Optimal control of development of oil fields for the Buckley-Leverett model

Atlas V. Akhmetzyanov
Institute of Control Sciences of Russian Academy of Sciences
Moscow, Russia
awa@ipu.ru

Alexei G. Kushner
Lomonosov Moscow State University
Moscow, Russia
Institute of Control Sciences of Russian Academy of Sciences
Moscow, Russia
kushner@physics.msu.ru

Valentin V. Lychagin
Institute of Control Sciences of Russian Academy of Sciences
Moscow, Russia
Arctic University of Norway
Tromso, Norway
valentin.lychagin@uit.no

Andrey A. Shevlyakov
Institute of Control Sciences of Russian Academy of Sciences
Moscow, Russia
aash29@gmail.com

***Abstract* — Optimal strategy of development of oil fields is constructed for one-dimensional the Buckley-Leverett model. The obtained results are intended for use in field development conventional or heavy oil alternating parallel rows of vertical production and injection wells and alternating trunks or vertical cracks hydraulic fracturing in the bottom-hole zones of horizontal production and injection wells.**

***Index Terms* Multiphase flow, Buckley-Leverett model, porous medium.**

INTRODUCTION

In this paper we consider the Buckley-Leverett model [1,2,3] for the filtration of a two-phase system (water – oil) under the assumption that the liquids are incompressible and immiscible, and without capillary forces.

We consider the case where there is one injection well and one producing well.

For this model, various optimal control regimes have been investigated, delivering the maximum production level at the second well.

THE MODEL

Let $u_i, i = 1{,}2$ be the filtration rate (that is, the volume of fluid passing through the unit surface per unit time, and s_i is the saturation (i.e., part of the volume) of the i-th component. By ρ_i, μ_i we denote the pressure and viscosity of the i-th component respectively.

Neglecting the capillary forces, we assume that $p_1 = p_2 = p$.

In addition, since $s_1 + s_2 = 1$, we assume that $s_1 = \sigma, s_2 = 1 - \sigma$.

The Buckley-Leverett system of equations consists of two sets of equations that reflect the Darcy law and the law of conservation of mass.

In our case, we assume that the Darcy law for each component has the form:

$$u_1 = -\frac{k}{\mu_1} f_1(\sigma) \mathrm{grad} p_1, \qquad (1)$$

$$u_2 = -\frac{k}{\mu_2} f_2(\sigma) \mathrm{grad} p_2, \qquad (2)$$

where k is a coefficient of hydraulic conductivity, and $f_i(\sigma)$ is a phase permeability.

The second group of equations that reflect the law of conservation of mass has the form:

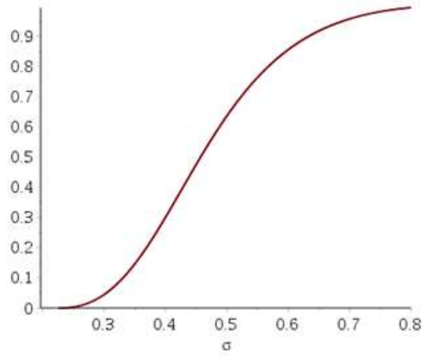

Fig. 1. Graph of the function $F(\sigma)$.

$$m\frac{\partial s_1}{\partial t} + \mathrm{div} u_1 = 0, \quad (3)$$

$$m\frac{\partial s_2}{\partial t} + \mathrm{div} u_2 = 0, \quad (4)$$

where m is the porosity of the medium, i.e. coefficient that indicates what part of the environment occupy the pores.

Adding Eq.3 and Eq.4 and, using the fact that

$$s_1 + s_2 = 1,$$

we obtain the following relation: $\mathrm{div}\, u = 0$, where $u = u_1 + u_2$ is a total filtration rate.

