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Possibilities and Problems of Modern Magnetotellurics

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Abstract—Approaches to the solution of three-dimensional inverse problems are considered in the paper. The main methods for regularization of inverse problems of electromagnetic sounding are reviewed. Basic scenarios of three-dimensional and multicriterion interpretation are considered.

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INTRODUCTION

The remarkable pioneer work by Andrey Tikhonov, which laid the foundation for the research in the field of the magnetotelluric (MT) prospecting and deep magnetotellurics, was published sixty years ago [Tikhonov, 1950]. Today, MT methods are widely adopted and successfully used for solving the fundamental and applied problems of geophysics including engineering, ecological, hydrogeological tasks, the problems of metallogeny and mineral genesis, and tectonic and geodynamic issues.

The progress in modern magnetotellurics is related with the striking advances that took place in the past decade in the methods and technology of the exploration and depth of geophysics. Instrumentation for field research ensuring reliable determination of MT and magnetovariational (MV) parameters has been designed. The MV sounding technique that has long been at the periphery of magnetotellurics has now become an important instrument for deep electromagnetic research, which is free of the distorting effects of local surface heterogeneities (geoelectric noise). New approaches are developed for the analysis and interpretation of the magnetovariation and MT data, which enhance the geological and geophysical informativeness of geoelectrics. Field measurements carried out in many tectonic provinces all over the world offered essentially new information concerning the structure of the sedimentary cover, solid crust, and the upper mantle [Berdichevsky and Dmitriev, 2002; 2008].

The prime challenge of modern magnetotellurics is the necessity to develop the methods and computational systems for efficient interpretation of the MT and MV sounding data. This work calls for new mathematical ideas and computational resources. The inverse problem of magnetotellurics is unstable. In order to obtain a stable solution, i.e., to regularize the problem, we need a priori information about the geoelectric medium under study. And here the geophysicist's role becomes of primary importance. It is the geophysicist who selects the geological hypotheses concerning the structure of the studied region, which have to be validated and refined in the context of the theory of ill-posed problems.

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ON THE INSTABILITY OF INVERSE PROBLEMS

An inverse problem is unstable since there are media with strongly different properties that can, due to integral effects, generate similar electromagnetic fields. A stable solution of a problem can be found if the solution is sought among a limited (compact) set of geologically reasonable models. This set forms an interpretation model. The interpretation model is constructed in accordance with a priori information and qualitative analysis of the measurement data. The described approach is based on the theory of regularization of ill-posed problems [Tikhonov, 1974; 1977].

When solving an inverse problem, we face a phenomenon called the paradox of instability. The stricter the constraints imposed on the interpretation model, the more stable the inverse problem and the poorer the degree of detail of its solution. At the same time, the more stable the inverse problem, the better its resolution. The resolution of the inverse problem and the detailedness of its solution are competing factors: the better the resolution, the poorer the detailedness. The inverse problem should be solved with the optimal balance between its stability, resolution, and detailedness [Dmitriev, 1987].

The informativeness of magnetotellurics essentially depends on the way constraints are placed on the set of the admissible solutions to the inverse problem that forms the interpretation model. Due to a priori information, these limitations can be introduced into the interpretation model either directly or as a functional that is constrained only on the set of the constructed interpretation models. This functional is a stabilizer of the inverse problem's solution, because it defines the stability of the solution. Thus, the inverse problem reduces to the minimization of the sum of two functionals: the model's misfit functional and the stabilizing functional. The stabilizing functional enters the summary functional with a weight that is called the regularization parameter. The weight is selected in accordance with the accuracy of the observed data [Tikhonov, 1974].

The methods for the solution of inverse problems are usually classified according to the form of the stabilizing functional applied in the solution. The classical functional is the smoothness functional defined as the norm of a gradient of the electric conductivity distribution. The set of the interpolation models in this case includes smoothly varying distributions of electrical conductivity, i.e., smoothed models. Therefore, this technique was called the smoothing method [Constable et al., 1987; Berdichevsky and Dmitriev, 2002; 2008].

If it is a priori known that there are sharp contrasts in the distribution of electric conductivity; a special stabilizer is applied that outlines the region with a high gradient in electric conductivity. Minimization of this stabilizer offers a more contrasting solution of the inverse problem. This method was called the focusing technique [Portniaguine and Zhdanov, 1999; 2007; Mehanee and Zhdanov, 2002; Zhdanov, 2002; Zhdanov and Tolstaya, 2004; 2006].

