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SOIL MECHANICS

DETERMINATION OF THE UNLOADING MODULUS OF DEFORMATION IN DISPERSIVE SOILS AND ITS ACCOUNTING IN DESIGNING

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The aspects of determining the moduli of unloading and reloading of dispersed soils were considered. Sample tests involving triaxial compression devices showed that these parameters depend on the stress-strain state. The ratios between the primary loading and reloading moduli accepted in design practice were significantly underestimated, which led to errors in determining the settlement of structures. In accordance with the results of the study, a methodology for determining the moduli of unloading and reloading was proposed.

Introduction

Most decisions of soil mechanics are based on the assumption of single loading of soil, in which elastic and plastic deformations occur [1, 2]. Thus, the active development of the underground space denotes the importance of considering the soil base in the field of unloading/reloading. When erecting buildings in deep foundation ditches, the base is unloaded, reaching hundreds of kPa, and in certain cases (underground parking without aboveground parts, tunnels, and subsurface metropolitan railway), the weight of the structure fails to reach the weight of soil excavated from the foundation ditch. In this case, unloading deformations calculated with a constant strain modulus become extremely large.

Although soil is an elastic-plastic body with the predomination of plastic strain, unloading deformations can play a significant role, especially in the construction of tunnels with a large laying depth.

The different physicomechanical natures of elasticity, as a property of a material that restores partially the shape and volume during stress removal, in rocky and dispersed soils determine the significant dependence of the elastic characteristics of dispersed soils on the type of stress state, the achieved level of strains, and the degree of approach to the ultimate state.

Currently, various methods account for unloading deformations. In Ref. [3], when calculating the subsidence, the use of the modulus of deformation during unloading $E_{e,i}$, which is determined experimentally, is proposed, or in the absence of experimental data, the equivalent $5E_i$ is obtained. This approach avoids overstating the calculated unloading deformations in deep foundation ditches. At the same time, no recommendations for the experimental determination of this parameter in dispersed soils are provided in the current regulatory documents [4]. Furthermore, for rocky [5] and semi-rocky soils [4], direct instructions are given on the stress range in which the modulus of elasticity along the unloading branch is necessarily computed.

Many contemporary models used in software systems apply various strain characteristics during loading and unloading. In this case, for primary loading, the nonlinearity of strain development is modeled based on the hyperbolic law [6] in hardening soil models [7] or the logarithmic law in the models

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of Cam Clay and Soft Soil family [8-10]; for unloading, the dependences of the linear-elastic Hooke model are used. The hardening soil model also uses the dependence of the unloading modulus E_{ur} on the actual mean effective stress p'. Moreover, hardening soil small-strain model is one of the several models describing the strain nonlinearity during unloading and reloading; however, the shape and slope of the unloading loop are independent of the degree of approach to the threshold state [11, 12]. The situation is also complicated by the absence of any authorial recommendations for determining the parameters of these models.

One work [7] included the hardening soil parameters determined for the near-wall sand of fluffy consistence. In this study, the authors established the relationship between the unloading and reloading modulus and the secant modulus at stress rate of 50% strength as follows:

$$E_{ur} = 3E_{50} . \tag{1}$$

This relation is approximately preserved for the strain moduli of sand of various densities [13]. Most users of the software package perceive this relationship as the only possible one and already use it for all types and varieties of dispersed soils; such practice is definitely erroneous.

Experimental data indicate that in dispersed soils, the unloading strain modulus (slope of the "loop") depends on the degree of approach to the threshold state. For small relative strains, the unloading moduli are an order of magnitude higher than those for strains corresponding to the threshold shear resistance. In modern models known to the authors, this fact is not considered.

The paper presents experimental data confirming the dependence of the unloading strain modulus on the selected unloading point and provides practical recommendations for determining this parameter. These recommendations can be considered when making changes to regulatory documents.

Materials and Methods

The work used the results of tests on samples of non-cohesive and cohesive dispersed soils in the fifth-generation triaxial compression devices manufactured by the Research and Production Enterprise Geotek, in accordance with the consolidated-drained tests with pore pressure control. The tests were conducted on the Upper Jurassic clay soils of marine genesis and the Lower-Middle Pleistocene sandy soils of the fluvioglacial genesis, which were selected in Moscow from the depths of 17.00 and 9.85 m, respectively.

Characterization of the physical properties of pulverescent sand included the density of soil particles ρ_s at 2.68 g/cm^3; natural humidity W_e of 13.6%; porosity coefficient e of 0.650; dry soil density ρ_d of 1.62 g/cm^3; particle contents of 5-2 mm (0.4%), 2-1 mm (0.7%), 1-0.5 mm (4.1%), 0.5-0.25 mm (18.0%); 0.25-0.10 mm (47.8%), and <0.10 mm (29.0%) [14].

