CALIBRATION OF INERTIAL MEASUREMENT UNITS ON A LOW-GRADE TURNTABLE WITH SIMULTANEOUS ESTIMATION OF TEMPERATURE COEFFICIENTS *

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Abstract

Key words: Inertial measurement unit, inertial sensors, calibration, temperature variations, optimal estimation, Kalman filter

The work concerns an improvement of the method for inertial measurement unit (IMU) calibration using a low-grade single-axis turntable with a horizontal axis of rotation. Conventional parameters of inertial sensors error model are estimated in this method. These parameters include null biases, errors of scaling factors, sensitive axis misalignments, dynamic drifts coefficients, and others, if necessary. At that, there is no need in any information provided by the turntable, either in angle or precise position measurements or rate measurements. This allows all desired parameters to be obtained in a simple procedure without precise equipment and without strict compliance with any predefined plan of operations.

The aim of this particular work is to demonstrate that the method can be modified in order to incorporate the effect of temperature variations of sensor error model parameters. Temperature coefficients are estimated simultaneously with the rest of parameters in the same calibration procedure as before, but with non-constant temperature.

Introduction

The aim of calibration of inertial measurement unit (IMU) is to determine a set of inertial sensor error parameters. Conventional set includes null biases of gyros and accelerometers, its scaling factor errors, sensitive axis mutual misalignments, and dynamic drift (g-sensitivity) coefficients, for some types of gyros. Calibration includes some procedures carried out with IMU itself installed in some test bench like turntable and logging measurement data. These data are then used to calculate the desired parameters. The method under consideration does not require any information from the test bench. Moreover, it does not require to strictly follow some particular plan of operations. Only some general conditions must be met.

Below we are going to demonstrate that using this method in almost the same procedure as before we can obtain estimates not only for inertial sensor error parameters mentioned above. Temperature coefficients for null biases and scaling factors can be estimated as well. Here we use models and notation of [1–3].

Calibration as an optimal estimation

Here we state the problem of sensor error parameters determination as an optimal estimation problem of Kalmantype [1–2]. Then we can use efficient estimation techniques after the problem is stated in proper way. The model of accelerometer errors (assuming that some trivial pre-calibration step has been performed to make sensor errors small enough) is the following:

$$f_z' - f_z = \Delta f_z^0 + \Gamma f_z + \Delta f_z^s, \qquad f_z' = \begin{bmatrix} f_1' \\ f_2' \\ f_3' \end{bmatrix}, \qquad f_z = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} \Gamma_{11} & 0 & 0 \\ \Gamma_{21} & \Gamma_{22} & 0 \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} \end{bmatrix}.$$

Here f_z' stands for sensor output,f_z is true specific force acting on the reference proof mass as projected onto instrument reference frame Mz,$\Delta f_z^0$is null biases written in a column matrix, and$\Gamma$is a matrix containing errors of scaling factors on its diagonal, and small angles of sensitive axis misalignments off the diagonal. We choose instrument reference frame in a way that elements of Γabove the main diagonal are zeros. For gyros (rate sensors) we have:

$$\omega_z' - \omega_z = -\nu_z^0 - \Theta\omega_z - D\frac{f_z}{g_e} - \nu_z^s,$$

$$\omega_z' = \begin{bmatrix} \omega_1' \\ \omega_2' \\ \omega_3' \end{bmatrix}, \qquad \omega_z = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}, \qquad \Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ \Theta_{21} & \Theta_{22} & \Theta_{23} \\ \Theta_{31} & \Theta_{32} & \Theta_{33} \end{bmatrix}, \qquad D = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}.$$

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Similar to the previous, we have ω_z' for sensor output, ω_z for true absolute angular rate vector as projected onto the instrument frame, ν_z^0 for null biases, Θ containing scale factor errors and misalignments, D for coefficients of dynamic drift, and g_e is some reference value of specific force (e.g. gravity force on Earth's equator). Minus sign on the right side is just a convention.

Attitude matrix L_y is computed using logged gyro data [4]. L_y is a calculated value of transition matrix from geodetic to instrument frame of reference. It contains small error that can be represented as a three small rotations incorporated into a vector β_x of small angles. Then we introduce error equations, which form a system with state vector containing sensor error parameters Δf_z^0, Γ, ν_z^0, Θ, D, and orientation errors β_x. Using notation from [1–3] we get linear approximation for system mechanization:

$$\dot{\beta}_x = \hat{u}_x \beta_x + L_y^T \left(\nu_z^0 + \Theta \omega_z + D \frac{f_z}{g_e} \right) + L_y^T \nu_z^s \qquad \beta_x(0) = \beta_0,$$
$$\dot{\nu}_z^0 = 0, \qquad \dot{\Theta} = 0, \qquad \dot{D} = 0, \qquad \Delta \dot{f}_z^0 = 0, \qquad \dot{\Gamma} = 0. \tag{1}$$

Measurements in the system are differences between calculated gravity vector and its measured value. This difference contains information about sensor errors and can be expressed in terms of error components as

$$z^{acc} = \hat{\beta}_x \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T + L_y^T \left(\Delta f_z^0 + \Gamma f_z \right) + L_y^T \Delta f_z^s. \tag{2}$$

Estimation of the system state vector in (1) using measurements (2) is done with conventional optimal Kalman filter. Observability of the system depends primarily on the IMU motion in calibration procedure, as coefficients near estimated values in equations (1), (2) depend on the motion. The best condition is achieved by three cycles of rotation, each around one of roughly horizontal instrumental axis [1–2].

