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Two-Color Localized Optical Vortices in a Nonlinear Waveguide with Zero Dispersion of the Group Velocity at the Second Harmonics

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Two-Color Localized Optical Vortices in a Nonlinear Waveguide with Zero Dispersion of the Group Velocity at the Second Harmonics

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Abstract. Using the method of averaged Lagrangian and diffraction approximation, an approximate analytical solution is obtained and analyzed, which describes the propagation of two-color vortex light bullets in a quadratic-nonlinear graded-index waveguide.

1. Introduction

The stable bunches of light energy localized in time and space are often called light bullets or spatiotemporal solitons [1]. For the formation of a light bullet in a homogeneous medium, the presence of focusing nonlinearity, dispersion, and diffraction is necessary. These factors are not always enough. Sometimes, for the formation of a light bullet, a spatial inhomogeneity of the medium is required. Such conditions are created, for example, in the graded-index focusing waveguides [2]. This applies to media with both quadratic [3] and cubic (Kerr) nonlinearities [4].

Light bullets can have different internal structures. This structure can be defined using a topological charge that takes integer values. The simplest bullets have a topological charge of zero. They can also be called fundamental spatiotemporal solitons. Otherwise, we can talk about vortex light bullets. They are of considerable interest from both fundamental and applied points of view [1].

To date, both single-color and two-color (parametric) vortex light bullets propagating in homogeneous media or in waveguides have been studied in sufficient detail [1, 5, 6]. The formation of two-color bullets occurs quite efficiently, provided that carrier frequencies of both components belong to spectral range with anomalous dispersion of the group velocity (DGV). Recently, it was found that fundamental two-color light bullets are capable of forming in a homogeneous medium, provided that the group dispersion at the fundamental frequency is negative, and at the second harmonics one is equal to zero [7]. A reasonable question arises: can vortex two-color light bullets be formed under such conditions? This work is devoted to an analytical consideration of this possibility in a nonlinear graded-index waveguide.

2. Modified Gross - Pitaevsky equation for soliton parametres

Let the light pulse propagates along the z -axis of the quadratic-nonlinear graded-index waveguide. Then the set of equations for the envelopes ψ_1 and ψ_2 of electric fields for the first and second harmonics, respectively, has the form [3, 8]

$$i\frac{\partial \psi_1}{\partial z} = -\frac{\beta}{2}\frac{\partial^2 \psi_1}{\partial \tau^2} + \alpha_1 \psi_1^* \psi_2 - \omega q_1 \psi_1 + \frac{c}{2n\omega}\Delta_\perp \psi_1 , \tag{1}$$

$$i\frac{\partial \psi_2}{\partial z} = \alpha_2 \psi_1^2 - 2\omega q_2 \psi_2 + \frac{c}{4n\omega}\Delta_\perp \psi_2 . \tag{2}$$

Here $\tau = t - z / v_g$, t is the time, v_g and n are the group velocity and the linear refractive index on the central axis of the waveguide, respectively, the same for both frequencies ω and 2ω, c is the speed of light in vacuum, $\Delta_\perp$ is the transverse Laplacian, $\beta = \partial v_g^{-1} / \partial \omega$ is the coefficient of DGV for the fundamental frequency ω (for second harmonic, this coefficient is zero), α_1 and α_2 are the coefficients of quadratic nonlinearity, determined through nonlinear susceptibilities $\chi_1^{(2)}$ and $\chi_2^{(2)}$ at the fundamental frequency and the second harmonics, respectively, using the expressions $\alpha_1 = 2\pi\omega\chi_1^{(2)} / cn$ and $\alpha_2 = 4\pi\omega\chi_2^{(2)} / cn$, the coefficients $q_{1,2}(\mathbf{r})$ determine the linear refraction of the waveguide:

$$q_{1,2}(\mathbf{r}) = \frac{n^2 - n_{1,2}^2(\mathbf{r})}{2cn} , \tag{3}$$

where $\mathbf{r}$ is the transverse radius vector originating on the central axis of the waveguide.

Since $n_{1,2}^2(0) = n$, then $q_{1,2}(0) = 0$.

In the case of a one-dimensional ($\Delta_\perp \psi_1 = \Delta_\perp \psi_2 = 0$) and homogeneous ($q_1 = q_2 = 0$) medium, the system (1), (2) has exact solutions in the form of temporal solitons [7]:

$$\psi_1 = \pm \frac{\beta}{\sqrt{\alpha_1 \alpha_2}\,\tau_p^2} \exp\left(i\frac{\beta}{2\tau_p^2} z \right) \operatorname{sech}\left(\frac{\tau}{\tau_p} \right), \tag{4}$$

$$\psi_2 = -\frac{\beta}{\alpha_1 \tau_p^2} \exp\left(i\frac{\beta}{\tau_p^2} z \right) \operatorname{sech}^2\left(\frac{\tau}{\tau_p} \right). \tag{5}$$

Here τ_p is a free parameter that has the meaning of the soliton temporal duration.

