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PERIODIC MOTIONS OF A RIGID BODY WITH A FIXED POINT IN THE GRAVITY FIELD OF TWO CENTERS

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Andoyer variables [1] are used to investigate the problem of existence of periodic motions of a rigid body with a fixed point in the gravity field of two stationary centers.

1. Consider the motion of a rigid body about a fixed point O_1 in the Newtonian gravity field of two fixed centers M_1 and M_2. We introduce the following coordinate systems: $OXYZ$ is a stationary coordinate system chosen so that the centers of attraction $M_1(X_1, 0, 0)$ and $M_2(X_2, 0, 0)$ lie on the OX-axis and the fixed point $O_1(0, Y_0, 0)$ of the body M lies on the OY-axis; O_1xyz is a coordinate system with the origin at the fixed point O_1, the axes of which are parallel to the axes of the $OXYZ$-coordinate system; $O_1\xi\eta\zeta$ is a moving coordinate system the axes of which are directed along the principal axes of inertia of the body M with respect to the fixed point. We described the motion of the body using the Andoyer elements

$$L, G, H, l, g, h \tag{1.1}$$

Here G denotes the magnitude of the vector $\mathbf{G}$ of the moment of momentum of the body rotation; L and H are the projections of the vector $\mathbf{G}$ on the $O_1\zeta$-axis and O_1z-axis of the body, respectively; l is the angle counted from the line of intersection of the intermediate plane P normal to the vector $\mathbf{G}$ with the $O_1\xi\eta$ plane of the body, to the $O_1\xi$-axis; h is the angle counted from the O_1x-axis to the line of intersection of the O_1xy plane and the plane P, g is the angle between the line of intersection of the plane O_1xy and the plane P, g is the angle between the line of intersection of the planes P, $O_1\xi\eta$ and the line of intersection of the planes P, O_1xy.

The motion in question is determined by the Hamiltonian

$$K = T - U_1 - U_2 \tag{1.2}$$

$$T = \frac{G^2 - L^2}{2AB}(A\cos^2 l + B\sin^2 l) + \frac{L^2}{2C}$$

$$U_s = -P_s(\xi_c\alpha_s + \eta_c\beta_s + \zeta_c\gamma_s) - \frac{3P_s}{2mR_s}(A\alpha_s^2 + B\beta_s^2 + C\gamma_s^2)$$

$$P_s = f\frac{m_s m}{R_s^2}, \quad s = 1, 2$$