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DEGRADATION, REHABILITATION, AND CONSERVATION OF SOILS

Modeling of Attrition of Soil Aggregates in Slope Flows

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Abstract—The erosive capacity of slope water flows, a key parameter in the quantitative assessment of soil erosion, is defined as the difference between the transporting capacity of the flow and the total content of its suspended load and bedload. Therefore, it is necessary to assess the factors and intensity of soil aggregates attrition in the water flow that determine the shares of suspended and dragged particles in the sediment load. The earlier simulation of the attrition of river sediments (H. Sternberg) and soil aggregates (G.I. Shvebs) fail to fully reflect the condition of interaction between soils and slope flows. The further attempts to describe the attrition process using empirical dependences have not given any significant improvements. A fundamentally different model of particle attrition based on the laws of mechanics allows us to describe the attrition of soil aggregates broken away by water flow differentiating the total load between the bedload and suspended load. The experimental verification of the model calculations appears to be satisfactory.

Keywords: attrition of aggregates, abrasion, aggregate breakage, sediment transport, leached chernozem (Luvic Chernozem (Pachic))

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INTRODUCTION

The sediments moved by a water flow considerably influence the intensity of soil erosion. Foster et al. [8] believe that the erosive capacity is defined as the difference between the transporting capacity of the flow and its turbidity (as the total content of sediment load in the flow). According to Mirtskhulava [5], if any signs of sediment accumulation are not seen in the rills on a slope, these sediments have no effect on erosion so that the sediment load should be taken into account only when the slope angle decreases enough for the sediment accumulation to appear. However, certain data demonstrate the effect of the concentration of sediment load in a flow on its transporting capacity in general and in its bottom layer in particular, which depends on the share of the transported soil aggregates [12].

As is shown later, the loads can play different roles and utilize different mechanisms influencing the erosion process depending on their type (bedload or suspended load) [3, 4]. Thus, the effect of loads on soil erosion is indescribable with a single dependence. In order to assess the integral role of sediments in erosion, it is necessary to know the amount of bedload and suspended load along the overall slope flow. Shvebs [6] pioneered in this area using a circulation flume with a constant flow velocity, which limits the value of his research. He studied attrition in a flow over 2–3 km; however, the slopes with such length are almost absent under natural conditions. He observed an explosive character of attrition of soil aggregates after running the first 100 m. Note also that Shvebs [6] proposed a very complex dependence with ambiguous estimates of soil characteristics.

Different methods are used to assess the attrition of soil aggregates and river sediments during their transport by water flows. Wang et al. [13] utilized a purely empirical approach to estimate the size and shape variation of soil aggregates during their attrition in a water flow depending on the way they run. Other researchers studied the attrition of model loads (marlstone and limestone rocks) in a flow using arbitrary equations and a discrete stochastic abrasion model [7] or the model relying on a physical description of abrasion and fragmentation with the help of statistical distributions [9].

Thus, the soil aggregates broken away are destroyed in a flow to the initial silt and clay particles; correspondingly, a model describing this process is in demand.

The goal of the work was to construct a model of attrition of soil particles (aggregates) relying on the laws of mechanics and allowing the attrition of the aggregates broken away by the water flow to be described with distinguishing between the bedload and suspended load in the total sediment content in the flow.

† Deceased.

Table 1. Destruction and attrition of soil aggregates depending on the length of their run in the flow, flow depth, and flow velocity

Diameter of aggregate, mm	Slope, degrees	Flow depth, mm	Remaining weighed sample (%) after running the below distance, m				
			10	20	50	100	200
2	2	10	75.7	68.2	37.6	18.9	6.23
2	2	15	83.8	76.7	28.7	16.7	12.3
2	5	5	–	87.7	72.5	48.9	27.1
2	5	10	58.3	34.8	27.2	4.85	5.30
2	5	15	82.3	73.6	32.4	4.15	0
4	2	5	74.9	69.3	57.7	37.1	21.7
4	2	10	74.6	72.6	42.8	22.7	9.20
4	2	15	74.9	36.2	30.2	22.6	12.4
4	3	5	79.8	54.3	49.5	30.0	10.7
4	3	10	77.8	57.5	42.5	25.2	8.60
4	3	15	78.2	62.7	42.5	40.6	17.3
4	5	5	69.4	59.8	46.4	15.0	5.15
4	5	10	72.6	77.4	49.1	28.0	6.10
4	5	15	68.8	52.7	23.2	9.10	6.96

