THE REFLECTION AND TRANSMISSION COEFFICIENT OSCILLATION MECHANISM FOR A RAYLEIGH WAVE IN AN ELASTIC WEDGE

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Simple theory is proposed to explain the observed multiple oscillations in the reflection and transmission coefficients for surface Rayleigh waves at the edge of an elastic wedge, which are dependent on the vertex angle.

Surface Rayleigh wave reflection from the edge of an elastic wedge is important in ultrasonic flow detection, seismology, and acoustoelectronics [1]. Although there are many papers (see review [2]), no exact solution has so far been obtained. Existing solutions are based on various approximations in perturbation theory [2-4] and apply only for fairly blunt wedges, while they give a poor description of the observed multiple oscillations in the reflection and transmission coefficients as functions of the wedge angle θ [1].

Here we propose a simple theory that describes these oscillations closely. A difference from most existing approximate approaches, which apply for $\theta \lesssim \pi$, the theory takes the other limiting case $\theta \gtrsim 0$. The wedge is considered as a set of two coupled surface-wave guides, while a Rayleigh wave incident normal to the edge is represented as the sum of symmetric and antisymmetric modes in this system. If the wedge is sufficiently sharp, these are the lowest symmetrical (quasi-longitudinal) and antisymmetrical (bending) Lamb waves in a plate of variable thickness 2h, and one can represent the surface displacements in the incident Rayleigh wave to a first approximation by means of the following phase integrals (the factor exp(-iωt) is omitted), with the x axis directed from the edge along the wedge face:

$$u_{inc}(x,\ \theta)=\frac{1}{2}u_0\exp\left[-i\int_{x_0}^{x}k_s(x',\ \theta)\,dx'\right]+\frac{1}{2}u_0\exp\left[-i\int_{x_0}^{x}k_a(x',\ \theta)\,dx'\right]. \qquad (1)$$

Here u_0 is the amplitude of the incident Rayleigh wave, while x_0 is the point at which the reflection coefficient is determined, with $k_s(x,\ \theta)$ and $k_a(x,\ \theta)$ the wave numbers of the symmetrical and antisymmetrical modes, whose dependence on x and θ is via $h(x,\ \theta) = x\,\mathrm{tg}(\theta/2)$, since $k_s = k_s(h)$ and $k_a = k_a(h)$. For sufficiently large $x < x_0$, namely for $2h(x,\ \theta) > \lambda_R$, where λ_R is the length of the Rayleigh wave at the given frequency ω, one has noted closely that $k_s = k_a = k_R$, where k_R is the wave number of the Rayleigh wave, with both modes in (1) propagating in phase. On approach to the edge, i.e., for $2h(x, \theta) \leq \lambda_R$, k_s and k_a become different one from the other, which results in a phase difference between the two modes. When each of the modes has been reflected from the edge, the process is repeated in the opposite direction. Clearly, the phase difference between the reflected symmetric and antisymmetric modes at $x = x_0$ is dependent on θ in this model, and it is the reason for the oscillations in the reflection and transmission coefficients as θ varies. Of course, this applies on the assumption that the reflection coefficients for the symmetric and antisymmetric modes are close to one in modulus, i.e., there is virtually no energy transfer from these lowest Lamb modes to higher ones, which can be treated as a consequence of emission into the bulk of the wedge. This assumption is quite obvious for a sharp wedge.

On the above basis, the reflection coefficient R and transmission coefficient T can be put as

$$R = \sin\psi_- \exp(i\psi_+ - i\pi/2),$$
$$T = \cos\psi_- \exp(i\psi_+), \quad (2)$$