Accordingly, equation (3) takes the form

$$m\sigma_t + div(F(\sigma)u) = 0 \quad (5)$$

where

$$F(\sigma) = \frac{f_1(\sigma)}{f_1(\sigma) + \mu f_2(\sigma)}, \qquad \mu = \frac{\mu_1}{\mu_2}.$$

Adding now Eq.1 and Eq.2 under the assumption of absence of capillary forces, i.e.

$$p_1 = p_2 = p,$$

we obtain Darcy's law for the total filtration rate:

$$u = -f(\sigma)\nabla p,$$

where

$$f(\sigma) = \frac{k}{\mu_1} f_1(\sigma) + \frac{k}{\mu_2} f_2(\sigma).$$

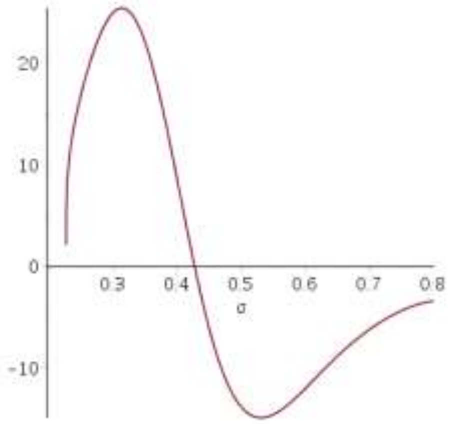

Fig. 2. Graph of the function $F''(\sigma)$.

We call the resulting system the Buckley–Leverett system (see [2]):

$$\begin{cases} m\sigma_t + UF'(\sigma)\sigma_x = 0, \\ u = -f(\sigma)\nabla p, \\ u_x = 0. \end{cases}$$

Note that knowing the total filtration rate u, we can restore the phase filtration rates u_1 and u_2 as follows:

$$u_1 = F(\sigma)u, \quad u_2 = (1 - F(\sigma))u_2.$$

In this paper we consider only the one-dimensional case, typical of filtration processes, when the development of oil fields is made using alternating rows of vertical and horizontal trunks of production and injection wells and alternating vertical cracks hydraulic fracturing in the bottom-hole zones of production and injection horizontal wells.

Then the condition

$$div\, u = u_x = 0$$

means that the total filtration rate depends on time only: $u = u(t)$. We introduce the function $\tau(t)$ such that

$$u = \dot{\tau}(t), \quad \tau(0) = 0.$$

In what follows we assume that $u > 0$, i.e. that the function $\tau(t)$ is monotonically increasing.

Under these assumptions, we can represent the saturation function $\sigma(t,x)$ in the following form:

$$\sigma(t,x) = a(\tau(t), x),$$

where the function $a(\tau, x)$ satisfies the analogue of Eq.5:

$$m\frac{\partial a}{\partial \tau} + F'(a)\frac{\partial a}{\partial x} = 0.$$

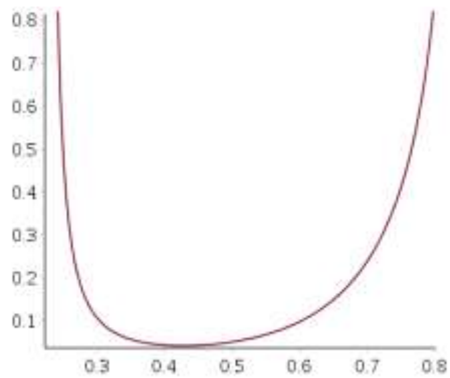

Fig. 3. Graph of the function $\phi(\alpha)$.

BOUNDARY VALUE PROBLEM

Suppose that the injection well is at the point x_0, and the producing well is at the point x_1, where $x_1 > x_0$.

Suppose also that the saturation $s_1 = \sigma$ is given at the point x_0, i.e. $\sigma(t, x_0) = \alpha(t)$ for some function α.

Then the solution $\sigma(t,x)$ of Eq.5, under the assumption that the function $\tau(t)$ is known, can be found by the method of characteristics.