A contrasting model can also be obtained in another way, if the region containing inhomogeneities is in advance divided into a limited number of blocks, each having a constant or smoothly varying distribution of electric conductivity [Varentsov, 2002; 2007]. In this case, a priori information about the structure of the medium is directly introduced into the interpretation model. The regularization of the problem is carried out through the minimization of the number of blocks. This method is called the block method. There are two modifications of the block method: the simple block method, when the only varying parameter is the conductivity of the blocks, and the full block method, when, in addition to the variations in the conductivity of blocks, their dimensions also may change. The study region should always be divided into a comparatively small number of blocks; otherwise the problem starts to become unstable.

An interesting approach to the solution of inverse problems is the hypotheses testing technique. In this method, a hypothetical model of the structure of the medium is constructed based on a priori information. The stabilizer is the misfit functional of deviation of the interpretation model from the hypothetic model. In this case, we found the solution of the problem that is closest to the hypothetical model. This technique is essentially dependent on the experience and the intuition of the interpreter geophysicist who constructs the hypothetical model [Berdichevsky and Dmitriev, 1991; 2002].

The choice of a particular method for solving the inverse problem largely depends on the real geological situation. Precisely this is a key factor defining the diverse approaches to the inverse problems in MT prospecting and in depth geoelectrics.

In the first case, the set of interpretation models reproducing the structure of the medium is described in rather simple terms. The inhomogeneous surface layer rests on a rather homogeneous half-space that contains local irregularities whose electric conductivity noticeably differs from that of the host medium. The main purpose is the identification of the heterogeneities located below the surface's inhomogeneous layer. The heterogeneities can be a priori assumed to be highly contrasting structures. The main impediment for the solution of the inverse problem is the surface heterogeneities whose effects must be eliminated. Evidently, the most effective method in this case is the focusing of the inverse problem solution.

In the second case, in depth magnetotellurics, the set of the interpretation models becomes much more complicated. Here, an inhomogeneous surface layer is again present, but this layer is underlain by a large heterogeneous zone composed of blocks with different conductivities. This zone is separated from the surface layer by a thick high-resistivity layer; therefore, the heterogeneous zone is excited mainly inductively. The blocks of this model are not necessarily highly contrasting. In this case, it is very difficult to define the constraints for the set of the interpretation models than in the first case. The simplest method applied for solving the inverse problem in depth geoelectrics is the block method, which is usually preceded by application of the smoothing method. The latter ensures a more adequate division of the region into several blocks. In some cases, when, based on the geological information, it is possible to suggest a hypothesis of the structure of the target region, the hypotheses testing method is employed [Varentsov, 2002; Spichak, 2005].

Due to the recent development of powerful computers and appearance of multiprocessor clusters, the possibility arose to solve three-dimensional inverse problems for large models defined by several million parameters [Newman and Alumbaugh, 1997; Mackie and Watts, 2004; Siripunvaraporn et al., 2005; Zhdanov, 2009]. Such large models can be used for the interpretation of the results of depth geoelectrics (see the paper by Zhdanov et al. in this issue).

When discussing the prospects of MT interpretation, we must bear in mind that the practical instability of the inverse problem grows with the increasing dimensionality and complexity of the interpretation model, i.e., with the amount of free parameters necessary for the adequate description of the region of interest. The more complex the interpretation model, the stronger constraints should be imposed on the model class to ensure a stable interpretation. That is why we will consider the methods for the solution of the inverse problem separately for MT prospecting and for depth geoelectrics.

TWO-DIMENSIONAL INVERSE PROBLEM

The evolution of the methods for solving the MT inverse problems developed from the simpler models toward the more complex media. The simplest case is the one-dimensional inverse problem. Many diverse techniques were proposed for its solution; they include the method of a minimum number of layers for the stratified media with piecewise-constant distribution of electric conductivity; the method of maximum smoothness with a fixed number of discontinuities in the electric conductivity; and the conductance technique. We may state that the issue concerning the solution to the one-dimensional inverse problem is completely settled.

