Delayed feedback control for the system with hyperbolic attractor

Sergey Belyakin, Aleksander Loskytov, Arsen Dzhanoev

Physics Faculty, Lomonosov Moscow State University, Moscow, Russia

Theory of hyperbolic attractor

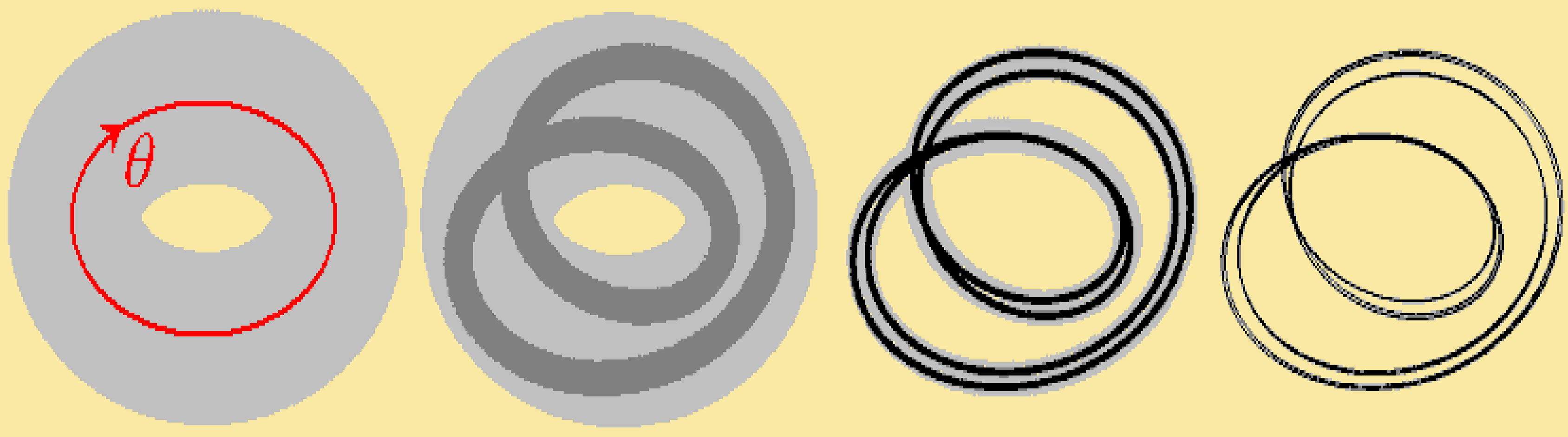
The set Λ is called a hyperbolic attractor of the dynamical system if Λ is closed, topologically transitive hyperbolic set and there exists the neighborhood $U \supset \Lambda$ such that $\Lambda = \bigcap_{n\geq 0} f^n U$

The well-known examples of the hyperbolic attractors are the Smale-Williams' solenoid and Plykin's attractor.

The Smale-Williams' solenoid is obtained by mapping the toroidal region $T = S^1 \times D^2$ into itself, where S^1 - unit circle and D^2 - unit disc in R^2 .

Then $f: T \to T$ $f(x, y, \varphi) = (\frac{1}{k}x + \frac{1}{2}\cos\varphi, \frac{1}{k}y + \frac{1}{2}\sin\varphi, 2\varphi)$, where *k*>2 defines the contraction "along the thickness" and defines the solenoid as a subset $T \subset R^3$.

