain is non-zero height between an initial position and a final position of chain ($h_1 > 0$). The second is given to the system momentum or enough gravity force. The chain was fed into a shallow ceramic plate in a random configuration, and the plate was elevated to a distance h_1 above the floor. The end of the chain was put on a floor at the initial moment of time for the beginning of chain motion (with zero initial velocity) and the initiation of the fountain formation.

We found that the phenomenon of fountain was observed in experiments with all chains and cord, but especially strong siphoning is characteristic of a metal ball chain with a diameter of 4.5 mm (Table 1). All experiments presented below were carried out with this chain.

In the second stage of the experiments we investigated the dependence of fountain height h_2 on the initial position h_1 of the chain. The position relative to the floor of the plate was varied; the fountain height was measured (figure 4).

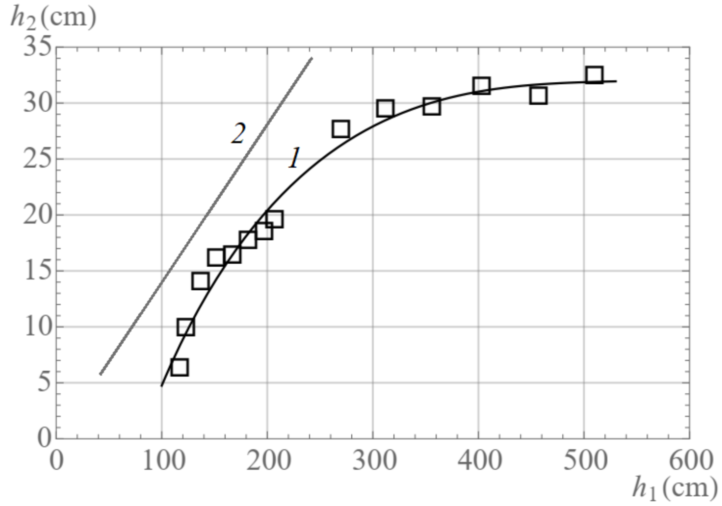


Figure 4. The dependence of fountain height h_2 on the initial position of chain h_1:
1 –experiment, 2 – the linear fit $h_2 = 0.14\,h_1$ obtained in [9] for the same chain

From the data (1) presented in figure 4, it follows that the dependence is essentially nonlinear. The figure also shows the linear dependence obtained in [9]: $h_2 = 0.14\,h_1$. Linear approximation of our experimental data can be used only for a range h_1 = 0 – 200 cm.

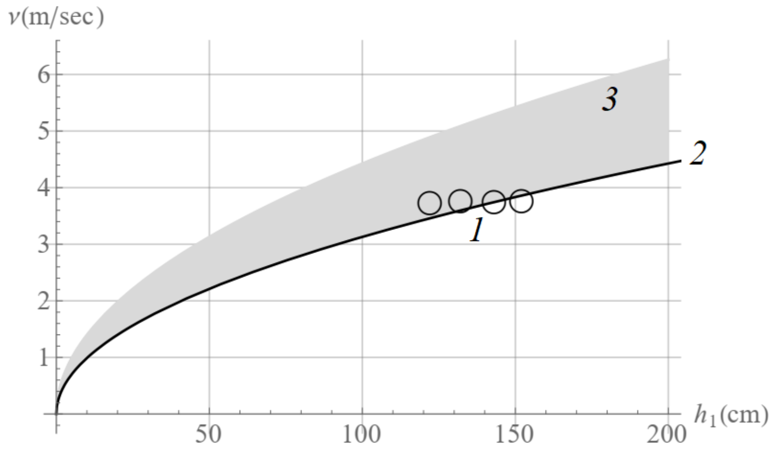


Figure 5. Comparison of our experimental data *1* with analytical expressions for steady velocity: *2* – $v = \sqrt{gh_1}$ (Eqs. (3) and (5)); *3* – $v = \sqrt{2gh_1/(2\alpha+1)}$, $\alpha \in (0, 0.5)$ (Eq. (4))

According to [9, 14], velocity of siphoning chain is constant on the steady-state process, and our experimental data confirm this conclusion.

The speed of the chain links was determined at different heights h_1 of the initial position, using high speed video (500 fps). Results are presented in Table 2 and Figure 5.

Table 2. Velocity of links at stationary chain siphoning.

Height h_1 ,m	1.22	1.32	1.43	1.52
Steady velocity, m/sec	3.74	3.75	3.78	3.78

To interpret our experimental results we use several theoretical models.

