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Gas-dynamic Discontinuity Conception

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Abstract: The aim of the study-to demonstrate the properties of the gas-dynamic discontinuity as a singularity the geometry of the Euler equations give on this basis, the definition of the intensity discontinuity. We have considered the gas-dynamic discontinuity conception. We demonstrated geometrical content of gas dynamics equation. The shock-wave process concept (as a transfer function of gas-dynamic variables space reorganization) was introduced. The basic types of gas-dynamic discontinuities: shock waves, compression shocks, centered depression and compression waves, contact discontinuities were considered. The discontinuity intensity concept was introduced. The basic formulae and discontinuity intensity calculation results were given.

Keywords: Compression wave, shock wave, shock-wave structure, shock-wave process

INTRODUCTION

The object of study-normal and contact the gas-dynamic discontinuity, as well as its intensity and the relationship between the parameters on both sides of gasdynamic discontinuities. Geometrical interpretation of the gas-dynamic characteristics of the space discontinuity as gas-dynamic variables. Shock-Wave Process (SWP) are gas-dynamic system reorganization processes which parameters f into a system with parameters $\hat{f}$:

$$f \to f_1 \tag{1}$$

Here, f and f_1 are sets of gas-dynamic variables before and after SWP. These sets include cinematic, thermodynamic and thermal variables which characterize the gas flow parameters:

- Cinematic (u -speed, w-acceleration)
- Thermodynamic (p-pressure, ρ-density, T-temperature)
- f_0-relevant deceleration parameters
- And entropy change $\Delta S = C_v \ln\vartheta/\vartheta$, where $\vartheta = p/p^\gamma$–Laplace-Poisson invariant (Loitsyansky, 1978) and $h$and h_0 -enthalpy, as well as thermal and physical parameters (thermal capacity c_p and c_v, $\gamma = c_p/c_v$- adiabatic index, viscosity index etc.), which can change during the SWP

SWP is understood as isentropic or shock waves, which arise in the outgoing streamline or are taken into it from outside.

Isentropic waves cover sound (acoustic fields) fields and Riemann waves ($\overline{R}$) or Prandtl-Meyer waves ($\overline{\omega}$), which lead to the stream line compression ($\overline{R_c}, \overline{\omega_c}$) or depression ($\overline{R_r}, \overline{\omega_r}$).

Shock waves ($\overline{D}$) are Gas-Dynamic Discontinuities (GDD), they can occur inside of the outgoing streamline with known parameters f before SWP, or they can be captured in the streamline from outside (Adrianov *et al*., 1995). Difference of waves and discontinuities is in the fact that waves have the width which can be measured from the front edge to the end edge. Gas-dynamic discontinuities are some kind of idealization of the area with parameters f bumping, replacing it by surfaces where gas-dynamic variables unevenly change, that is, GDD is first kind discontinuity. Shock wave real width, as Prandtl showed (Deitch, 1974), comparable with the width (λ) of molecule free path and can be big enough in diluted gas. In solid medium it is insignificant and model of mathematical discontinuity with zero length is adequate. A criterion of gas rarity is Knudsen number $Kn = \lambda/L$, where λ is average length of free path of molecules in gas, L is characteristic parameter of the streamline (for example, streamline body length, pipeline diameter, free jet diameter). From all has been said it follows that the important thermo-dynamic difference of simple waves and discontinuities is behavior of entropy in streamlines going through these discontinuities.

Let us mention *tangential*τ and *contact* ($\overline{K}$) *discontinuities* (entropic waves). Throughτ and ($\overline{K}$) gas cannot overflow, these discontinuities separate moving gas flows with different thermo-dynamic variables,

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excluding static pressure. So, tangential and contact discontinuities are not SWP. In tangential discontinuity, velocity vector is collinear to the discontinuity`s plane.