Namely, the characteristic equations for equation (13) have the form

$$\dot{\tau} = 1, \qquad \dot{x} = \frac{1}{m}F'(a), \qquad \dot{a} = 0$$

and we see that

$$x_1(\tau_1) - x_0(\tau_0) = \frac{F'(a_0)}{m}(\tau_1 - \tau_0), \quad (6)$$

where a function $a_0 = a(\tau_0, x(\tau_0))$ is constant along the characteristic.

Consider a characteristic starting at the point (x_0, t_0) and ending at the point (x_1, t_1). Then from Eq.6 we obtain:

$$x_1 - x_0 = \frac{F'(a_0)}{m}(\tau(t_1) - \tau(t_0)),$$

or

$$\tau(t_1) - \tau(t_0) = \phi\big(\alpha(t_0)\big), \quad (7)$$

where

$$\phi(\alpha) = \frac{m(x_1 - x_0)}{F'(\alpha)}.$$

In addition, the saturation value of the first phase $\sigma(t, x_1) = \beta(t)$ at the second well is equal to $\beta(t_1) = \alpha(t_0)$.

By $\pi_0(t)$ and $\pi_1(t)$ we denote the value of the pressure gradient at the first and second wells, respectively:

$$\nabla p|_{x=x_0} = \pi_0(t), \qquad \nabla p|_{x=x_1} = \pi_1(t).$$

Then from Darcy's law it follows that

$$U|_{x=x_0} = \tau'(t) = -f(\alpha(t))\pi_o(t),$$
$$U|_{x=x_1} = \tau'(t) = -f\big(\beta(t)\big)\pi_1(t),$$

or

$$f\big(\alpha(t)\big)\pi_0(t) = f(\beta(t))\pi_1(t).$$

In particular, the condition $u > 0$ is satisfied if and only if the functions $\pi_0(t)$and $\pi_1(t)$ take only negative values.

Variational problem

Suppose that the quantity of water cV is given, where V is the volume of the well pumped into the first well for a fixed time T, i.e. assume that

$$I_w = \int_0^T \alpha(t_0)dt_0 = c.$$

Then amount of oil available for production in the second well is

$$I_o = \int_{T_1}^{T_2} (1 - \beta(t_1))dt_1,$$

where the time intervals $T, T_1, T_2$are related by the relation Eq.7, namely

$$\tau(T_1) = \phi(\alpha_0),$$
$$\tau(T_2) = \tau(T) + \phi(\alpha_1),$$

where $\alpha_0 = \alpha(0)$, $\alpha_1 = \alpha(T)$.

Let us find the injection mode at the first well (i.e., the function $\alpha(t)$), such that the value of I_0 should be maximal for the given value of I_w.

Taking into account Eq.7, we obtain

$$dt_1 = \frac{\tau'(t_0) + \phi'(\alpha(t_0))\alpha'(t_0)}{\tau'(t_1)}dt_0.$$

Accordingly, the integral I_w takes the following form:

$$I_w = \int_0^T (1 - \alpha(t_0))\frac{f(\alpha(t_0))\pi_0 - \phi'(\alpha(t_0))\alpha'(t_0)}{f(\beta(t_1))\pi_1(t)}dt_0.$$

Further, $\beta(t_1) = \alpha(t_0)$ and assuming that

$$\pi_1(t) = \text{const} = \pi_1,$$

we rewrite I_w in the following form:

$$I_w = \frac{1}{\pi_1}\int_0^T (1 - \alpha(t))(\pi_9(t) - G(\alpha(t_0))\alpha'(t_0))dt_0,$$

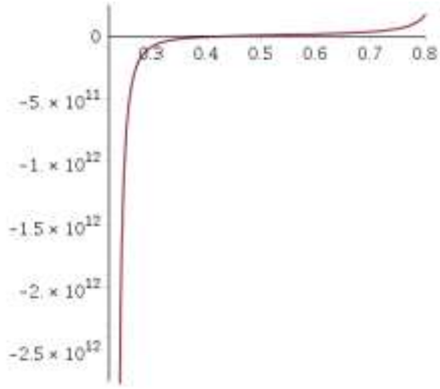

Fig. 4. Graph of the function G(α).