Evolution of hyperbolic attractors



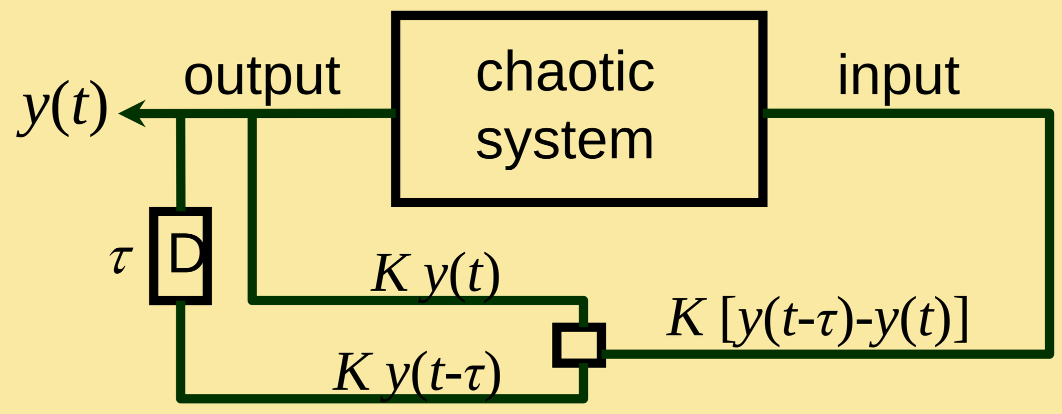
System of equations that has hyperbolic attractor

$$\dot{x}_1 = (1 - a_2 + \tfrac{1}{2}a_1 - \tfrac{1}{50}(1-a_1)^2)x_1 - \tfrac{1}{2}\varepsilon_1(x_2^2 - y_2^2),$$
$$\dot{y}_1 = (1 - a_2 + \tfrac{1}{2}a_1 - \tfrac{1}{50}(1-a_1)^2)y_1 - \varepsilon_1 x_2 y_2,$$
$$\dot{x}_2 = (a_1 - 1)x_2 - \varepsilon_2 x_1,$$
$$\dot{y}_2 = (a_1 - 1)y_2 - \varepsilon_2 y_1,$$
$$a_1 = x_1^2 + y_1^2, a_2 = x_2^2 + y_2^2, \varepsilon_1 = 0.01, \varepsilon_2 = 0.1.$$

Delayed feedback

$$\dot{x}_1 = (1 - a_2 + \tfrac{1}{2}a_1 - \tfrac{1}{50}(1-a_1)^2)x_1 + \varepsilon_1(x_2^2 - y_2^2),$$
$$\dot{y}_1 = (1 - a_2 + \tfrac{1}{2}a_1 - \tfrac{1}{50}(1-a_1)^2)y_1 - \varepsilon_1 x_2 y_2 + D_{t,t-\tau},$$
$$\dot{x}_2 = (a_1 - 1)x_2 - \varepsilon_2 x_1,$$
$$\dot{y}_2 = (a_1 - 1)y_2 - \varepsilon_2 y_1,$$
$$a_1 = x_1^2 + y_1^2, a_2 = x_2^2 + y_2^2.$$
$$D_{t,t-\tau} = K(y_{1,t-\tau} - y_{1,t})$$

