Control System Design for Plasma Unstable Vertical Position in a Tokamak by Linear Matrix Inequalities

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***Abstract*—The paper deals with the design and simulation of control systems for an unstable plasma vertical position of the linear model using Linear Matrix Inequalities (LMI) approach. It is necessary to suppress minor disruptions and stabilize a system output during a plasma discharge. The problem of state feedback stabilization for different LMI regions (left half-plane, vertical strip region, and intersection of sector and left half-plane) has been solved, as well as a H_2 state feedback LMI controller has been designed, and the systems have been simulated. In addition, a H_2 system with desired pole region placement has been designed and modeled. The last designed control system and a modal system with three-time multiple pole were compared in the numerical simulations on the base of a control power criterion. The dependence of the control power on plasma position reference and disturbance was calculated. The estimation of the acceptable disturbance in the presence of the plant input-output constraints was done by means of ellipsoid techniques.**

***Index Terms*—tokamak, plasma vertical instability, LMI design, feedback stabilization, H_2 control, control power assessment, ellipsoid estimation**

I. Introduction

The most promising devices for obtaining nuclear fusion energy are tokamaks [1]. There are two fusion megaprojects in the world: ITER (Cadarache, France) and DEMO. ITER is an experimental nuclear fusion reactor being under construction, DEMO is a nuclear fusion station that is intended to be built. The energy in tokamaks is derived from a high-temperature plasma. All the operational tokamaks must work at high plasma pressure, current, and temperature. The vertical elongation of the plasma in a tokamak corresponds to increase of plasma pressure for the same toroidal magnetic field. But pushing the plasma into such high-performance regime causes the plasma vertical instability. Such instability poses a risk to tokamak equipment: for example if the plasma shifts significantly in a vertical direction the first wall may be damaged. However, from the perspective of the fusion power reactor a plasma separatrix location near to the first wall during the diverter phase of the plasma discharge, while the plasma is unstable, is the most promising, attractive, and favorable. So the plasma vertical position control problem has to be solved by the most reliable way in order to continue progress toward producing energy from fusion.

The paper is organized as follows. The problem setting is given in Section 2. Section 3 discusses the *D*-stability control problem. Section 4 presents H_2 optimization LMI conditions and controller. The H_2 design with desired pole regions is discussed in Section 5. In Section 6 we estimated control power peaks at reference steps and disturbance drops. Section 7 establishes the acceptable disturbance. Finally, Conclusion gives some future work ideas.

II. Statement of the Control Problem

The goal of this work is to develop and study plasma vertical position stabilization systems on the sample of the simplest plasma vertical movement model of the T-15 tokamak (being constructed in Kurchatov Institute, Moscow, Russia) [2-4] in case of minor disruption disturbances during the plasma discharge using the LMI approach.

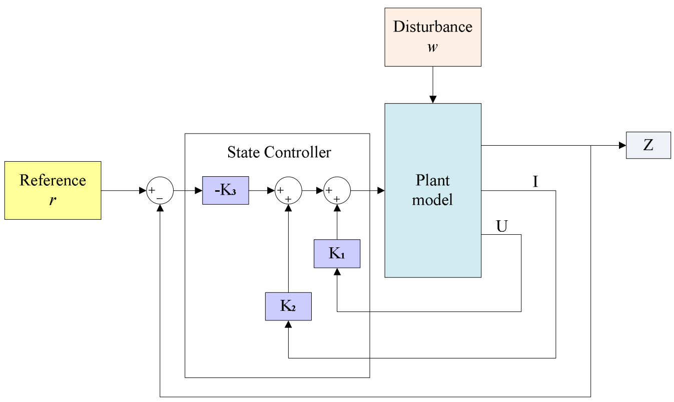


Fig. 1. Block diagram of the closed-loop plasma vertical position control system.

Nowadays LMI techniques are developing very intensively and make possible to create stable numerical procedures for control systems design as well as to state control problems from the common point of view [5, 6].

The identified linear plasma model was used in a plasma control system. It was derived from the DINA plasma physics code [7] tuned to the T-15 tokamak nominal regime and relative to the quasi-stationary phase of the plasma discharge [2]. The feedback system structure is shown in Fig. 1.