Characterization of the physical properties of heavy semi-solid clay included ρ_s of 2.69 g/cm^3; W_e of 48.9%; natural soil density ρ of 1.61 g/cm^3; ρ_d of 1.08 g/cm^3; e of 1.488; humidity on limit of liquidity W_L of 87.3%; humidity at the limit of plasticity W_P of 45.1%; plasticity index I_P of 42.2; liquidity index I_L of 0.09; water saturation coefficient S_r of 0.88.

The samples measured 50 mm in diameter and 100 mm in height. A sandy soil sample was formed by dry layer-by-layer filling, reaching a predetermined porosity coefficient (e = 0.650), followed by forced water saturation through the lower drain until water appeared in the upper drain. A clay soil sample was carved from a natural monolith.

After installing the samples in a triaxial compression chamber, a stepwise comprehensive compression was performed to domestic load with closed drains, and consolidation was performed at the same load. Lateral pressure σ_3 for the sand and clay samples measured 141 and 251 kPa, respectively.

The consolidation times for the clay and sand samples were 24 and 6 h, respectively.

The deviatoric loading of the samples was performed in the kinematic mode. The strain rates were 0.1 and 0.002 mm/min for the sand and clay samples, respectively. For sandy soil, nine stages of unloading were conducted, whereas four stages were considered for clay soil. Unloading was performed when

TABLE 1

Unloading stages	Unloading start criteria for			
	sandy sample		clay sample	
	D, kPa	% of D_{max}	D, kPa	% of D_{max}
1	51	11	101	23
2	100	22	201	47
3	151	33	301	70
4	200	43	391	91
5	251	54	–	–
6	301	65	–	–
7	350	76	–	–
8	401	87	–	–
9	441	95	–	–

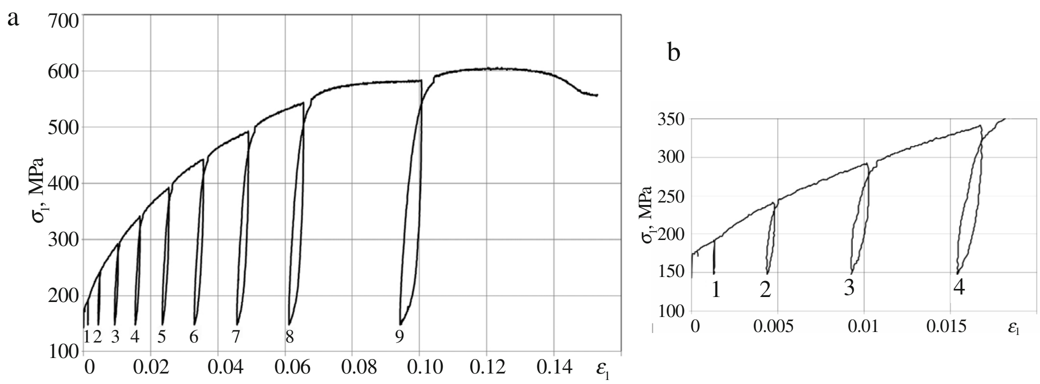

Fig. 1. Experimental dependence of relative strain ε on vertical stress σ_1 under conditions of axisymmetric triaxial compression for sandy soil (a) and increased initial section (b).

TABLE 2

Indices	Loop No.								
	1	2	3	4	5	6	7	8	9
E_{ur}, MPa	801.6	292.0	158.7	145.4	130.8	110.7	104.0	92.7	69.1
q/q^*	0.086	0.201	0.313	0.422	0.534	0.645	0.755	0.868	0.955

the deviator reached the maximum value D_{max}. Table 1 provides the criteria for the start of the unloading stage. Unloading was also performed in the kinematic mode at the same rate as the initial loading of the sample. The end of unloading was recorded when the deviator reached 5-10 kPa, which maintained the rod contact with the upper die, after which reloading was started. Determination of D_{max} was carried out on a twin sample that was tested in accordance with a similar scheme without unloading.

Discussion

The obtained experimental dependences (Fig. 1) show that the discharge loops have different slopes and areas depending on the degree of approximation of the deviator stress q to the threshold q*. A noticeable slope occurred only during unloading at strains of 1% or higher and with the subsequent slope increase (the modulus decrease).

In the initial section, at strain levels lower than 1%, the non-linearity of the unloading branch was manifested, which confirmed the regularity of this result.

Based on the results of processing, unloading strain moduli were obtained for all nine loops (Table 2).