The situation becomes more complicated when we proceed to the two-dimensional inverse problem. The simplest case here is the class of two-dimensional media composed of quasi-one-dimensional layers. Here, to ensure the stability of the inverse problem, we must require that the number of layers is fixed and that the lateral variations of the layers are rather slow. At the same time, the inverse problem of depth geoelectrics in the class of multilayered two-dimensional media with sharp horizontal inhomogeneities is essentially complicated by the local and regional distortions of the MT field, and thus it becomes strongly unstable. The solution of this problem raises questions concerning the normal cross-section, near-surface distortions, static shift of apparent resistivities, asymmetry of structures, galvanic screening of deep conductive zones, effects of the conductive faults, different resolution of TM and TE field modes, and different sensitivity of the impedance and the magnetic response functions. Presently, extensive research is being conducted in this field. New promising approaches have appeared, which pave the way for a successful two-dimensional interpretation of the MT and MV data obtained in regions with linear tectonics [Berdichevsky and Zhdanov, 1984; Berdichevsky and Dmitriev, 2000; 2002; 2003; 2008].

Two-dimensional problems in the exploration magnetotellurics are rather simple: the task is to determine the shape of the cross-section of a cylindrical inclusion embedded into a layered medium. The number of layers is usually small, and the inclusion is located within a single layer. The solution of such inverse problems is rather simple. The instability of the solution appears as the variations in the boundaries of the inclusion; it can be easily overcome if we require the boundary to be smooth and the contrast in the conductivity at the boundary to be sharp.

It is already in a two-dimensional MT inverse problem that we first face a multicriterion inversion. The two-dimensional forward problem yields the impedance of two modes and the tipper, i.e., three complex quantities. Here, the question arises: which one of these quantities is it preferable to use in the inverse problem? Generally, we should choose the quantity that has minimum errors and maximum sensitivity to the target structures. However, the impedance of an H-polarized field in the MT data is highly sensitive to the near-surface inhomogeneities in electric conductivity and poorly sensitive to the deep structures buried in the high-resistivity host medium. On the contrary, the tipper and the impedance of an E-polarized field are more sensitive to deep structures, particularly at low frequencies [Berdichevslii and Dmitriev, 2000; 2003].

Thus, the tipper (or the impedance of an E-polarized field) is most suitable for determining the parameters of a deep inhomogeneity, whereas the impedance of an H-polarized field is preferable if the analysis is targeted at studying the near-surface structures. To achieve this result, an iterative process is applied. At each step of this process (iteration), corresponding approximations to the parameters of the deep inclusion and the near-surface inhomogeneities are determined. This approach is called a *successive inversion* [Dmitriev, 2005; 2006].

Here, a question arises on whether it is possible to minimize the total functional comprising the impedance and tipper misfits instead of applying the iterative procedure? Obviously, such an approach is applicable if the weights defining the contributions of each of the misfits are chosen correctly. Generally, the optimal weight depends on the parameters of the medium; i.e., the misfit functional is taken as a norm of the model data's misfit from observations with a weighting function. This results in the fact that the more sensitive the functional to a given parameter the larger the weighting function. This is in fact an implementation of the successive inversion. Examples of the practical application of this approach are presented in [Zhdanov et al., 2010].

It is the two-dimensional inverse problems that were used for testing various inversion techniques with different forms of a stabilizer. The efficiency of these methods was demonstrated and the field of their application was outlined [Berdichevsky, 2008]. We may say that the two-dimensional inverse problems are studied in sufficiently fine detail, although the scenarios of their solutions are rather odd, and there is no unified interpretation scheme based on different methods that would incorporate both the MT and MV data. The mathematical and computational difficulties in the solution of two-dimensional inverse problems appear to have been overcome.

The only remaining problem is the possibility for using the two-dimensional models in the approximation of real situations. Deviations of real geological structures from two-dimensionality introduce noticeable errors into the interpretation. As a result, the qualitative structure of the medium is reproduced quite well, but the quantitative estimates contain considerable errors.

SOLUTION OF INVERSE PROBLEMS FROM A COMPUTATIONAL STANDPOINT

As mentioned above, the solution of an inverse problem usually reduces to the variational problem for the distribution of electric conductivity that delivers a minimum to the functional containing the misfit of the data from the stabilizer. In order to calculate the misfit functional, it is necessary to solve the corresponding forward problem. At each minimization step, the inverse problem should be linearized in the vicinity of the previous solution. This allows one to calculate the correction to the solution from the system of linear equations at each step of the calculation. Thus, the solution of the inverse problem reduces to the repeated solution of the forward problem.