We consider the plant linear model (plasma in a tokamak) in a state-space form with the disturbance input:

$$\dot{x} = Ax + B_1 u + B_2 w,\ y = Cx, \tag{1}$$

where $x = [U\ I\ Z]^T$ is the fully accessible state ($U(t)$ is the voltage, $I(t)$ is the current of the control coil, $Z(t)$ is the plasma's vertical displacement, $P(t)=U(t)I(t)$ is the control power), $u(t)$ is the control, $y(t)$ is the output, $w(t)$ is the additive disturbance;

$$A = \begin{bmatrix} -\dfrac{1}{T_a} & 0 & 0 \\ \dfrac{K_c}{T_c} & -\dfrac{1}{T_c} & 0 \\ 0 & \dfrac{K_p}{T_p} & \dfrac{1}{T_p} \end{bmatrix},\ B_1 = \begin{bmatrix} \dfrac{K_a}{T_a} \\ 0 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 0 \\ \dfrac{K_p}{T_p} \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$T_a = 3.3$ ms, $T_p = 20.8$ ms, $T_c = 46.7$ ms are the time constants of an actuator model, a plasma model, and a horizontal field (HF) control coil model respectively, $K_a = 2000$, $K_p = 1.78\,{cm}/{kA}$, $K_c = 11.11\,{1}/{\Omega}$ are gain constants [3]. The plasma model *unstable* pole is equal to $+\,{1}/{T_p}$.

The pair (A, B_1) in (1) is controllable: $rk\begin{bmatrix} B_1 & AB_1 & A^2 B_1 \end{bmatrix} = 3$, and the pair (A, C) is observable: $rk\begin{bmatrix} C^T & (CA)^T & (CA^2)^T \end{bmatrix} = 3$ [6, 10].

We seek a state feedback controller $K = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix}$ for different statements of LMI problem and determine thus the system's control law as

$$u = Kx. \tag{2}$$

III. D-Stabilization For Different LMI Regions

A. $D_{\alpha,\beta}$ region

We consider the following strip region: $D_{\alpha,\beta} = \{x + iy \mid -\beta < x < -\alpha\}$. We can study first the left half-plane when $\beta = \infty$. Let us design the state feedback control law (2) for the model (1) such as all the eigenvalues of the matrix of the closed-loop system are located in this strip region and the system is stable. In this case, the LMI conditions for the D-stability are as follows [6]:

$$\begin{cases} Y > 0, \\ AY + YA^T + B_1 F + F^T B_1^T + 2\alpha Y < 0, \\ AY + YA^T + B_1 F + F^T B_1^T + 2\beta Y > 0. \end{cases}$$

The system is stable if and only if there exists matrix Y satisfying these LMI conditions. We solve the auxiliary convex optimization with LMI constraints

$$\min t,\ H(x) < Q(x) + tI$$

where x and the scalar t are the decision variables and obtain the parameter matrices F and Y. The feedback control matrix $K = FY^{-1}$ is based on the LMI solution.

During the numerical simulation the reference step signal $Z_{ref} = r\left(1(t) - 1(t - T_r)\right)$, where $1(t)$ is the Heaviside function, r = 0.02 m, T_r = 0.2 s, and the disturbance $w = I_0\left(1(t) - 1(t - T_w)\right)$, I_0 = 1000 A, T_w = 0.1 s have been applied to the plant model. The *stable* poles of the closed loop system are located in the strip region $D_{250,350}$ and equal $\{-294 + 595i,\ -294 - 595i,\ -278\}$. Simulation results are shown in Fig. 2. The plasma vertical position deviation Z_d caused by the disturbance is $Z_d = 0.005$ m, the maximum vertical control coil current is $I_{VCC\ \max} = 5025$ A, the highest power is $P_{\max} = 2.6 \times 10^7$ W.

