CALIBRATION OF STRAPDOWN INERTIAL NAVIGATION SYSTEMS ON HIGH-PRECISION TURNTABLES

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Abstract

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Consideration is given to the problem of strapdown inertial navigation systems (SINS) calibration. The calibration means the determination of instrument error parameters of sensing elements by processing experimental results using special-purpose turntables. There is a calibration procedure, developed in the Laboratory of Control and Navigation of Lomonosov Moscow State University, which enabled to estimate instrument error parameters of inertial sensors only by means of the values of such sensors, and is used for calibration with high-precision turntables. It is demonstrated that additional angle information, provided by high-precision turntable sensors, allows sufficient improvement of calibration accuracy.

Calibration of the SINS sensing elements is the necessary stage of industrial processes, which takes place before system operation. The calibration involves determining the parameters of a mathematical model of instrument errors for inertial sensors for the following compensation of these errors in alignment and navigation mode. A mathematical model of error is given a priori.

Traditional calibration methods performed using special-purpose turntables mostly consist of a complex sequence of SINS turns. In this case, navigation system is usually installed in series at different positions. Such an experiment is difficult to implement, because it requires feedback from the turntable, data processing takes place in the course of the experiment, and the plan must be changed depending on the results of intermediate calculations. Moreover, in some known methods the angular rate of Earth serves as calibration signals for the angular rate sensors, which does not provide good conditioning of problem estimation. Sometimes, as the calibration is carried out separately for accelerometers and angular rate sensors, there is a problem to agree coordinate frames associated with different types of sensors.

The Laboratory of Control and Navigation of Lomonosov Moscow State University offered a fundamentally different SINS calibration method that eliminates these disadvantages. It is shown that the information on orientation of the SINS coordinate frame obtained using angular rate sensors and accelerometers in rotation of SINS on the turntable is sufficient for simultaneous calibration of the both sensing units of SINS. The possibility of constructing a workable algorithm based only on this information is shown in the earlier studies of the laboratory, where a large-scale study was conducted dedicated to the use of low-grade turntables, which confirmed the possibility of estimating the parameters of the used instrument errors' model, i.e., full calibration. The model used presents common instrumental errors of each of the accelerometers, including zero offset error, scale factor error, non-orthogonality errors and high-frequency noise component, which is assumed to be the white noise. Appropriate calibration algorithm is constructed so that it does not use information from the turntable sensors, which makes it possible to use coarse calibration turntables without tracking and positioning systems. For the evaluation, the Kalman filter is used.

The paper is dedicated to applying the above method to the problem of calibration on high-precision turntables using information on turntable platform orientation towards the enclosure. However, in this case, errors of angle measurement and errors of data flows time synchronization should be considered, which means that processing algorithms become more complicated.

Mathematical models of the calibration algorithm

A strapdown inertial navigation system consists of three single-axis accelerometers, three angular rate sensors and an on-board computer. Accelerometers' and angular rate sensors' axes are rigidly positioned in such a way that they form an orthogonal frame accurate to the instrument errors. This frame $Mz_1z_2z_3$ is called hereinafter a body frame. In this case M is a location of the reduced proof mass of accelerometers. In projections of the axis of the coordinate frame, an external force f_z and its angular rate ω_z are measured:

$$f_z' = f_z + \Delta f_z, \qquad \omega'_z = \omega_z - \nu_z,$$

where $\Delta f_z = (\Delta f_{z1}, \Delta f_{z2}, \Delta f_{z3})^T$ – accelerometer errors vector, $\nu_z = (\nu_{z1}, \nu_{z2}, \nu_{z3})^T$ – angular rate sensors errors vector. It is also assumed that the instrument errors of each accelerometer include a zero-signal error (zero error), bias, scale factor error and high-frequency component which is white noise. Given the above the vector of the instrument errors of the accelerometers $\Delta f_z = f_z' - f_z = (\Delta f_{z1}, \Delta f_{z2}, \Delta f_{z3})^T$ can be written as follows: $\Delta f_z = \Delta f_z^0 + \Gamma f_z + \Delta f_z^s$, where $\Gamma = \begin{pmatrix} \Gamma_{11} & 0 & 0 \\ \Gamma_{21} & \Gamma_{22} & 0 \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} \end{pmatrix}$, $\Delta f_z^s = (\Delta f_{z1}^s, \Delta f_{z2}^s, \Delta f_{z3}^s)^T$ are high-frequency components (white noise). Angular rate sensors' instrument errors follow the similar model: $\nu_z = \nu_z^0 + \Theta\omega_z + \nu_z^s$, where $\Theta = \begin{pmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ \Theta_{21} & \Theta_{22} & \Theta_{23} \\ \Theta_{31} & \Theta_{32} & \Theta_{33} \end{pmatrix}$.