where $\psi_\pm = \int_0^{x_0} [k_a(x, \theta) \pm k_s(x, \theta)]\,dx + (\pi + \Phi_a \pm \Phi_s)/2$, while Φ_s and Φ_a are the phase shifts for the symmetric and antisymmetric modes arising from reflection. The theory of oscillations in a thin plate shows [5] that $\Phi_s = 0$ and $\Phi_a = \pi/2$ for a plate of constant thickness. To extend this theory to the case of not excessively sharp angles, it is necessary to determine the dependence of Φ_s and Φ_a on θ. This can be done most simply by interpolation with a straight line joining the values of Φ_s and Φ_a for $\theta = 0$ and $\theta = \pi$. The values of Φ_s and Φ_a for $\theta = \pi$ are readily obtained by considering the colliding motion of two identically polar Rayleigh waves (symmetrical mode) and two oppositely polar ones (antisymmetric mode) on the surface of a half-space. Clearly, for $\theta = \pi$ we have $\Phi_s = 0$ and $\Phi_a = \pi$, so the $\Phi_s(\theta)$ and $\Phi_a(\theta)$ relationships in this approximation will take the form $\Phi_s(\theta) \equiv 0$, $\Phi_a(\theta) = (\pi + \theta)/2$. Then we replace integration with respect to x by integration with respect to h in (2) to get the moduli of the reflection and transmission coefficients for $x_0 \to \infty$, which are the ones of interest:

$$|R| = |\sin[\delta/\mathrm{tg}(\theta/2) - (\pi - \theta)/4]|,$$
$$|T| = |\cos[\delta/\mathrm{tg}(\theta/2) - (\pi - \theta)/4]|, \quad (3)$$

where $\delta = \int_0^\infty [k_a(h) - k_s(h)]\,dh$ is a dimensionless parameter dependent on Poisson's ratio σ for the medium. As analytic expressions in explicit form for $k_s(h)$ and $k_a(h)$ are lacking, it is convenient to calculate δ by approximating the corresponding dispersion curves derived from numerical calculations [6]. We do not consider this simple but fairly cumbersome procedure here and give the value of δ for duralamin ($\sigma = 0.35$); $\delta \approx 2.75$.

Figure 1 shows the observed |R| and |T| for duralamin specimens [1], while the solid lines show the dependence of these calculated from (3). On the whole, the theoretical relationships describe the experimental results quite well; in

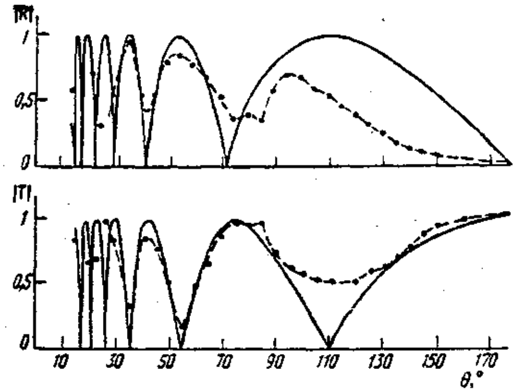

Fig. 1. Dependence of $|R|$ and $|T|$ for a Rayleigh wave on wedge angle θ.

particular, they reflect the observed reduction in the oscillation period as θ decreases and the correspondence between the peaks in the reflection coefficient and the minima in the transmission one, and vice versa. Good qualitative agreement is also obtained for large θ, where the theory is certainly not applicable, in particular because it does not incorporate the fairly considerable bulk emission occurring at these angles (according to (2) and (3), $|R|^2 + |T|^2 = 1$, always). The deviation of this sum from one reflects the accuracy of this approximate approach. Experimental data [1] indicate that $|R|^2 + |T|^2$ deviates appreciably from one at angles greater than 50°, which approximately sets the upper limit to this theory. As regards very small θ, we consider that there are too few experimental points for a detailed comparison of theory and experiment, since the oscillation periods become comparable with or less than the measurement steps.

We now consider a new effect that this simple theory predicts, namely that inclined incidence at an angle α for a Rayleigh wave can be reduced to the above case of normal incidence for a wedge with an equivalent vertex angle $\theta' = 2\,\mathrm{arctg}\cdot[\mathrm{tg}(\theta/2)\cos\alpha]$. Therefore there should be oscillations in $|R|$ and $|T|$ as the angle of incidence α varies even for a fixed θ.

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