where

$$G(\alpha) = \frac{\phi'(\alpha)}{f(\alpha)} = -\frac{m(x_1 - x_0)}{f(\alpha)F'(\alpha)^2}F''(\alpha).$$

Let

$$L(t_0, \alpha, \dot{\alpha}) = \frac{1}{\pi_1}(1 - \alpha)(\pi_0(t_0) - G(\alpha)\dot{\alpha}).$$

Then the desired optimal value $\alpha(t)$ is a solution of the following variational problem:

$$\int_0^T L(t_0, \alpha, \dot{\alpha})dt_0 \to max, \ \int_0^t \alpha dt_0 = c = \text{const}. \quad (8)$$

The Euler-Lagrange equation for this problem has the form:

$$\frac{\partial L}{\partial \alpha}(L - \lambda\alpha) - \frac{d}{dt}\frac{\partial L}{\partial \dot{\alpha}} = \lambda,$$

where λ is Lagrange's multiplier, or

$$\frac{\pi_0(t)}{\pi_1} = \lambda.$$

Thus, we obtain the following result.

Theorem. *The boundary conditions σ that solve the variational problem Eq.8 for the maximum volume of incoming oil to the second well, subject to a predetermined injection time T and the amount of injected water c, and a constant pressure gradient π_1 at the second well, should be such that the pressure gradient at the first well will be a constant too: $\pi_0(t) = \pi_0 = const$.*

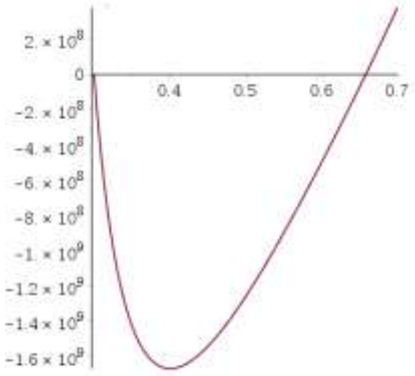

Fig. 5. Graph of the function $H(\alpha)$, $\alpha(0) = 0.3$.

Corollary. For the optimal injection mode the following conditions are fulfilled:

$$\pi f(\beta(t_1)) = \pi f(\beta(t_0)).$$

Assuming now that the gradient$\pi_0(t) = \pi_0 = \text{const}$, we calculate the integral I_0:

$$I_0 = \frac{\pi_0}{\pi_1}(T - c) - \frac{1}{\pi_1} H(\alpha_1),$$

where

$$H(\alpha) = \int_{\alpha(0)}^{\alpha} (1 - \alpha) \frac{\phi'(\alpha)}{f(\alpha)} d\alpha.$$

CONCLUSION

The constructed method allows us to find the optimum strategy of management of developments of oil fields. This method can be used for domains with many wells and in case of not isothermal filtration too [4].

ACKNOWLEDGMENT

This work was supported by the Russian Science Foundation (project No 15-19-00275).

REFERENCES

[1] Buckley, S.E., Leverett, M.C. “Mechanism of fluid displacement in sands”, in Trans. AIME, SPE, 1942, 146, 107-116.

[2] Barendlatt, G.L., Entov, V.M., Ryzhik, V.M. “Theory of fluid flows through natural rocks”. Kluver Acad. Publ., 1989, 208 p.

[3] Allen M.B., Behie G.A., Trangenstein J. A. “Multiphase Flow in Porous Media”, Springer US, 1988.

[4] Akhmetzyanov, A.V., Kushner, A.G., Lychagin, V.V. “Mass and heat transport in the two-phase Buckley-Leverett model”, in J. Geom. Phys, 113 (2017), 2-9.