MATERIALS AND METHODS

Gas-dynamic discontinuity image as features of reflection of projection of gas-dynamic parameters variety: Generalization of the surface concept is variety. The variety is random set of points in the form of aggregation of finite number of Euclidean space domains with the given local coordinates. According to the modern representation, gas-dynamic variables form multidimensional hyperspace and the Euler equations describing the perfect gas streamline specify hypersurface (Arnold, 1976) which curvature is specified by gas-dynamic unevenness N_i (unisobarity N_1, curvature of streamline N_2 and vorticity N_3):

$$N_1 = \frac{\partial \ln P}{\partial s}, N_2 = \frac{\partial \vartheta}{\partial s}, N_3 = \varsigma = \frac{\partial \ln P_0}{\partial n}$$

In axisymmetric case, the Euler equations written with the help of unevenness look in the natural coordinate system connected with the streamlines as follows:

$$\frac{M^2-1}{\gamma M^2} N_1 + \frac{\partial \vartheta}{\partial n} + \frac{\sin \vartheta}{y} = 0,$$
$$\gamma M^2 \frac{\partial \ln V}{\partial s} = -N_1,$$
$$M^2 N_2 = -\frac{\partial \ln P}{\partial n}$$

In these equations:
n = The normal length/streamlines
s = Arc length along streamline
P = Pressure
ϑ = Velocity vector angle
P_0 = Total pressure
ζ = Vorticity

The first expression is the continuity equation. The second and third ones are projections of motion equation on the natural coordinate system axis connected with the streamlines. As we know, supersonic flows can contain domains where parameters change unevenly. Within the perfect gas model, in such cases they say about existence of Gas-Dynamic Discontinuities (GDD).

Let us consider, for simplicity, the one-dimensional equation of perfect gas motion (Euler equation):

$$\frac{du}{dt} + u\frac{\partial u}{\partial x} = 0$$

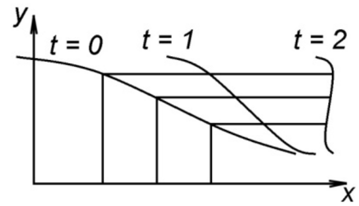

Fig. 1: Euler equation solution with the help of characteristics

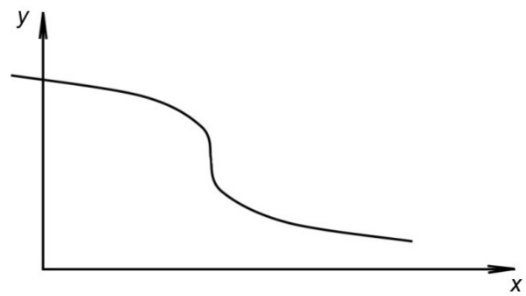

Fig. 2: Integral surface ceases to be function graph y(x)

This equation describes the velocity field of particles freely moving along the straight line. The particle free motion law is of the form $x= \varphi(t)= x_0+ut$, where u is particle velocity. Function φ meets the Newton equation. By definition $d\varphi/dt = u\ (t,\ \varphi)$. Differentiating the last ratio with respect to t we result the equation:

$$\frac{\partial \varphi}{\partial t^2} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

In such a way, the motion description by the Euler equation for the field is equivalent to description by the Newton equation for particles. We know that partial quasi-linear equations can be solved with the help of characteristic formation. Every variety meets its own characteristic field. The characteristics are phase curves of the characteristic field. The equation of the Euler equation characteristics is equivalent to the Newton equation (Arnold, 1990).

In such a way, the wave propagation problem can be solved by construction of characteristics along which material particles move. Figure 1 show how the Euler equation can be solved with the help of characteristics.

On surface y-x initial function $y = u_0(x)_{t\ =\ 0}$ is specified. Characteristic equations $t' = 1$, $y' = 0$, $x' = y$. At moments $t = 1$, $t = 2$ etc., the solution is constructed by means of transport along the characteristics at the initial moment of time. The integral surface in different ways is projected on plane *x-t* (Fig. 2). Reflection *y(x)* ceases to be the function graph, i.e., there are values x which meet a few values y. The curve of the projecting critical a value (tangent to the surface is vertical) has a return point (Fig. 3).