We note that the computation of the correction to the solution takes approximately the same time as the solution of the forward problem. Therefore, we may estimate the time of the inverse problem's solution if we assume that each minimization step takes twice as much time as the solution of the forward problem. This is the reason why two-dimensional inverse problems are readily solved on sufficiently powerful professional computers, whereas the real-time solution to three-dimensional inverse problems is possible only on supercomputers or big clusters.

ON THE THREE-DIMENSIONAL INVERSE PROBLEMS OF MAGNETOTELLURICS

The main research trend in modern magnetotellurics is the transition from two-dimensional inversion to three-dimensional inversion. This transition has a well-substantiated basis that includes the methods developed for mathematical modeling of electromagnetic fields in heterogeneous media [Wannamaker, 1991; Smith, 1996; Newman and Alumbaugh, 1997; Hursan and Zhdanov, 2002; Sasaki, 2004; Avdeev, 2005; Zhdanov et al., 2007; Endo et al., 2009; Zhdanov, 2002; 2009]. The alculation of fields for media with a rather complex structure takes several hours of computations on supercomputers or clusters; in these calculations, the medium is divided into up to a few million blocks. All this allows us to study the sensitivity of MT fields to various inhomogeneities in rather fine detail.

Due to the instability of the inverse problems, we have to put several constraints on the set of the interpretation models. The higher the dimensionality of a problem, the larger the number of constraints that should be imposed. This is associated with the fact that, e.g., in two-dimensional models, in addition to the usual constraints imposed by the stabilizer, we have another very strong constraint, which dictates that the model has practically no variations along the given direction. When passing to the three-dimensional inverse problems, we discard the condition of two-dimensionality. Therefore, it is necessary to introduce some additional constraints to compensate for the rejection of the two-dimensionality of the model.

The first such constraint is, as a rule, an introduction of a layered background medium. In other words, it is assumed that everywhere outside an inhomogeneous 3D cube ($x| \le l$, $|y| \le L$, $0 \le z \le H$) the medium has a layered structure and is characterized by a given normal distribution of electric conductivity $\sigma^N(z)$. The layered background is inferred from measurements at the margins of the region of study and is understood as the medium that best approximates the experimental data on average. This locally inhomogeneous set of interpretation models blocks off the use of models in which the current leakage from distant regions with another heterogeneous structure is possible.

A further limitation of the set of the interpretation models occurs via the imposition of constraints governing the changes of electric conductivity within a heterogeneous region. The most common approach is an averaged model in which a stabilizer in the form of a quadratic norm of the gradient of electric conductivity is used. This approach enables one to capture the areas of enhanced and reduced electric conductivity. Further, we can make these areas more contrasting by applying the focusing technique.

When sounding the zones containing elongated structures, the most suitable interpretation models are the set of models with limited two-dimensionality; i.e., it is implied that the distribution of conductivity within the heterogeneous region is two-dimensional, although limited in length along the axis of two-dimensionality. In this case, the stability of the inverse problem is close to the stability of the two-dimensional model, because here only one additional parameter, namely the elongation of the heterogeneity, is introduced.

A special place in the three-dimensional inverse problems belongs to the set of quasi-layered interpretation models where it is assumed that the derivative of the electric conductivity across the Earth's surface is limited. In this case, the quasi-one-dimensional method of solution is applied. This method is most efficient in structural tasks. Here, the solution of the three-dimensional inverse problem reduces to the repeated solution of a one-dimensional problem, with the subsequent introduction of corrections via iterations. This method, which can be easily implemented on conventional professional computers, allows real-time determination of the output parameters. However, this technique yields excessively smoothed results and thus is usually applied in the preliminary interpretation, which enables one to identify the characteristic features of the structure of the medium.

The main drawback of the quasi-one-dimensional method is oversmoothing of the results. Due to this, the resulting geoelectric cross section offered by this method in case of abrupt variations in the properties of the medium yields only an initial guess that approximates the actual distribution of conductivity. Then, the solution of the inverse problem is refined on the basis of the hypotheses' testing method.

Adequate initial approximation of the geoelectric cross section is of key importance in the hypotheses' testing technique. This initial approximation is the basis for constructing the hypothetical distribution of electric conductivity. Precisely such an adequate initial approximation is provided by the quasi-one-dimensional interpretation technique. In our opinion, the combination of the quasi-one-dimensional method with the hypotheses' testing technique is a promising way for the construction of systems for the three-dimensional interpretation of data [Berdichevsky and Dmitriev, 2009; Dmitriev, 2005].