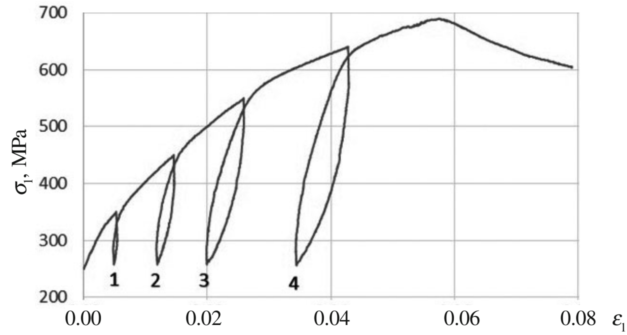

Fig. 2. Experimental dependence of relative strain ε on vertical stress σ_1 under conditions of axisymmetric triaxial compression for pulverescent-clay soil.

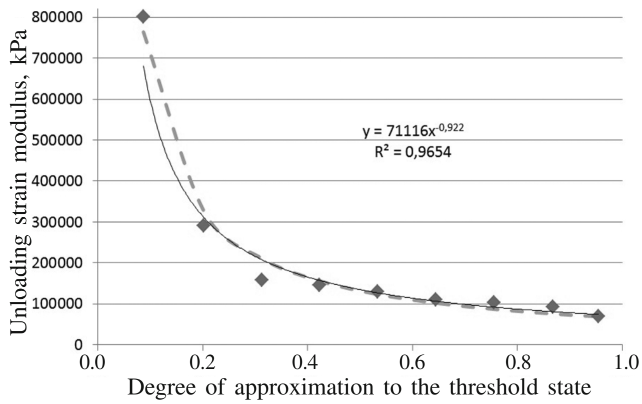

Fig. 3. Dependence of the unloading strain modulus on the degree of approximation to the threshold state and its approximation by a power-law dependence for sandy soils.

Similar tests were performed for clay samples of heavy semi-solid consistency, in which a similar decrease in the unloading modulus was registered with the approximation to the threshold condition (Fig. 2).

For the four loops of unloading and reloading, the indicators were E_{ur} 146.7, 66.4, 49.0, and 45.1 MPa, with q/q^* of 0.23, 0.47, 0.7, and 0.91, respectively.

Results

As shown in Fig. 3, the dependence of the strain modulus on the degree of approximation to the threshold state is described by an exponential function. Given that the exponent is close to −1, the inversely proportional dependence can be used, whose coefficient is the magnitude of the unloading strain modulus on the loop, which is as close as possible to the threshold state (as shown by the dotted line).

Such behavior of non-cohesive soil samples can be explained by the mechanism of elastic behavior of a dispersed medium. With low (absent) shear strains, the soil skeleton is stable, and its total stiffness is due solely to the stiffness of the contacts between the individual particles. Given the measuring capabilities of the test equipment and with the stiffness of individual particles being considerably greater than the stiffness of the soil as a whole, the dependence observed indicates the inelastic behavior of the soil. However, with the development of shear deformations, rearrangement of the skeleton particles and wedging of several particles by others occurred.

This finding leads to the appearance of reactive stresses that return the particles to their initial position. During unloading, these stresses push out individual particles and cause significant deforma-

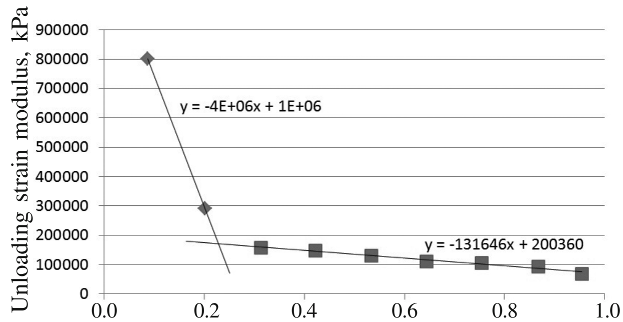

Fig. 4. Approximation of the relation of unloading strain modulus and the degree of approximation to the threshold state by a bilinear dependence for sandy soils.

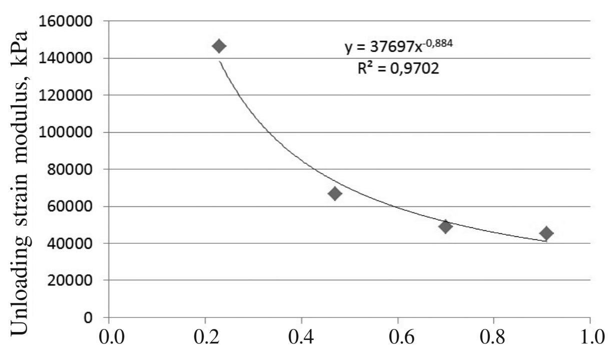

Fig. 5. Dependence of the unloading strain modulus on the degree of approach to the threshold state and its approximation by a power-law dependence for pulverescent-clay soils.

tions of the sample as a whole. On the graph, the trend is presented as an increase in the slope of the unloading loop, which resulted in the decreased modulus of the unloading strain (Fig. 3).