Pyragas Method of stabilization of chaotic system

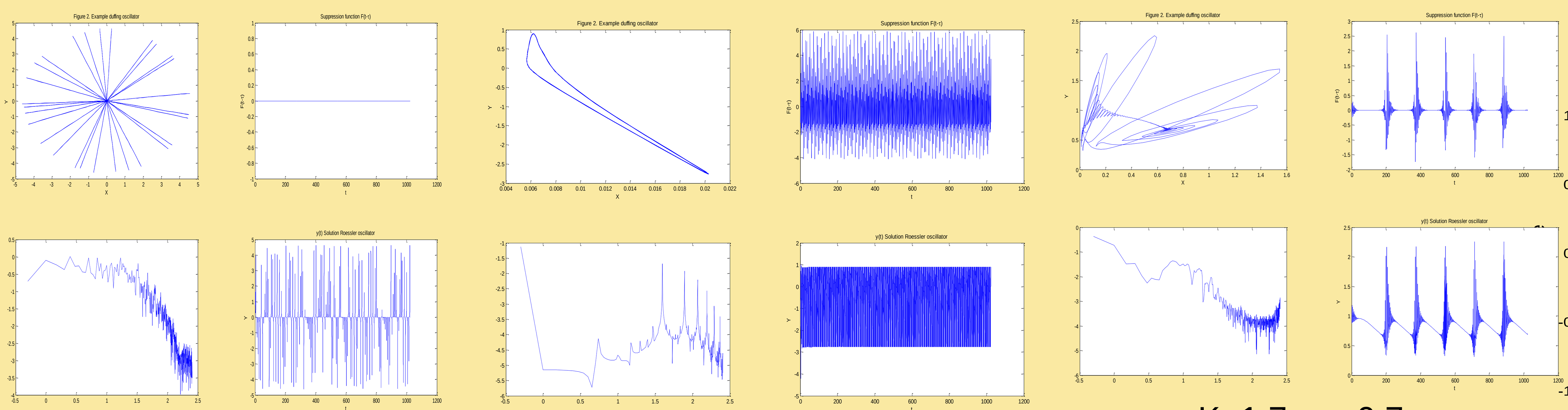


Pyragas Method

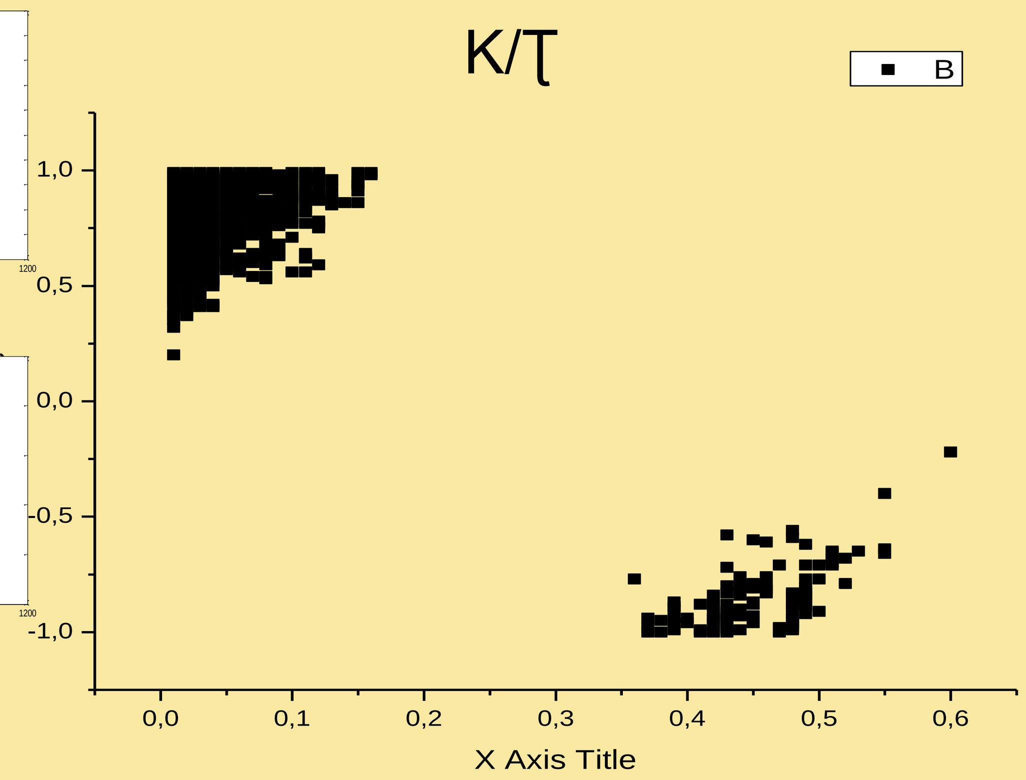
The smooth family of the nonlinear systems with control of ODE