EMPIRICAL METHODS AND SOLUTIONS

Two sets of experiments were performed to get the quantitative data on the destruction of soil aggregates in a slope flow necessary for further verification of the corresponding model [1, 2]. A hydraulic flume 10 m long was used in the experiments. The bottom of the flume was filled with gravel and covered with a mixture of loam and white glue to imitate a rough slope flow bed. The experiments were conducted according to the following scheme. The water-stable soil aggregates were selected by wet soil sieving and air-dried. The water-stable soil aggregates with diameters of 2 and 4 mm (10 aggregates) were weighed and moistened in a capillary manner on a sheet of filter paper for 10–12 h. Each portion of the aggregates was placed into the flow in the head part of the flume; a sieve (mesh, 0.25 mm) was installed at the end of the flume. The soil aggregates were washed from the sieve into a cup and again placed into the flow at its head. This procedure was repeated until the aggregates passed a certain specified distance (10, 20, 50, 100, and 200 m). The preserved particles of aggregates were dried on a paper filter and weighed. The difference between the initial and final weights of a sample was regarded as the attrition and disruption losses of the aggregates to a size smaller than 0.25 mm. The experiments in each set were performed in three–six replicates.

In the first set of experiments, the effects on attrition of the size of aggregates and microorganisms (yeast cultures) inoculation to soil were studied [2]. These experiments demonstrate that the Sternberg equation [11] fails to describe the attrition of soil aggregates since this equation gives satisfactory result only for the fragments of rocks in riverbeds. The attrition coefficient in this equation is a constant for the rock fragments of different size and depends only on the mechanical properties of a particular rock. As for the soil aggregates, this coefficient appeared to be a variable and decreased with the length of the path. This phenomenon was assessed using a series of empirical dependences [2].

Another set of experiments focused on the effect of the flow velocity and depth on the attrition of aggregates depending on the length of their run [1]. Visual observations have shown that the soil aggregates move in the flow in a jumpwise manner (saltation) and are broken at the moment they touch the flow bed. In accordance with the laws of mechanics, the force of stroke of aggregates on the flow bed is proportional to the squared velocity of the aggregate movement, which, according to the measurements, almost coincides with the mean flow velocity. Based on this premise, an empirical equation describing the breaking of soil aggregates during their travel along the slope in a rill network was proposed. In general, the proposed equation satisfactorily describes the attrition of soil aggregates in a wide range of hydraulic conditions. The correlation coefficient for the experimental and calculated data for the aggregates with diameters of 2 and 4 mm at a flow depth of 10 and 15 mm and slope gradients of 2, 3, and 5° was 0.92, which is a good value since the variation in the rate of attrition of the aggregates is rather high (Table 1).

However, the data on the attrition of aggregates at a flow depth of 5 mm, i.e., at a relative depth of 1–2 units (H/d, where d is the diameter of aggregate and H, flow depth), drastically differs from the data for the deeper flows. Correspondingly, the proposed equation described only particular cases of attrition of aggregates, namely, when the flows are several times deeper as compared with the diameter of aggregates. Thus, the problem of devising a universal model for the attrition of soil aggregates in slope flows remains unsolved despite that it is most relevant as a stage in the development of physically grounded model for soil erosion. This forced us to continue theoretical studies aiming to search for the general solution of the problem relying on the concept of the mechanism underlying the destruction of aggregates in a flow that resulted from the conducted experiments.

THEORETICAL SOLUTION OF THE PROBLEM

Sternberg [11] was the first to theoretically study the attrition of rock fragments; he derived a differential equation based on the assumption that the loss in a particle weight (P) was proportional to its weight in water and the change in its path travelled in a flow (x), i.e.,

$$dP = -CPdx. \tag{1}$$

The following function is the solution for the Sternberg equation at $C = \text{const}$:

$$P = P_0 e^{-Cx}, \tag{2}$$

where P_0 and P are the initial and final weights of the particle, respectively; C, a coefficient depending on the rock hardness, flow velocity, and movement of the particle in the flow, as well as on other physical properties of the particle and the rock forming the flow bed; and x, the distance run by the particle.

Defining the weight of the particle via the volume of the sphere with an effective diameter D, we get the following equation:

$$D = D_0 e^{-\frac{c}{3}x}. \tag{3}$$

Schoklitsch [10] processed the experimental data and proposed the below equation for the C coefficient in the Sternberg equation:

$$C = C_1 v^{0.25}\left(\frac{D+15}{15}\right), \tag{4}$$

where v is the velocity of particles in the flow, m/s; C_1 is the coefficient determined by the rock hardness; and D, diameter of particles, mm.

However, note that Eq. (4) in addition to diameter (D) contains also an unknown velocity of the particle (v). Moreover, the power coefficient in Eq. (2) according to Larionov et al. [2] depends on x at a power of n ($n = 0.64$).