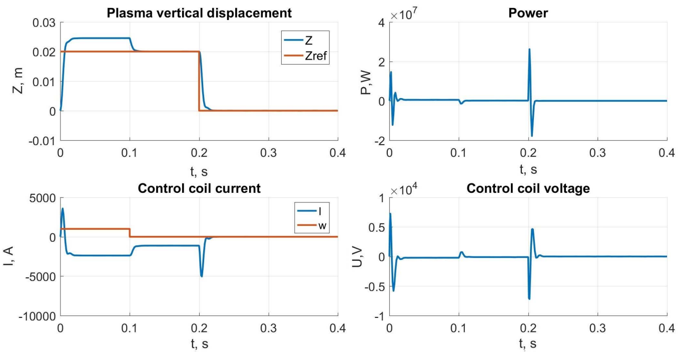


Fig. 2. Closed-loop plasma vertical position control system performance at $H_{\alpha,\beta}$ stabilization.

B. $D_{\alpha,r,\vartheta}$ region

The next D-region to deal with is the intersection of a sector and the left half-plane (Fig. 3):

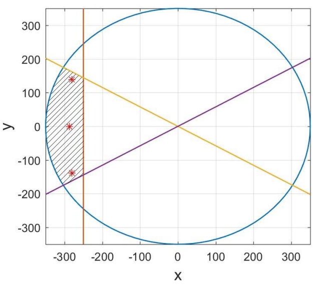


Fig. 3. LMI $D_{\alpha,r,\vartheta}$ region.

$$D_{\alpha,r,\vartheta} = \left\{ x+iy \mid x < -\alpha < 0,\ |x+iy| < r,\ |y| < x\tan\vartheta \right\}.$$

We search the state feedback controller confining the poles of the closed loop system to this region. The LMI conditions, which allow solving the control problem, are as follows:

$$\begin{cases} AP + PA^T + B_1W + W^TB_1^T + 2\alpha P < 0, \\ \begin{bmatrix} -rP & AP + B_1W \\ PA^T + W^TB_1^T & -rP \end{bmatrix} < 0, \\ \begin{bmatrix} AP + PA^T + B_1W + W^TB_1^T & AP - PA^T + B_1W - W^TB_1^T \\ -AP + PA^T - B_1W + W^TB_1^T & AP + PA^T + B_1W + W^TB_1^T \end{bmatrix} \circ \\ \circ \begin{bmatrix} \sin\vartheta & \cos\vartheta \\ \cos\vartheta & \sin\vartheta \end{bmatrix} < 0, \end{cases} \quad (3)$$

where the symbol $\circ$ is the Hadamard product. The controller $K = WP^{-1}$ is derived from parameter matrices W and P.

During the numerical simulation the reference step signal r = 0.02 m, T_r = 0.2 s, and the disturbance w = -1000 A, T_w = 0.1 s have been applied to the plant model. The stable poles of the closed loop system are located in the $D_{250,350,30}$ region (Fig. 3) and equal $\{-280+138i,\ -280-138i,\ -287\}$. Simulation results are shown in Fig. 4: $Z_d = 0.0005$ m, $I_{VCC\ \max} = 2850$ A, $P_{\max} = 4.45\times10^6$ W.

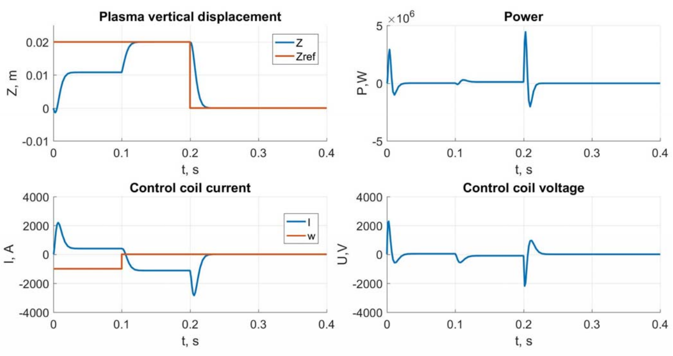


Fig. 4. Control system performance at $D_{\alpha,r,\vartheta}$ stabilization.

IV. H_2 Stabilization

In this section we are considering the H_2 state feedback control problem. We have to design the controller for the system such that the effect of the disturbance to the system output is prohibited to a desired level and the closed-loop system is stable.