To take into account the transverse inhomogeneity of the medium, we use the results obtained by means of method of the averaged Lagrangian [3, 7, 8]. For this, in accordance with (4) and (5), we choose trial solutions in the form

$$\psi_1 = \pm \frac{\beta}{\sqrt{\alpha_1 \alpha_2}} \mu^2 e^{i\Phi} \operatorname{sech}(\mu\tau), \tag{6}$$

$$\psi_2 = -\frac{\beta}{\alpha_1} \mu^2 e^{2i\Phi} \operatorname{sech}^2(\mu\tau). \tag{7}$$

Here μ and Φ are functions of the coordinates obeying the set of equations (see details in [3, 7, 8])

$$\frac{\partial\rho}{\partial z}+\nabla_\perp\left(\rho\nabla_\perp\varphi\right)=0\,, \tag{8}$$

$$\frac{\partial\varphi}{\partial z}+\frac{\left(\nabla_\perp\varphi\right)^2}{2}+\frac{c}{2n\omega}\beta\rho^{2/3}+\frac{c}{n}q(\mathbf{r})=2g^2\frac{\Delta_\perp\sqrt{\rho}}{\sqrt{\rho}}\,, \tag{9}$$

where

$$\rho=\mu^3\,,\qquad\qquad \varphi=-\frac{c}{n\omega}\Phi\,, \tag{10}$$

$$q=\frac{3q_1+2q_2}{5}\,, \tag{11}$$

$$g=\sqrt{\frac{7\pi^2/30+17}{90}\frac{c}{n\omega}}\approx 0.463\frac{c}{n\omega}\,. \tag{12}$$

Let us define the complex function

$$Q=\sqrt{\rho}\exp\left(i\frac{\varphi}{2g}\right). \tag{13}$$

Then the system (8), (9) is equivalent to the modified Gross - Pitaevsky equation

$$i\frac{\partial Q}{\partial z}=-g\Delta_\perp Q+\frac{c\beta}{4gn\omega}|Q|^{4/3}Q+\frac{cq}{2gn}Q\,. \tag{14}$$

For an unambiguous dependence of the function Q on the azimuthal angle ϕ of the cylindrical coordinate system, it is necessary to satisfy equality $2g=c/n\omega$ [8] (see (10) and (13)). Then, taking into account the approximate nature of the averaged Lagrangian method, instead of (12) and (14), we write

$$g=\frac{c}{2n\omega}\,, \tag{12a}$$

$$i\frac{\partial Q}{\partial z}=-\frac{c}{2n\omega}\Delta_\perp Q+\frac{\beta}{2}|Q|^{4/3}Q+\omega q(\mathbf{r})Q\,. \tag{14a}$$

3. Vortex light bullets in the diffraction limit

Let us define the dispersion l_d and diffraction l_D scale lengths by means of relations

$$l_d=\frac{2\tau_p^2}{|\beta|}\,,\qquad\qquad l_D=\frac{n\omega}{c}R_0^2\,, \tag{15}$$

where R_0 is the characteristic transverse size (aperture) of the optical pulse.

In the diffraction limit

$$l_D << l_d \tag{16}$$

the second term on the right-hand side of equation (14a) can be neglected. Then we arrive at the nonstationary Schrödinger equation for an imaginary quantum particle

$$i\frac{\partial Q}{\partial z} = -\frac{c}{2n\omega}\Delta_\perp Q + \omega q(\mathbf{r})Q, \tag{17}$$

where the z - coordinate plays the role of time, and the function $\omega q(\mathbf{r})$ plays the role of potential energy (see (11) and (3)).

Let the axially symmetric dependence of the refractive indices $n_{1,2}$ on the transverse coordinate r of the waveguide have the form

$$n_{1,2} = \sqrt{n^2 - (n^2 - 1)r^2 / a^2}, \tag{18}$$

where a is the radius of waveguide.
Then we have

$$\omega q(\mathbf{r}) = \omega \frac{n^2 - 1}{2cn}\frac{r^2}{a^2}. \tag{19}$$

We will find a solution of the equation (14a) in the following form

$$Q(r, \phi, z) = G(r)e^{-i\gamma z}e^{im\phi}, \tag{20}$$

where γ is a constant to be determined, m is an integer.