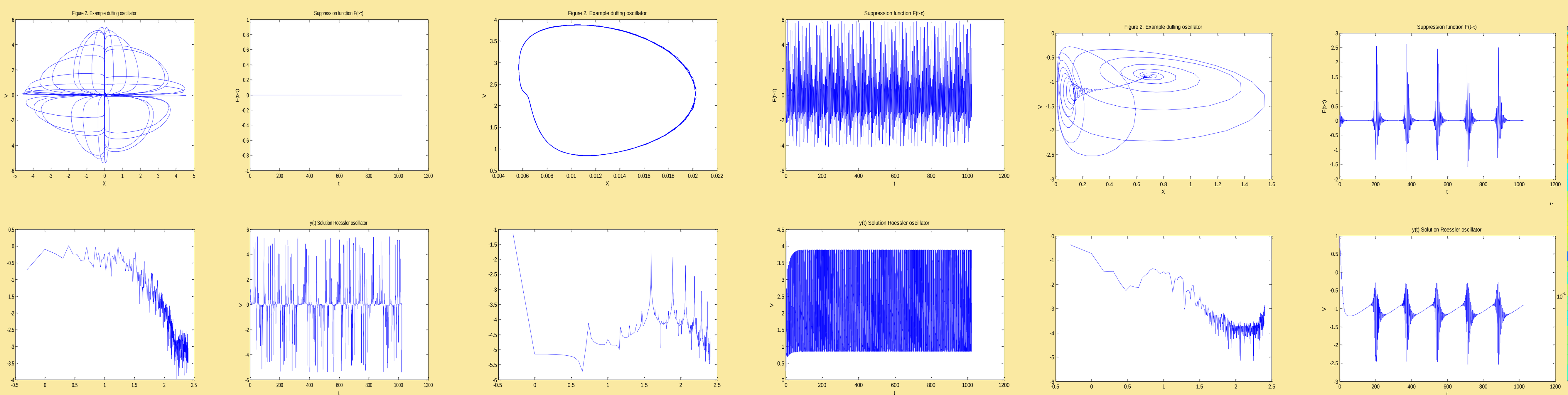
$$\dot{x} = F(x, \mu, u)$$
$$x \in M \subset R^m, \mu \in L \subset R^k, u \in U \subset R^n, F \in C^\infty$$

that depends on control parameters vector: *u* . Let us need to stabilize the unstable limit circle $x^*(t,\mu^*)$ of the period *T*, which is the solution of the system of the family when *u=0* and $\mu = \mu^*$. Let, under the same values of the parameters *u=0* и$\mu = \mu^*$, the family have a regular or singular attractor. Then the stabilization of the circle $x^*(t,\mu^*)$ is found by the feedback law with delay of the form $u(t) = K(x(t) - x(t-T))$, where *K* — matrix of the coefficients. Initial condition for *x(0)* we choose in a sufficiently small neighborhood of the circle's orbit. The solution *x(t)* of the system: $\dot{x} = F(x(t), \mu^*, K(x(t) - x(t-T))$ with the feedback when $\mu = \mu^*$ might converge to the relevant unstable circle $x^*(t,\mu^*)$.