Violation of the solution uniqueness can be interpreted as free passage of particle flows through

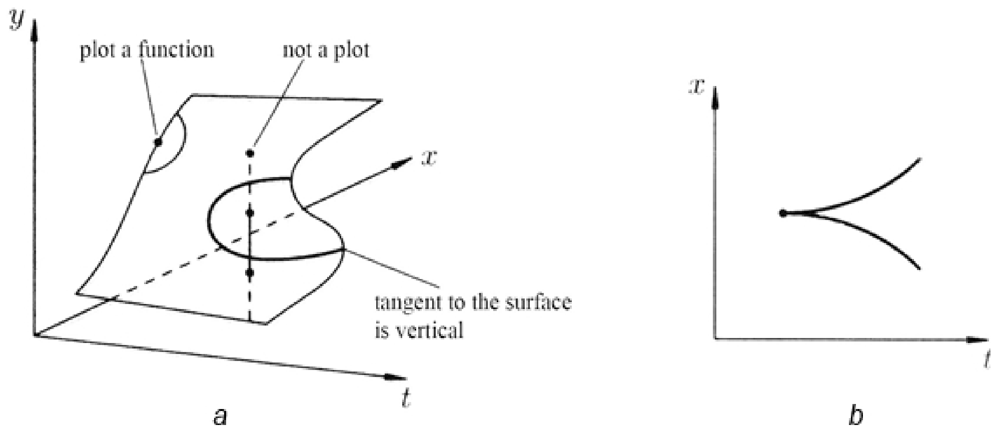

Fig. 3: GDD-singularity of reflection of parameter variety projection

each other. On the other hand, for the particles large density, you cannot neglect their interaction.

In this case, the Euler equation should be replaced by the Burgers equation which takes into consideration gas particles interaction inside the shock wave:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \varepsilon\frac{\partial^2 y}{\partial x^2}$$

For small ε, it brings closer the Euler equation in the domains of the parameters smooth change. On the shock wave left and right, the streamline is described with the Euler equations, inside the shock wave (gas-dynamic discontinuity) -with an equation similar to the heat conduction equation.

So, the shock wave or Gas-Dynamic Discontinuity (GDD) is a reflection singularity of gas-dynamic parameters variety projection (Fig. 3). The GDD interaction forms Shock Wave Structures (SWS).

Parameter connection at discontinuities and waves: The SWP intensity is characterized by the ratio of static pressure $J = p_1/p$ after (p_1) and before (p) SWP. The values J_c>1characterize the streamline compression in $\overline{R_c}$, $\overline{\omega_c}$ orin ($\overline{D}$) –waves and values J_r<1–its expansion (depression). A particular case of $\overline{D}$–wavesarestationary waves ($\overline{D_0}$) in supersonic outgoing streamline (Mach number $M = v/a>1$), when the wave velocity is $D = 0$. Such stationary waves are often called compression shocks ($\overline{\sigma}$).

Sometimes, stationary discontinuities ($\overline{D_0}$) and compression shocks ($\overline{\sigma}$) are considered shock wave synonyms. It is incorrect, as shock waves go in the medium and compression shocks are at rest in the supersonic flow. In $\overline{D}$-waves, the flow total heat σ content h_0 changes and for the shock passage the value remains the same ($H_0 = h_{01}/h_0 = 1$) (Uskov, 2000). The same note is true for the Riemann progressing waves and stationary Prandtl-Meyer waves in supersonic flows. The density ration $E = \rho/\rho$ is connected with the wave intensity by the Laplace-Poisson isentropic (invariant) ($\vartheta = const$), that is:

$$\vartheta/\vartheta = JE^{\gamma} = 1 \quad (2)$$

Or the Rankine-Hogoniot shock adiabat:

$$E = \frac{1+\varepsilon J}{J+\varepsilon} \quad (3)$$

where, $\varepsilon = \lim_{J\to\infty} E = \frac{\gamma-1}{\gamma+1}$. From (1, 2) you can see that $\Delta S = 0$ in isentropic waves and $\Delta S>0$ in shock waves, where formula (3). The waves through which gas flows are called normal waves, as compared with tangential discontinuities τ and K, which are not crossed with the streamlines.