The results of experiments suggest that it is necessary to revise the theory by Sternberg [11] and to construct a new model for the attrition of a particle that relies on the laws of mechanics. Recall the corresponding definitions. The change in the mass of a particle (m) per time unit is equal to the velocity of its loss (m'), i.e.,

$$\frac{dm}{dt} = -m'. \tag{5}$$

Newton's second law, i.e., the change in momentum (mv) per time unit, is equal to the effect of the external forces (F),

$$m'\frac{dv}{dt} = F. \tag{6}$$

Evidently, the selection of m' and F determines the law of attrition. Consider below some variants of how m' and F may be specified.

Variant 1.

(a) Let the velocity of attrition of a particle be proportional to its weight in water and the attrition coefficient be proportional to the velocity of the particle (v), i.e.,

$$m' = \frac{\mu}{g}vP = \mu v(\rho_p - \rho)\frac{4}{3}\pi r^3, \tag{7}$$

where g is the acceleration of gravity; ρ_p is the density of the particle; ρ, density of water; μ, coefficient of the loss in mass; and r, radius of the particle.

(b) Assume also that the external force is the sum of all forces of impact pressure and dry friction, i.e.,

$$F = C_x\pi r^2\rho(u - v)^2 - Kg(\rho_p - \rho)\frac{4}{3}\pi r^3, \tag{8}$$

where C_x is the motion drag coefficient; K, friction coefficient; and u, mean flow velocity. Substituting Eqs. (7) and (8) to Eqs. (5) and (6), respectively, put down the latter as

$$\frac{dm}{dt} = -\mu\left(1 - \frac{\rho}{\rho_p}\right)mv, \tag{9}$$

$$m\frac{dv}{dt} = C_x\pi r^2\rho(u - v)^2 - Kg\left(1 - \frac{\rho}{\rho_p}\right)m. \tag{10}$$

Eq. (10) allows the maximum radius of the particle (r^*) that will not be carried by the flow ($v = 0$) to be determined:

$$r^* = \frac{\frac{3}{4}C_x\frac{\rho}{\rho_p}u^2}{Kg\left(1 - \frac{\rho}{\rho_p}\right)}. \tag{11}$$

In other words, the initial radius of the particle (r_0) that can be moved by the flow must meet the condition $r_0 < r^*$.

Consider Eq. (9). Dividing both parts by v and taking into account that $dx = vdt$, we get

$$\frac{dm}{dx} = -\mu\left(1 - \frac{\rho}{\rho_p}\right)m. \tag{12}$$

Multiplying both parts of Eq. (12) by g, we get Sternberg Eq. (1):

$$\frac{dP}{dx} = -CP, \text{ where } C = \mu\left(1 - \frac{\rho}{\rho_p}\right),$$

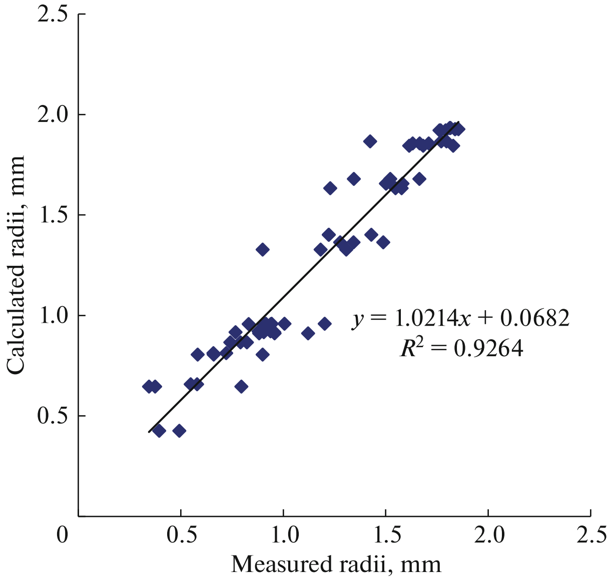

Fig. 1. Comparison of the radius of soil aggregates calculated using Eq. (19) and the measured values after running the distance of 10, 20, 50, 100, and 200 m.

with the dependence of attrition on the relative density of the particle.