The influence of the disturbance w to the system output z is determined by $z(s) = \underbrace{C\left(sI - (A+B_1K)^{-1}\right)B_2}_{G(s)} w(s)$. We search the state feedback control law such that $\|G(s)\|_2 < \gamma$. The following LMI problem provides the controller's matrices W and P:

$$\begin{cases} AP + PA^T + B_1W + W^TB_1^T + B_2B_2^T < 0, \\ \begin{bmatrix} -Z & CP \\ PC^T & -P \end{bmatrix} < 0,\ trace(Z) < \gamma^2. \end{cases} \quad (4)$$

The minimal attenuation level γ is found and the optimization problem: $\min\limits_{P,Z,W,\rho} \rho\ (\rho = \gamma^2)$ is solved. The controller gain matrix is given by $K = WP^{-1}$.

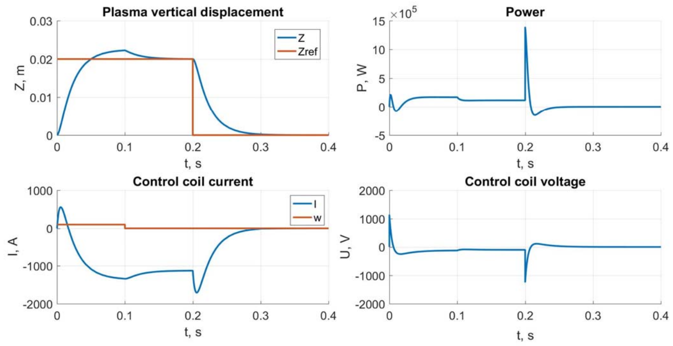


Fig. 5. Control system performance at H_2 stabilization.

During the numerical simulation the reference step signal r = 0.02 m, T_r = 0.2 s, the disturbance w = 100 A, T_w = 0.1 s have been applied to the plant model. Simulation results are shown in Fig. 5: $Z_d = 0.002$ m, $I_{VCC\ \max} = 1708$ A, $P_{\max} = 1.38\times10^5$ W. The corresponding minimal attenuation level is $\gamma = 11.86$.

V. H_2 Design with Desired Pole Region

In this section, we study the H_2 optimization with $D_{\alpha,r,\vartheta}$ pole location problem. The main point is to satisfy the conditions (3) and (4) simultaneously and to seek a feedback control vector $K = WP^{-1}$ such that all the requirements are met.

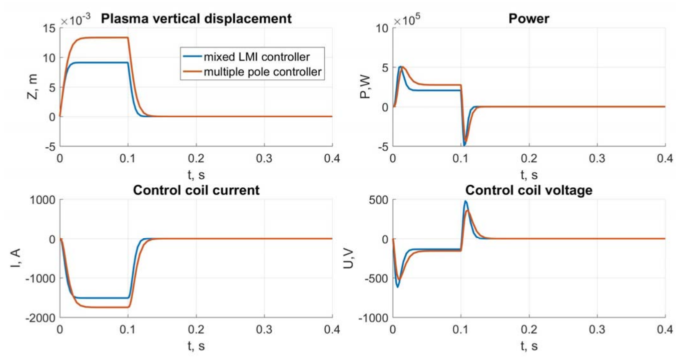


Fig. 6. Control system performance at $D_{\alpha,r,\vartheta}$ stabilization.

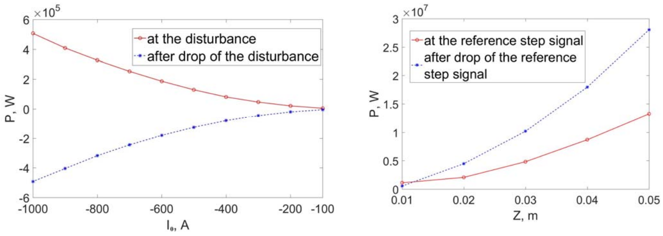
During the numerical simulation the disturbance w = -1000 A, T_w = 0.1 s has been applied to the plant model. Fig. 6 shows the results of comparison of the designed control system and a modal system with three-time multiple pole [3] based on the control power criterion. Firstly, we solved the H_2 design with desired pole regions problem and found the optimal controller

K. After that, we tuned the modal system with three-time multiple pole by changing the pole so that both systems had the same power in peaks and then compared the systems. One can see that the system with the LMI controller better rejects the disturbance during the plasma discharge; the advantage in *Z* displacement is about 30% with the equal control power peaks.