The possibility of using bilinear dependence is of interest (Fig. 4). The graph shows that a significant decrease in the unloading modulus was registered until the deviator reached 20% of the threshold value, after which the modulus almost remained unchanged. This finding can be explained by the development of shears in the soil skeleton, violating the elastic work of cementation bonds (if any) and cohesion.

The decrease in the unloading modulus of cohesive soils as they approach the threshold state can be explained by the destruction of contacts [15]. Moreover, several of the irreversible (cementation and phase) contacts are hypothetically destroyed, and as they approach the threshold state, they are increasingly replaced by reversible (coagulation) contacts. This assumption explains the return of the load-strain curve during load-unload-reload cycles with an increase in the area of loops in pulverescent-clay soils of solid and semi-solid consistency and provides the possibility of several loading-unloading cycles within one experiment.

We can also describe the decrease in the modulus by an exponential function (Fig. 5), similar to that determined for sand. The exponent in this case is also close to unity, but its magnitude cannot be neglected due to a noticeable deterioration in the approximation.

The variable unloading strain modulus E_{ur} can be defined as follows:

$$E_{ur} = E_{ur}^{*} \frac{q^{*}}{q}, \tag{2}$$

where E_{ur}^* is the unloading modulus that is determined as close as possible to the threshold state; q is the current deviator; q^* is the threshold value of the deviator.

For pulverescent-clay soils, the exponent of the correlation equation must be maintained and the dependence used as follows:

$$E_{ur} = E_{ur}^*(q / q^*)^x, \tag{3}$$

where x is the exponent determined experimentally in one triaxial test.

The dependences obtained enable the determination, with a high degree of certainty, of the unloading and reloading moduli at various degrees of approximation to the threshold state.

Methodologically, the determination of one modulus (after having performed unloading once during the test) with a known deviator at the unloading point is sufficient, and other values can be determined analytically or experimentally. With another approach, the stress level at which unloading can be performed is established either when a relative vertical strain of 2%-3% is achieved (which approximately corresponds to the level of deformations of the soil mass under the structures and reflects an economically feasible version of the design solution) or at a stress level of 80%-90% of the maximum load determined experimentally on a twin sample or calculated from archival data and the type of dispersed soil. With this approach, the values of the unloading modulus are in the work reserve of the structure, which are more costly when comparing the types of foundation. In the first approach, in consideration of the identified patterns and given that the hardening soil model uses a hyperbolic dependence to describe nonlinear primary loading,

$$\varepsilon_1 = \frac{1}{E_i} \frac{q}{1-(q / q_a)}, \tag{4}$$

the dependences for changing the unloading modulus obtained for dispersed soils can be mathematically and easily integrated into the model. In this case, Poisson's variable coefficient of unloading v_{ur} should also be used. However, this issue requires additional research.

Conclusions

Despite the use of mathematical apparatus following the theory of continuous media for dispersed soils, in most cases, the soils show no elasticity due to internal bonds. Their elastic behavior depends on the changes in the relative position of the particles during loading and therefore, on the degree of approach to the threshold state. In this regard, the use of a single unloading strain modulus to describe the deformability on the unloading/reloading branches, as regulated by normative documents and adopted in numerical models, is incorrect.

Selecting an approach to determine the unloading strain modulus depending on the expected stress state of the base is appropriate. In cases where the base undergoes significant shear deformations, the unloading strain modulus should be determined based on the second approach (performing unloading as close as possible to the tensile strength). If shear deformations of no more than 1%-2% are expected, the first approach is recommended as economically appropriate.

The proposed method enables the determination of the unloading moduli, with a high degree of approximation, at any degree of approximation to the threshold state. Technically, this method enables one triaxial compression test for direct determination of the parameters of the hardening soil model under laboratory conditions, that is, to determine the modulus at 50% strength E_{50} by a continuous test along the primary loading branch and after exceeding 50% to unload and determine the unloading modulus E_{ur}. Unloading of an estimated 75% shear resistance is recommended for this type of soil to achieve confidently 50% strength and prevent specimen fracture. Strength parameters unknown before the test can be considered on the basis of preliminary tests or in accordance with archival data.

The results of preliminary studies indicate the inadmissibility of using the empirical dependences of the unloading modulus on the primary loading modulus, which is proposed for the sands under study.

Experimental data show that the ratio between the moduli varied from 11 to 3 for pulverescent-clay soil and from 95 (!) to 7.5 for sandy soil. Eqs. (2)-(4) are currently widely used in the practice of geotechnical design, leading to an overestimation of the design deformations of unloading of all types of dispersed soils.

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