In equations above it is assumed that there is no temperature dependence in sensor error parameters. A conventional approach to deal with temperature variations of real sensor errors is to conduct calibration procedures at several temperature points. While calibrating at each temperature point the temperature is assumed to be constant. After calibration parameters are calculated for these temperature points, parameter values are interpolated using piecewise-linear or spline functions.

In order to let temperature transition processes to fade to achieve constant temperature inside the IMU, one have to wait up to hours before calibration starts. Moreover, temperature points should cover all IMU operational temperature range. For example, for the range from –60 to +60 degrees the number of temperature points can be up to twelve (–60, –45, –30, –15, –10, –5, +5, +10, +15, +30, +45, +60).

Adding temperature variations

To take temperature variations of sensor errors into consideration, let us modify models above. First, we introduce temperature sensor outputs for each inertial sensor: T_{f1}, T_{f2}, T_{f3}, for accelerometers and $T_{\omega 1}$, $T_{\omega 2}$, $T_{\omega 3}$ for gyros. These quantities are arranged into matrices:

$$T_f = \begin{bmatrix} T_{f1} & 0 & 0 \\ 0 & T_{f2} & 0 \\ 0 & 0 & T_{f3} \end{bmatrix}, \qquad T_\omega = \begin{bmatrix} T_{\omega 1} & 0 & 0 \\ 0 & T_{\omega 2} & 0 \\ 0 & 0 & T_{\omega 3} \end{bmatrix}.$$

Then we state the following:

- temperature is measured with appropriate accuracy in order to compensate temperature variations of inertial sensor errors;
- temperature changes inside the range where sensor errors have nearly linear temperature variations;
- temperature variations of sensitive axes misalignments and dynamic drifts (if present) can be neglected.

Under the above assumptions we modify models of sensor errors introducing temperature variations:

$$f_z' - f_z = \Delta f_z^0 + T_f k_{\Delta f} + \Gamma f_z + T_f K_\Gamma f_z + \Delta f_z^s, \qquad k_{\Delta f} = \begin{bmatrix} k_{\Delta f1} \\ k_{\Delta f2} \\ k_{\Delta f3} \end{bmatrix}, \qquad K_\Gamma = \begin{bmatrix} K_{\Gamma 11} & 0 & 0 \\ 0 & K_{\Gamma 22} & 0 \\ 0 & 0 & K_{\Gamma 33} \end{bmatrix},$$

$$\omega_z' - \omega_z = -\nu_z^0 - T_\omega k_\nu - \Theta\omega_z - T_\omega K_\Theta \omega_z - D\frac{f_z}{g_e} - \nu_z^s, \qquad k_\nu = \begin{bmatrix} k_{\nu 1} \\ k_{\nu 2} \\ k_{\nu 3} \end{bmatrix}, \qquad K_\Theta = \begin{bmatrix} K_{\Theta 11} & 0 & 0 \\ 0 & K_{\Theta 22} & 0 \\ 0 & 0 & K_{\Theta 33} \end{bmatrix}.$$

Then we add new components into the state vector of previous system. Addedcomponentsare $k_{\Delta f}$, k_ν, i.e. temperature coefficients of biases, and temperature coefficients of scaling factors, i.e. non zero elements of$K_\Gamma$andK_Θ. Thus, instead of (1) we now have new mechanization equations

$$\dot{\beta}_x = \hat{u}_x \beta_x + L_y^T \left(\nu_z^0 + T_\omega k_\nu + \Theta\omega_z + T_\omega K_\Theta \omega_z + D\frac{f_z}{g_e} \right) + L_y^T \nu_z^s \qquad \beta_x(0) = \beta_0,$$
$$\dot{\nu}_z^0 = \Delta\dot{f}_z^0 = \dot{k}_{\Delta f} = \dot{k}_\nu = 0, \qquad \dot{\Theta} = \dot{D} = \dot{\Gamma} = \dot{K}_\Theta = \dot{K}_\Gamma = 0. \tag{3}$$

Measurement model is properly changed, also:

$$z^{acc} = \hat{\beta}_x [0 \quad 0 \quad g]^T + L_y^T \left(\Delta f_z^0 + T_f k_{\Delta f} + \Gamma f_z + T_f K_\Gamma f_z \right) + L_y^T \Delta f_z^s \tag{4}$$

As before, we use Kalman filtering to obtain estimates for the whole set of unknown parameters. In this new model of calibration coefficients in (3) and (4) near estimated parameters depend not only on IMU motion, but on temperature variation also. In order to ensure observability, the temperature of inertial sensors have to be non constant. It should change in a range of about several degrees, and should be independent of mechanical motion. This is easily arranged in real calibration, since usual temperature variations are nearly linear, or periodical ramp, or asymptotic during self heating of the IMU.