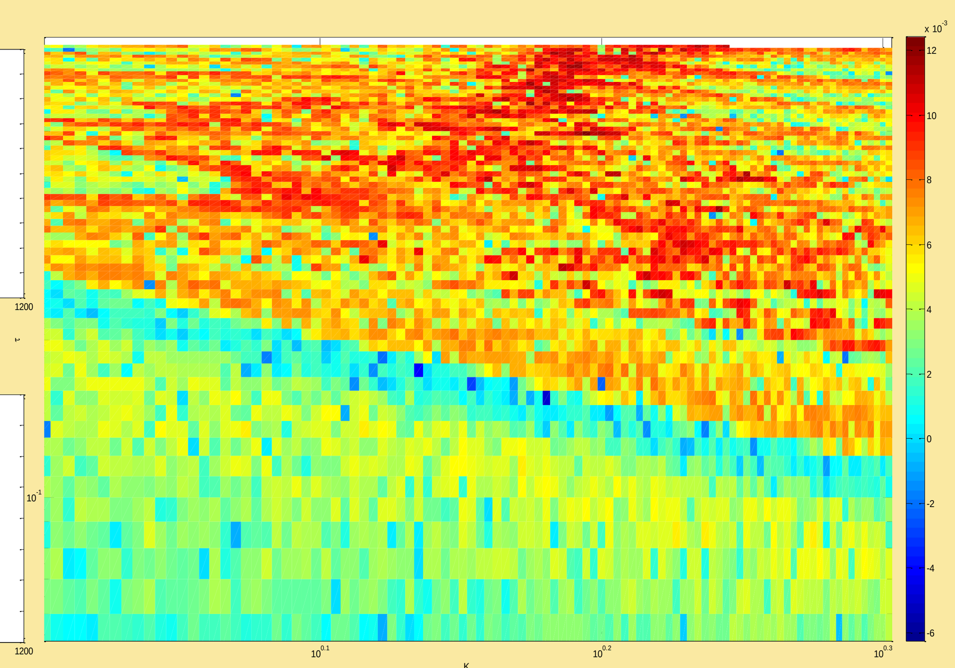


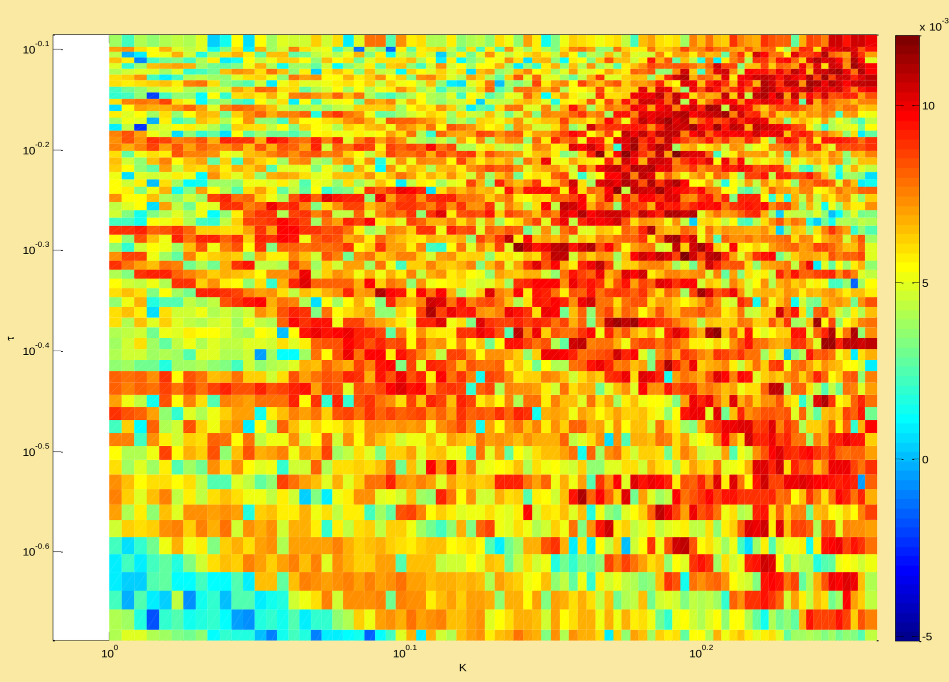
y_1 / x_1 K=0 K=-1.7 , τ =0.7 K=1.7 , τ =0.7





y_2 / x_1 K=0 K=-1.7 , τ =0.7 K=1.7 , τ = 0.7





References:

1. Kuznetsov S. P., Seleznev E. P, JETP 102, 355, 2006.
2. Pyragas K., Continuous control of chaos by self–controlling feedback. Phys. Lett. A 170 (1992) 421 – 428.
3. Kuznetsov S. P., Pikovsky A., Autonomous coupled oscillators with hyperbolic strange attractors. Physica D 232 (2007) 87 – 102
4. Mikhailov A. S., Loskutov A. Yu., Foundations of Synergetics . Chaos and Noise. Springer . Berlin and Moscow. April 1996.