Here, a question naturally arises concerning the interrelation between the quasi-one-dimensional method and the method of smoothed models, since both methods define models with a smooth distribution of electric conductivity. The basic distinction of these methods is the different form of the stabilizing functional used. The stabilizer in the method of smoothing models is the rms norm for the gradient of electric conductivity. We note that in this case, the derivatives are minimized all at once within the entire inhomogeneous region.

Unlike the smoothing model method, in the quasi-one dimensional approach the stabilizer contains differences of the electric conductivity's distributions in the x and y coordinates in the neighboring points. Hence, the quasi-one-dimensional method makes use of the local constraint on the variations in the electric conductivity in contrast to the method of smoothing models, where a general integral constraint on the derivatives of conductivity is applied.

Another distinction of these methods is the different way the start model of interpretation is used. The natural start model in the quasi-one-dimensional method is the introduced normal cross section. Successively interpreting the data in the observation points, we gently vary the geoelectric structure of the region from the normal cross section at the edge to the actual cross section in the center of the target area. The start model in the method of smoothing models should be a homogeneous zone framed by the normal cross section. The interpretation in the homogeneous zone under study will reveal the areas characterized by increased electric conductivity that are adjacent to the normal cross section.

Another problem in three-dimensional inversion is the influence of small three-dimensional inhomogeneous inclusions contained in the near-surface layer, which introduces noticeable distortions into the MT field [Tikhonov and Dmitriev, 1969]. There are two ways to filter out this geological noise.

First, by applying the two-dimensional spline approximation of the observed data over the given area, we may calculate the smoothed values of impedance. The anomalies generated in the MT field by deep heterogeneous structures are smoother than the anomalies produced by surface heterogeneities; therefore, spatial filtering eliminates geological noise, while retaining information about the deep structure of the medium.

A more strict method was recently developed by Zhdanov et al., [2010]. In this method, the normalization of MT curves is introduced directly into the process of the solution of the inverse problem. For this purpose, a three-dimensional block with an unknown electric conductivity is introduced beneath each measurement point. This procedure does not heavily increase the number of blocks used in the inverse problem, while at the same time makes it possible to allow for the shift of the MT curves caused by the effects of the small three-dimensional near-surface inhomogeneous inclusions.

Thus, we may state that the problem of the three-dimensional interpretation of MT data has generally been solved in terms of its computational aspects, although the solution of complicated problems requires the use of supercomputers. However, besides computational aspects, the problem also has its technological specificity. This is, primarily, the necessity of areal measurements. Presently, areal MT measurements are only implemented as particular campaigns with special allocation of funds.

An example of a large-scale project on areal MT measurements is the *EarthScope* project. This is a ten-year-long US national program in the field of Earth sciences. The main goal of the *EarthScope* project is to conduct regional seismic and MT measurements all over the USA, in order to study the deep geological structure of the North American continent. It is planned in the scope of this project to cover the entire territory of USA by a network of MT stations spaced 60–100 km from each other. By the end of 2009, 262 deep MT soundings had been carried out in the western states of America including the states of Oregon, Washington, Idaho, Montana, and Wyoming. The data of these MT soundings contain unique information about the deep geoelectric structure of the crust and the upper mantle of North America. Three-dimensional interpretation of these data is an extremely challenging scientific problem. The initial results on the three-dimensional MT interpretation of the *EarthScope* data are presented in [Patro and Egbert, 2008; Zhdanov et al., 2010].

Here, we should emphasize that in case of a simple set of the interpretation models there is no need to carry out full-scale areal measurements on a grid with a constant step. Measurements at a certain set of points of a given area are quite sufficient. This is related to the fact that the number of parameters describing the interpretation model is much fewer than the number of blocks the three-dimensional model is divided into. The number of blocks of a three-dimensional medium for the inverse problem usually ranges from several hundred thousands to a few million, whereas the number of generalized parameters defining the structure of the medium usually does not exceed a few hundred. Due to this fact, it is quite sufficient to measure the frequency responses of a MT field at a few dozen points within the region of study. Evidently, these measurement points should be located in the maximally informative places. The locations of measurement sites are usually selected by a geophysicist based on a priori information about the structure of the region. However, in future works it will be necessary to involve experiment-planning methods that will allow researchers to choose the optimal new measurement points on the basis of the available data obtained at points of previous observations. For a faster solution of this problem, one may apply the quasi-one-dimensional method in order to get a qualitative insight into the structure of the region of study. The quasi-one-dimensional method is readily implemented on conventional PCs and, thus, can be applied in the field.