As non constant temperature is now not unwanted, but desired condition of the experiment, there is no need to wait until temperature stabilizes inside IMU. So the whole procedure becomes shorter. In addition, since we know not only the values of sensor error parameters at each temperature, but also we know its temperature coefficient, we are now able to extrapolate dependencies outside the calibration temperature range. This allows to reduce the number of calibration operations for a given IMU operational temperature range.

Results

To verify new modifications a real IMU data were processed. The unit under test packs micro-machined gyros and accelerometers of performances as follows:

- bias stability 0.03 deg/sec for gyros, 5×10^{-3}meters/sec^2 for accelerometers;
- noise density 0.05 deg/sec/√Hz for gyros, 2×10^{-3}meters/sec^2/√Hz for accelerometers.

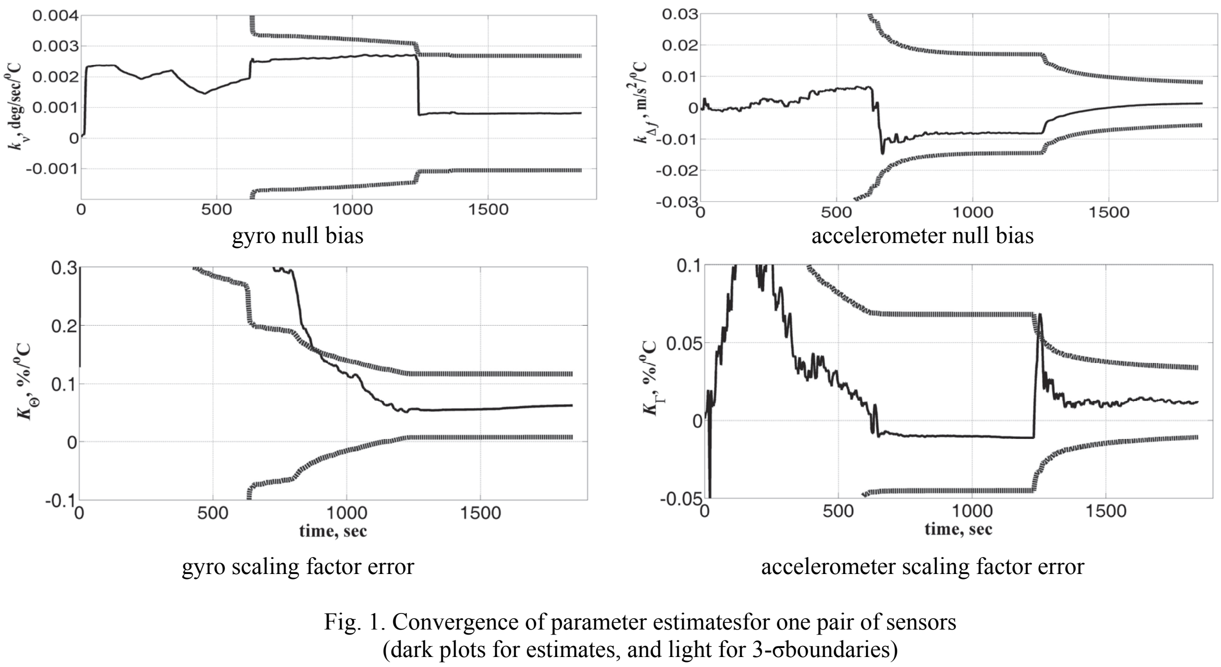

Fig. 1. Convergence of parameter estimatesfor one pair of sensors
(dark plots for estimates, and light for 3-σboundaries)

As mentioned before the procedure included three cycles of rotation around a horizontal rotation axis of a turntable at room temperature with self heating IMU. The rate of rotation was approximately 10±1 degrees per second. In the first two cycles temperature changed up to 2.5 degrees Celsius, and around 1 degree in the third cycle.

Temperature coefficients of biases and scaling factors were estimated simultaneously with the rest of parameters. Results are shown in Fig. 1. There is observability present for parameters under consideration, as estimation error covariances converge. Successive compensation of estimated temperature variations indicates that estimates for temperature coefficients are close to real ones.

Conclusions

Estimation of temperature coefficients of null biases and scaling factors are added to the method of IMU calibration on a low-grade single axis turntable. These coefficients are estimated simultaneously with the rest of parameters in a calibration at non-constant temperature. New algorithms have been tested in a real data processing and have proved to work well.

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