Let's introduce a reference frame Mx, which is rigidly bound with a geographic vertical. The Mx_1 axis contacts a parallel passing through the M point and is oriented to the East , the Mx_2 axis lies in the meridian plane and is oriented to the North, the Mx_3 axis is opposite in its direction to the gravitational vector. Denote the angular rate vector of the Mx frame as u_x. Denote the orientation matrix of model frame, which is a numerical image of *Mz* frame*,* with respect to *Mx:* $\dot{L} = \hat{\omega}_z' L - L\hat{u}_x$, $L(f_0) = L_0$, where $\hat{\omega} = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix}$. The error equations in Mx frame take the following form:

$$\begin{aligned} \dot{\beta}_1 &= \omega_3\beta_2 - \omega_2\beta_3 + \nu_1 \\ \dot{\beta}_2 &= -\omega_3\beta_1 + \omega_1\beta_3 + \nu_2 , \\ \dot{\beta}_3 &= \omega_2\beta_1 - \omega_1\beta_2 + \nu_3 \end{aligned}$$

where $\beta_x = (\beta_1, \beta_2, \beta_3)^T$ — vector of kinematic errors, $\nu_x = (\nu_1, \nu_2, \nu_3)^T = L^T \nu_z$.

The measurement vector is introduced $w = f_z' - f_x = (E + \hat{\beta}) f_x + L^T \Delta f_z - f_x = \hat{\beta}(0, 0, g)^T + L^T \Delta f_z$.

Therefore, the problem is reduced to the generation of state vector estimates of the linear dynamic system using z measurement vector, which is linearly connected with the state vector components. The mathematical model of instrument errors has a linear dependency on the collection of unknown parameters, set by constants. Each of such *c* parameters meets the shaping equation $\dot{c} = 0$ and together with the components of β vector it forms the state vector of the examined system.

Program motions of turntable platform

In order to get the maximum possible conditioning of the problem solution, there was suggested a calibration procedure with three cycles. Within each cycle, the SINS instrument axes are sequentially coinciding (accurate to installation errors) with the rotation axis of turntable platform. In this case the rotation axis approximately lies in horizontal plane. To be specific it is assumed that the rotation axis is close to the direction of Mx_1 axis. That means that Mz_1, Mz_2, Mz_3 are coinciding with the Mx_1 axis within each of the cycles. It is also assumed that the installation was performed in accordance with the type of initial matrix L, stated below. The angular rate is selected in the form of piecewise constant function. According to the adopted rotation program, the following relations occur: during the first cycle $\Omega_z^{прог} = (\Omega, 0, 0)^T$, during the second cycle $\Omega_z^{прог} = (0, \Omega, 0)^T$, during the third cycle $\Omega_z^{прог} = (0, 0, \Omega)^T$, where $\Omega = const$.

Using turntable information as adjusting measurements

The high-precision turntables offer an opportunity to extend the correction vector due to additional information on orientation angles of turntable platform in relation to the turntable enclosure. Now let's determine the corresponding relations using some additional frames. The basis frame Mp is rigidly connected with the turntable enclosure. The turntable is usually mounted on the foundation and is oriented in geographical coordinate grid, which means that ideally it coincides with the Mx^0 frame. Let's define orientation of the Mp frame with regard to Mx^0 as the small turn constant vector $\gamma_p = (\gamma_{p1}, \gamma_{p2}, \gamma_{p3})^T$. For some high-precision turntables , the γ_p turns out so small that it could be neglected. The Mz^* frame is rigidly connected with the turntable platform. During installation of the SINS enclosure on the platform, the Mz instrument frame will be coincident with the Mz^* frame, accurate to installation errors. Orientation of the Mz frame regarding the Mz^* frame is defined by the small turn constant vector $\delta_z = (\delta_{z1}, \delta_{z2}, \delta_{z3})^T$. Orientation matrix of the turntable platform Mz^* frame of the relative turntable basic frame Mp is L_{z^*}. In turn, matrix L_{z^*} can be determined by $\kappa_1^*, \kappa_2^*, \kappa_3^*$ values – angles of the three sequential turns, equivalent to ψ heading angle, γ bank angle, θ pitch angle in aviation. Output information of turntable angular sensors are the values $\kappa_1^{*'} = \kappa_1^* + \rho_1$, $\kappa_2^{*'} = \kappa_2^* + \rho_2$, $\kappa_3^{*'} = \kappa_3^* + \rho_3$, where ρ_1, ρ_2, ρ_3 are the instrument errors in measuring these angles.

In addition, it is highly important to consider the fact that the data delivered by the turntable and the SINS information are not synchronized, each has its own time-scale and various discreteness. Denote turntable data delay with regard to the SINS information by τ .