This type of solution corresponds to optical vortices [1]. As a result, we arrive at the eigenvalue problem

$$\frac{d^2G}{dr^2} + \frac{1}{r}\frac{dG}{dr} - \frac{m^2}{r^2}G - \kappa r^2 G = -\varepsilon G, \tag{21}$$

where

$$G(r) \to 0 \text{ при } r \to \infty, \tag{22}$$

$$\kappa = \left(\frac{\omega}{ca}\right)^2 \left(n^2 - 1\right), \tag{23}$$

and, the eigenvalue ε is determined by the expression

$$\varepsilon = \frac{2n\omega}{c}\gamma. \tag{24}$$

The eigenfunctions satisfying condition (22) have the form

$$G = G_0\left(\frac{r}{R_0}\right)^m L_l^m\left(\frac{r}{R_0}\right)\exp\left(-\frac{r^2}{2R_0^2}\right). \tag{25}$$

where G_0 is the some constant,

$$R_0 = 0.40\frac{\sqrt{a\lambda}}{(n^2 - 1)^{1/4}}, \tag{26}$$

$\lambda = 2\pi c / \omega$ is the wavelength corresponding to the frequency ω, $L_l^m(\xi)$ are the Laguerre polynomials, $l = 0, 1, 2, \ldots$ A parameter γ is determined by the expression

$$\gamma = \frac{\sqrt{n^2 - 1}}{na}\left(2l + 1\right). \tag{27}$$

From (25), (20), (13), and (10) we find

$$\mu = \frac{1}{\tau_p}\left(\frac{r}{R_0}\right)^{2m/3}\left[L_l^m\left(\frac{r}{R_0}\right)\right]^{2/3}\exp\left(-\frac{r^2}{3R_0^2}\right), \qquad (28)$$

$$\Phi = \gamma z - m\phi. \qquad (29)$$

Taking the topological charge of the vortex at the fundamental frequency $m = 1$, for the fundamental soliton mode of the waveguide ($l = 0$) we have

$$\mu = \frac{1}{\tau_p}\left(\frac{r}{R_0}\right)^{2/3}\exp\left(-\frac{r^2}{3R_0^2}\right), \qquad (30)$$

$$\Phi = \gamma z - \phi. \qquad (31)$$

In this case, the topological charge of the vortex at the second harmonic is equal to two.

Thus, the expressions (6), (7), (30), and (31) under condition (16) describe the propagation of two-color vortex light bullets in a nonlinear graded-index waveguide. These bullets are localized both in space and in time. In this case, near the central axis of the waveguide ($r << R_0$) for the intensities of both components of the vortex we have $I_{1,2} \sim |\psi_{1,2}|^2 \sim r^{8/3}$.

4. Concluding remarks

Thus, the using of the averaged Lagrangian method and the diffraction approximation (16) made it possible to obtain an approximate analytical solution of the system of equations (1), (2) in the form of two-color localized optical vortices (two-color vortex light bullets).

Due to the diffraction approximation, we neglected the nonlinear term in the equation (14a). As a result, the study was reduced to solving a linear eigenvalue problem. It is important to note that the original problem is highly non-linear. In particular, the second harmonic generation is a essentially nonlinear effect. In addition, it is due to the nonlinearity that the vortex bullet is localized in the direction of its propagation. In this case, transverse localization occurs due to the linear refraction of the pulse in the focusing gradient waveguide. I.e. in the diffraction limit (16), it is possible to effectively separate the longitudinal and transverse dynamics of a light pulse.

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References

[1] Kivshar Yu S and Agrawal G P 2003 *Optical Solitons / From Fibers to Photonic Crystals* (New York: Academic Press)
[2] Shtyrina O V, Fedoruk M P, Kivshar Yu S, and Turitsyn S K 2018 *Phys. Rev. A* **97** 013841
[3] Sazonov S V 2020 *Laser Phys. Lett.* **17** 095401
[4] Sazonov S V 2019 *Phys. Rev. A* **100** 043828
[5] Kalinovich A A, Komissarova M V, Zakharova I G, and Zagursky D Yu 2019 *Proc. SPIE* **11026** 110260M
[6] Kalinovich A A, Bryantsev B S, Zagursky D Yu, and Zakharova I G 2020 *Proc. SPIE* **11358** 113581W-1
[7] Sazonov S V and Komissarova M V 2020 *JETP Lett.* **111** 320
[8] Sazonov S V, Kalinovich A A, Komissarova M V, and Zakharova I G 2019 *Phys. Rev. A* **100** 033835