At the same time, a question arises concerning the uniqueness of the solution to the three-dimensional inverse problem, in case of a few measurement points only. The uniqueness of the solution is attained as a result of the limitations imposed on the set of the interpretation models. Here, the role of the interpreter–geophysicist, who defines the set of the interpretation models based on his own hypothesis about the structure of the studied region, is particularly important. At the same time, it is always helpful to use the hypotheses' testing method, where the stabilizer is the misfit functional for the deviation of the desired model from the hypothesized one.

CONCLUSIONS

In summary, we highlight ten key points of the MT and MV inversion.

1. Regardless of the processing power of a computer, the inversion should be conducted in an interactive mode, which ensures the feedback between the interpreter–geophysicist and the computer. It is the geophysicist who defines the set of the interpretation models and selects the strategy of inversion; and this choice is decisive for successful interpretation. An important element of the inversion is the testing of hypotheses. Inversion in the set of simple interpretation models can be conducted automatically by the computer.

2. When solving the inverse problem, it is necessary to specify the normal cross section of the geological medium outside the region of observations. The normal background is introduced as a mathematical abstraction consistent with the a priori information and the measurements at the boundary of the target region.

3. The inverse problem is unstable (incorrect). It is informative if its solution is sought on a limited (compact) set of geologically plausible models. This set forms the interpretation model. The interpretation model is constructed in accordance with the a priori information and the qualitative structural analysis of the data offered by observations.

4. The electromagnetic field in the conductive Earth has a diffusive nature. Obviously, the MT field may yield only a smoothed image of the real geoelectric medium. The sharp geoelectric contrasts buried deep in the Earth can be artificially introduced into the interpretation model either based on a priori information or hypothetically. The most complete and informative interpretation is provided by a series of inversions focused on different elements of the interpretation model. Partial inversions are informatively linked with each other: the result of the previous inversion is inherited by the subsequent inversion via the start model and the stabilizer, which ensures the closeness of the result to the start model.

5. When solving the unstable inverse problem, we face a phenomenon called the paradox of instability. The narrower the limits of the interpretation model, the more stable the inverse problem and the poorer the detailedness of its solution. However, on the other hand, the more stable the inverse problem, the higher its resolution. The resolution of the inverse problem and its detailedness are antagonistic factors: the higher the resolution, the poorer the detailedness. The inverse problem should be solved with the optimum balance between its stability, resolution, and detailedness.

6. The inverse problems of magnetotellurics are multicriterion problems. In the interpretation of MV and MT response functions, we deal with the complex-valued matrices of tipper, magnetic tensor, impedance tensor, and phase tensor. These response functions have a different sensitivity to the different parameters of the interpretation model and different tolerance to the near-surface distortions. The best approach to the solution of a multicriterion inverse problem is a reasonably constructed sequence of partial inversions, each focusing on different elements of the interpretation model. The sequence of the inversions is organized in such a way that the results of the previous inversion are transferred to the subsequent inversion in the form of its start model.

7. The MT effects reflect the integral influence of the inhomogeneities contained in the geoelectrical medium. Hence, large compact bodies may appear as an alternation of cells with high and low resistivity. This mosaic should be considered as an indication of the instability in the form of one of the possible equivalent solutions. This model solution can be smoothed based on geological considerations, provided that the model misfits do not increase.

8. The adequacy of the inverse problem's solution can be best estimated from the comparison of the measured and the model local response functions at each observation point. Those elements of the obtained solution, whose elimination does not increase the model's misfit, are considered as unnecessary (as artifacts) and are excluded from the solution.

9. Regional and deep MT studies are usually implemented at long regional profiles that ensure the possibility of quasi-one-dimensional or two-dimensional interpretation. The admissibility of such interpretation should be validated by the a priori and a posteriori analysis of three-dimensional effects.

10. The three-dimensional inverse problems are the most complex ones. The number of parameters defining the structure of the medium becomes much larger; and the requirements for the processing power of computers and their memory increase. The technological difficulties become much stronger, and the stability of solutions deteriorates. The geoelectric models become too complicated. Nevertheless, some positive experience has been gained already in the solution of three-dimensional inverse problems on clusters. The use of modern supercomputers will enable the transition to more complex systems of three-dimensional MT interpretation in the nearest future.

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