Variant 2. Assume that the velocity of particle attrition is proportional to the particle weight in water with the coefficient proportional to the squared velocity,

$$m' = \mu\left(1 - \frac{\rho}{\rho_p}\right)mv^2 = \frac{\mu}{g}Pv^2.$$

The external force is the sum of the drag force in Eq. (8), which is proportional to the difference between the velocities of the flow and the particle:

$$F = C_x\pi r^2\rho(u - v)^2 - Kg\left(1 - \frac{\rho}{\rho_p}\right)m. \qquad (13)$$

Taking into account the assumption on the type of m and F, Eqs. (5) and (6) take on the following form:

$$\frac{dm}{dt} = -\mu\left(1 - \frac{\rho}{\rho_p}\right)mv^2, \qquad (14)$$

$$m\frac{dv}{dt} = C_x\pi r^2\rho(u - v)^2 - Kg\left(1 - \frac{\rho}{\rho_p}\right)m. \qquad (15)$$

Note that the process of attrition and, as a consequence, the change in the radius of a particle is, as a rule, rather slow. Correspondingly, the velocity of a particle also changes rather slowly. This fact allows the inertial term of Eq. (15) to be neglected and the velocity to be determined using the below equation:

$$C_x\pi r^2\rho(u - v)^2 - Kg\left(1 - \frac{\rho}{\rho_p}\right)\rho_p\frac{4}{3}\pi r^3 = 0.$$

Thus,

$$\frac{v}{u} = 1 - \sqrt{\frac{r}{r^*}}. \qquad (16)$$

Thus, we put down Eq. (14) as

$$\frac{dr}{dt} = -\frac{\mu}{3}\left(1 - \frac{\rho}{\rho_p}\right)rv^2.$$

Dividing both parts of the last equation by v and taking into account Eq. (16) and that $vdt = dx$, we get

$$\frac{dr}{dx} = -\frac{\mu}{3}\left(1 - \frac{\rho}{\rho_p}\right)u\left(1 - \sqrt{\frac{r}{r^*}}\right)r. \qquad (17)$$

Dividing both parts of Eq. (17) by r^* and inputting the new variable $y = \sqrt{\frac{r}{r^*}}$, arrange Eq. (17) for integration:

$$\frac{dy}{(1 - y)y} = \frac{\mu}{6}\left(1 - \frac{\rho}{\rho_p}\right)udx = -\lambda dx,$$

where $\lambda = -\frac{\mu}{6}\left(1 - \frac{\rho}{\rho_p}\right)u$. After the integration, the general solution can be put down as

$$\frac{y}{1 - y} = \frac{y_0}{1 - y_0}e^{-\lambda x}, \qquad (18)$$

where $y_0 = \sqrt{r_0/r^*}$, while r_0 is the initial radius of the particle.

Solving Eq. (18) for y, we get $y = \frac{Ae^{-\lambda x}}{1 + Ae^{-\lambda x}}$, where $A = \frac{y_0}{1 - y_0}$, and the below final solution taking into account the input designation:

$$r = r^*\left(\frac{Ae^{-\lambda x}}{1 + Ae^{-\lambda x}}\right)^2 = r^*\left(\frac{\sqrt{\frac{r_0}{r^*}}}{1 - \sqrt{\frac{r_0}{r^*}}}e^{-\lambda x}\left(1 + \frac{\sqrt{\frac{r_0}{r^*}}}{1 - \sqrt{\frac{r_0}{r^*}}}e^{-\lambda x}\right)^{-1}\right)^2. \qquad (19)$$

The last equation satisfactorily describes all experimental data obtained earlier [1].

A comparison of the radius of aggregates calculated using Eq. (19) and the experimental values measured at different lengths (10, 20, 50, 100, and 200 m) of their run in a flow shows a statistically significant correlation ($R^2 = 0.926$, $P = 0.95$), thereby suggesting a considerably higher predictive power of the theoretical equation as compared with the empirical one (Fig. 1). The calculations demonstrate that the slope coefficient (1.0214) does not differ from unity in a statistically significant manner ($P = 0.95$) and the estimate of the standard error of regression coefficient is 0.036. The estimate of the free term in the regression equation (0.0682) does not differ from zero in a statistically significant manner ($P = 0.95$). In addition, the theoretical equation describing the attrition of soil aggregates turned out applicable to the shallow flows, where the empirical equation fails.

CONCLUSIONS

(1) A theoretical analysis of the attrition of soil particles in a flow gives two solutions. The first solution leads to the known Sternberg equation and demonstrates that the abrasion coefficient depends on a relative density of the particle;

(2) The second solution, relying on the dependence of attrition on the squared velocity of the particle, leads to an equation of principally different type. This equation has made it possible to describe all our experimental data obtained earlier; and

(3) The last equation is applicable to describe the attrition of soil aggregates broken away during erosion and to distinguish between the bedload and suspended load in the total load, which is necessary to calculate the soil erosion along the profile of a slope.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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