Set the measurements from the turntables by means of a difference of model values of the instrument frame's orientation angles and measured values of the frame orientation angles, connected with the turntable. They have the following form:

$$\begin{aligned}
w_1^* &= -\beta_3 - \gamma_{p3} - \delta_{x3} - \rho_1 - \Omega_{x3}\tau,\\
w_2^* &= -\beta_1 \cos\kappa_1' - \beta_2 \sin\kappa_1' + \gamma_{p1}\cos\kappa_1' + \gamma_{p2}\sin\kappa_1' - \delta_{x2}\sin\kappa_1' - \delta_{x1}\cos\kappa_1' -\\
&-\rho_2 - \sin\kappa_1'\Omega_{x2}\tau - \cos\kappa_1'\Omega_{x1}\tau,\\
w_3^* &= \beta_1 \sin\kappa_1' - \beta_2 \cos\kappa_1' - \gamma_{p1}\sin\kappa_1' + \gamma_{p2}\cos\kappa_1' - \delta_{x2}\cos\kappa_1' + \delta_{x1}\sin\kappa_1' -\\
&-\cos\kappa_2'\rho_3 - \cos\kappa_1'\Omega_{x2}\tau + \sin\kappa_1'\Omega_{x1}\tau,\\
\delta_x &= L_y^T\delta_z.
\end{aligned}$$

Further we assume that high-precision turntables (for instance …) are used, therefore parameters γ , ρ can be neglected.

Thus, the calibration problem is reduced to estimation of constant values $\nu_{z_i}^0, \theta_{ij}, \Delta f_{z_i}^0, \Gamma_{ij}$ $, \delta_{z_i}, \tau$ by means of the Kalman filtering method using measurements and additional measurements, delivered by the high-precision turntable,

Analysis of covariance

During the analysis of covariance, the following close to real a priori characteristics for instrument errors parameters in the form of root-mean-square values were selected: σ_{ν^0} = 0.5 °/hour, $\sigma_{\Delta f^0}$ = 0.02 m/sec² , $\sigma_{\Gamma ii} = \sigma_{\Theta ii} = 10^{-3}$, $\sigma_{\Gamma ij} = \sigma_{\Theta ij} = 3'$ (i≠j). It is expected that they are not correlated with each other. During simulation, standard deviations of white noises in the angular-rate sensors σ_{ν^s} are taken to be equal to 0.1 °/hour at the frequency of 1 Hz, and standard deviations of noises in accelerometers are $\sigma_{\Delta f^s} = 0.001$ m/sec² at the frequency of 1 Hz. Data errors from the turntable are as follows: $\sigma_{\delta_i} = 1^\circ, \sigma_\tau = 0.05$ s, $\sigma_{\Delta\kappa_s^i} = 15''$ at the frequency of 1 Hz. The turntable is set for such program motions (demonstration version) that each cycle has the duration of 15 minutes. Piecewise-constant angular rate is selected. A period of a single cycle is selected in such a way that an angle is changed with the angular speed of 10°/sec within the half of cycle duration, and further change of this angle with the speed of -10°/sec.

The results are evaluated not using the information from the turntable

	ν_z^0 ,°/hour	θ_{ii}	θ_{ij} ,'	Δf_z^0 , m/s²	Γ_{ii}	Γ_{ij} ,'
σ	0.01	5.1*10^{-7}	0.1	9*10^{-5}	1.36*10^{-5}	0.1

The results are evaluated using the information from the turntable

	ν_z^0 ,°/hour	θ_{ii}	θ_{ij} , '	Δf_z^0 , m/s²	Γ_{ii}	Γ_{ij} , '
σ	0.003	$9.7*10^{-8}$	0.05	$7.7*10^{-5}$	$1.36*10^{-5}$	0.05

In order to compare calibration results using the high-precision turntable measurements and the results without such measurements, we carried out simulation of dispersion equation of autonomous navigation errors on the "snake-like" pattern. In the initial conditions for covariance matrix of SINS errors in the instrument errors' unit we used the values resulting from calibration, and demonstration errors were also determined by calibration results. During 1 hour we have simulated a flight of an aircraft using this pattern. As the factor of calibration quality we selected the value $\rho = a\sqrt{\sigma^2_{\Delta\lambda cos\phi} + \sigma^2_{\Delta\phi}}$, where a is a length of a major semi-axis of navigation ellipsoid, and $\Delta\phi$, $\Delta\lambda$ are the latitude and longitude determination errors . After calibration without the use of the high-precision turntable measurement, autonomous navigation error was 1900 m, if new measurements were used, 1100 m. Therefore, the use of new measurements enables to enhance the SINS calibration quality and improve the accuracy of the autonomous navigation by 1.7 times, without complication of calibration plan and time increase.

Conclusions

1. The analysis of covariance results of the proposed calibration methods not including the turntable information and including such information demonstrates their high efficiency, so these methods can be recommended for use.
2. Additional information provided by the turntable improves calibration results and it should be used when applicable.

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