The classical Meshchersky's problem [15] of a chain falling from the upper platform to the lower one (figure 6), an expression for the steady velocity of the chain links is

$$v = \sqrt{gh_1} \tag{3}$$

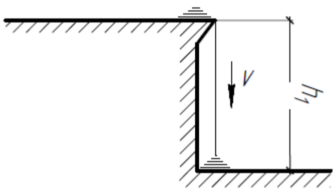


Figure 6. The classic problem of a chain falling solved by Meshchersky (1904)

In the simplified model [16], the chain was undergone to the ideal constraint which makes the chain to have the semicircle shape of the upper part of the fountain. From the Lagrange equation, the velocity of steady chain motion is

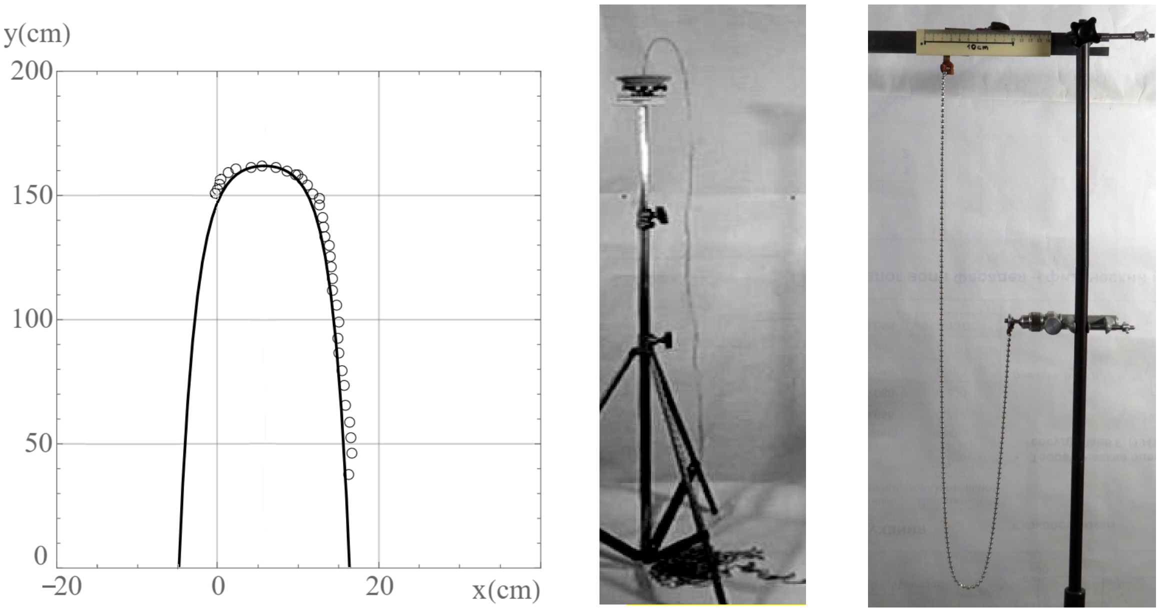
$$v = \sqrt{2gh_1/(2\alpha+1)},\ \alpha \in (0, 0.5) \tag{4}$$

We observed the fountaining of the cord (Table 1). In [13] an equation for the velocity of movement of chain links was obtained using the applied thread mechanics

$$v = \sqrt{gh_1} \tag{5}$$

Our experimental results and analytical expressions (3–5) are presented in Figure 5. We found that different models describing the chain motion give similar speeds. It is worth noting that the expression (3) was obtained in solving classical problems of mechanics, in which significant simplifications are used.