VI. Dependence of Control Power Maximum on Reference and External Disturbance

It is important for an actuator design to estimate maximums of control power, current, and voltage at external disturbance $w(t)$ and reference $r(t)$ actions. The external disturbance $w(t)$ in the control system in Fig. 1 and in Equation 1 reflects the disturbance of the minor disruption type in tokamaks. Such type of the disturbance in plasma physics [1] is caused by drops of the relative plasma pressure β_p and the internal inductance l_i. The control power $P(t)$ peaks as the function of drops of $w(t)$ are shown in Fig. 7a. The system spends about 0.5 MW control power in peak to reject the additive disturbance of 1 kA.

One has to know: how big power is needed to move plasma from one position to another one. Therefore, in Fig. 7b one can see the dependence of the control power peaks on reference position steps. It is clear that to move plasma in 2 cm needs about 2 MW control power in peak and 5 MW to move it back when the reference step changes to zero.



a b

Fig. 7. Dependences of the control power peaks on a) external disturbance drops and b) reference steps.

VII. Acceptable Disturbance Estimation

Optimal rejection of the effects of nonrandom exogenous disturbances on the behavior of linear systems is a classical problem, which has numerous approaches to solution [5, 8, and 9]. Invariant (bounding) ellipsoids characterize uncertainty in the system state (output) caused by the presence of exogenous disturbances. LMIs are considered to be the most appropriate technical tool in the implementation of this approach.

Remain the essentials of the invariant ellipsoids approach [10] and consider a continuous time system of the form

$$\dot{x} = Ax + B_2 w, \quad z = Cx, \quad x(0) = x_0, \tag{5}$$

where $A \in \mathbb{R}^{n\times n}$, $B_2 \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$ are known fixed matrices, $x(t) \in \mathbb{R}^n$ is the state vector, $z(t) \in \mathbb{R}^l$ is the system output, and $w(t) \in \mathbb{R}^m$ is the exogenous disturbance, which is bounded at every time instant in the Euclidean norm:

$$\|w(t)\| \le 1 \quad \forall t \ge 0 . \tag{6}$$

Assume that system (5) is Hurwitz stable, the pair (A, B_2) is controllable, and C is a full row rank matrix.

From now onward, $\|\cdot\|$ denotes the spectral matrix norm, and the matrix inequalities are understood in the sense of sign-definiteness.

Definition 1 [5, 10]. *The ellipsoid*

$$E_x = \left\{ x \in \mathbb{R}^n : x^T P^{-1} x \le 1 \right\}, \quad P > 0, \tag{7}$$

is said to be invariant for system (5) and (6), if the condition $x(0) \in E_x$ *implies* $x(t) \in E_x$ *for all* $t \ge 0$.

The following result is valid.

Theorem 1 [5, 8]. *Ellipsoid (7) is invariant for the system (5), (6) with* $x(0) = 0$ *if and only if its matrix P satisfies the following linear matrix inequalities:*

$$AP + PA^T + \alpha P + \frac{1}{\alpha} B_2 B_2^T \le 0, \quad P > 0, \tag{8}$$

for some $\alpha > 0$.

This formulation easily extends to the case of nonzero initial conditions $x(0) = x_0$: we simply require $x(0) \in E_x$ which, by the Schur lemma [11] is equivalent to the LMI

$$\begin{pmatrix} 1 & x_0^T \\ x_0 & P \end{pmatrix} \ge 0$$

which are to be appended to the basic LMI constraints (8).