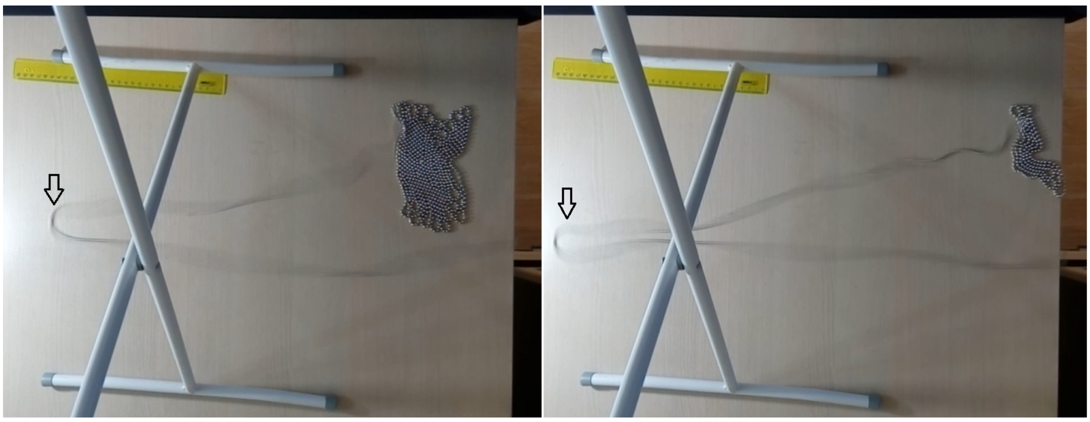
Based on the high-speed video of chain fountain, the coordinates of the points of the trajectory were calculated. An approximating function was chosen. In figure 7,a points belong to the trajectory of the chain links with a height $h_1 = 1{,}5$ m, the curve is a graph of the function $y = -\alpha \cdot \mathrm{Cosh}\left(x/\alpha\right)$. According to the data of the experiment, the parameter α does not depend on the height h_1. The parameter lies in the interval $\alpha \in (2; 2{,}5)$, and its average value is $\langle \alpha \rangle = 2.2$ for the metal balls chain (Table 1, No 1).



a) b)

Figure 7. a) The trajectory of the chain links in the steady process of fountaining $h_1 = 1,5$ m. b) The catenary is a form that a chain takes with fixed ends under the force of gravity

The chain links trajectory is an inverted “catenary” – a line shaped like a heavy chain with fixed ends in a gravity field (figure 7, b). This form is theoretically substantiated in [9].



a) b)

Figure 8. Metal chain fountain in a horizontal plane: a) the beginning of the experiment; b) the ending of the experiment. Arrow points fountain

We also conducted a series of experiments that confirmed the possibility of chain fountaining in the horizontal plane – figure 8. The chain was placed on a horizontal table; the end of the chain was put on the floor. The fountain was observed in the horizontal plane, and over the time the layers of the chain began to move opposite to the direction of movement of the chain.

4. Concluding remarks

We have presented experimental results of metal chain motion that do not agree with classical theoretical models. We conducted experiments with the falling U-folded chain and with chain fountain. The velocity and acceleration of chain motion were measured and estimated. The trajectory of the fountaining chain links has been investigated experimentally. A comparison of the experimental data with existing theoretical models is made.

Although both experiments are very effective and illustrative when demonstrated to students, their theoretical description requires further development.

References

[1] Aslanov V S, Ledkov A S 2012 Dynamics of tethered satellite systems *Oxford WP* 356 p.
[2] Love A E H 1921 Theoretical Mechanics (3rd ed.) *Cambridge U.P., Cambridge* 326 p.
[3] Lamb H 2009 Dynamics (reprinted) *Cambridge U.P., Cambridge* 368 p.
[4] Cayley A 1857 On a class of dynamical problems *Proc. R. Soc. London* **8** pp 506–511
[5] Calkin M G, March R H 1989 The dynamics of a falling chain *Am. J. Phys.* **57** pp 154-157
[6] Schagerl M, Steindl A, Steiner W, Troger H 1997 On the paradox of the free falling folded chain *Acta Mech.* **125** pp 155–168
[7] Virga E G 2015 Chain paradoxes *Proc. R. Soc. A* **471** 20140657
[8] Mould S 2013 Self-siphoning beads URL: http://stevemould.com/siphoning-beads/
[9] Biggins J, Warner M 2014 Understanding the chain fountain *Proc. R. Soc. A* **470** 20130689
[10] Pantaleone J 2017 A quantitative analysis of the chain fountain *Am. J. Phys.* **85** pp 414–421
[11] Yokoyama H 2018 Reexamining the Chain Fountain URL: https://arxiv.org/abs/1810.13008
[12] Wong C W, Yasui K 2006 Falling chains *Am. J. Phys.* **74** (6) pp 490–496
[13] Domnyshev A A 2018 On the metal balls chain spouting (in Russian) *Politekhnichesikj molodyozhnyj zhurnal MGTU* 12(29)
[14] Belyaev A, Sukhanov A, Tsvetkov A 2016 Gushing metal chain *Front. Mech. Eng.* **11**(1) pp 95-100
[15] Meshchersky I V 1952 Works on the mechanics of bodies of variable mass (in Russian) *Moscow GITTL* 280 p.
[16] Zubelevich O 2015 The Chain fontain again URL: http://arxiv.org/abs/1503.06663v5