Of the most interest in applications is usually an estimate of possible values of the system output, rather than its state. Having at hand an invariant ellipsoid (7) for the state, it is immediate to see that the corresponding output vector z belongs to the bounding ellipsoid defined as

$$E_z = \left\{ z \in \mathbb{R}^l : z^T \left(CPC^T \right)^{-1} z \le 1 \right\}.$$

We now turn to the design problem and consider the system

$$\dot{x} = Ax + B_1 u + B_2 w, \quad z = Cx \quad x(0) = x_0, \tag{9}$$

where $u(t) \in \mathbb{R}^p$ is the control input, the matrix $B_1 \in \mathbb{R}^{n\times p}$ is fixed and known, the matrix pair (A, B_1) is controllable, and the rest of the variables and matrix coefficients have the same meaning as above. It is natural to require the following constraint on the control input:

$$\|u(t)\| \le \bar{u} \quad \forall t \ge 0 . \tag{10}$$

Our goal is to estimate the maximal range

$$\|w\| \le \bar{w}$$

of the exogenous disturbances w for which we guarantee that the system output z remains in the ball

$$\|z\| \le \bar{z}$$

with the bounded control input (10) in the form of linear static state feedback $u = Kx$.

This problem can be treated in the framework of invariant ellipsoids approach and the technique of linear matrix inequalities. The proposed approach leads to the following result.

Theorem 2. *Let $\hat{w}$ be the solution of the optimization problem*

$$\max \overline{w}$$

subject to the constraints

$$\begin{pmatrix} AP + PA^T + \alpha P + B_1 Y + Y^T B_1^T & \overline{w} B_2 \\ \overline{w} B_2^T & -\alpha I \end{pmatrix} \leq 0,$$

$$\begin{pmatrix} P & Y^T \\ Y & \overline{u}^2 I \end{pmatrix} \geq 0, \quad \left\| CPC^T \right\| \leq \overline{z}^2,$$

where the maximization is performed in the matrix variables $P = P^T \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{p\times n}$, *the scalar variable* $\overline{w}$ *and the scalar parameter* α.

Then the output z *of the system (9), (10) remains in the ball* $\|z\| \leq \overline{z}$ *for all time instants, and for all unknown-but-bounded disturbances such that* $\|w\| \leq \hat{w}$.

Note that for any fixed α the problem obtained is nothing but the minimization of the linear function under constraints represented by linear matrix inequalities, i.e., it is an SDP, which is a convex optimization problem.

For the plant (1) with

$$\overline{z} = 0.02 \text{ m}, \quad \overline{u} = 1 \text{ V},$$

Theorem 2 leads to

$$\hat{w} = 1.5461 \cdot 10^3 \text{ A}.$$

Therefore, we can guarantee that the output of the system (1) for all admissible $|w| \leq 1.5461 \cdot 10^3$ A remains within stripe $|z| \leq 0.02$ m (under control input $\|u\| \leq 1$).

Conclusion

The simulations and LMI solutions were performed in MATLAB/Simulink environment.

The next stages of the work are supposed to design and simulate *robust* control systems for the plasma model (1) by LMIs in the MATLAB environment in the presence of plasma model parameter uncertainties as well as LMI control systems in *discrete time* with the usage of digital state controllers.

The LMI approach and the experience obtained in the paper given may be applied to any vertically elongated tokamak for plasma unstable vertical position stabilization where the actuator connected to the horizontal field coil may be approximated by linear dynamical or statical model. The example of such tokamak may be the ASDEX Upgrade machine (Germany) at which the vertical plasma position is stabilized by the multiphase thyristor rectifier that is approximated by the inertial stable unit of the first order [12, 3]. The same approach may be valid for ITER (France) [13]. Another example is the spherical Globus-M tokamak (Russia) [14] at which the actuator for plasma vertical stabilization is a current inverter operating in a self-oscillation mode [15]. This special regime is organized in line with the possibility to approximate the current inverter by only a static gain.

The application of the LMI approach may be done for another plasma control problems in tokamaks for instance for design and analysis of plasma current and shape control systems. This concerns a set of modern tokamaks with vertically elongated plasmas such as USDEX Upgrade, ITER, Globus-M, DIII-D (USA), JET (GB), TCV (Switzerland), EAST (China), KSTAR (South Korea), and others [16].

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