Thermodynamic variables are connected with the help of state equations, for example of the Clapeyron-Mendeleev perfect gas state:

$$\frac{p}{\rho T} = const = \frac{8340}{\mu} \quad (4)$$

where, μ is the gas molar mass.

The deceleration parameters change in SWP characterize the relation of total pressures $J_0 = p_0/p_0$ and total heat content $H_0 = h_0/h_0$ (Kochin *et al*., 1963):

$$J_0 = \left(\frac{H_0^{\gamma}}{J E^{\gamma}}\right)^{\frac{1}{\gamma-1}} \quad (5)$$

From (5) and the energy change equation in the non viscous non conducting gas:

$$\rho\frac{dh_0}{dt} = \frac{\partial p}{\partial t} \quad (6)$$

Which takes in to consideration кото рое the total enthalpy change h_0 only because of the pressure force work the two important conclusions follow:

- Instationary flows ($\partial\rho/\partial t$ = 0) along the streamline, the total heat content does not change. It means that in stable flows (H_0 = 1) and formula (5) look as follows:

$$J_0 = (JE^{\gamma})^{\frac{1}{\gamma-1}} \tag{7}$$

Formula (7) describes the total pressure loss factor in the supersonic stable waves (compression shocks ($\bar{\sigma}$) and in the Prandtl-Meyer isentropic waves, where $JE^{\gamma} =$ 1.

- In non-stationary flows, the total enthalpy changes ($H_0 \neq 1$), therefore in the Riemann waves and progressing shock waves changes of J_0 and H_0 are connected with the formula (5), at the same time, the most easy in simple ($\bar{R}$) waves:

$$J_0 = H_0^{\frac{\gamma}{\gamma-1}} \tag{8}$$

RESULTS AND DISCUSSION

Shock intensity: It is easy to introduce the intensity concept for a compression shock. Considering a direct compression wave as an individual shock wave, you can write the following expression for its intensity (Uskov and Mostovykh, 2012):

$$J_m = (1+\varepsilon)M_1^2 - \varepsilon \tag{9}$$

where, M_1-Mach number before the compression shock ($M_1 = u_1/a_1$). Dependence of J_m on Mach number (9) is given in Fig. 4.

Contact discontinuity intensity: We introduce the concept of contact discontinuity intensity $\bar{K}$. Despite of the fact that generally the gas-dynamic parameters at $\bar{K}$undergo discontinuity (excluding static pressure), they are not free. The parameters of the streamlines divided by $\bar{K}$ are connected with equations following the Clapeyron-Mendeleev equation:

$$\frac{\rho_1 R_1 T_1}{\rho_2 R_2 T_2} = 1 \tag{10}$$

Deceleration parameters on both sides $\bar{K}$ are different but interconnected, as well as the entropy values.

There are three options of an arbitrary discontinuity breakup, resulting in the formation of a multidirectional and separated by a contact discontinuity following formations:

- **Two shock waves:** The problem of the collision of counter masses of gas (Fig. 5a). This case arises in a collision of the two gas streams moving in opposite directions
- **Shock and rarefaction waves:** The problem of equalizing pressure (Fig. 5b). Occurs in a contact of two media with different pressure.
- **Two centered rarefaction waves:** The problem of the expansion of the two masses of gas (Fig. 5c). The case where two initially contacting masses of gas moving in opposite directions.

Formation of two outgoing waves as a result of an arbitrary discontinuity breakup is due to the need to perform dynamic compatibility conditions on the contact discontinuity separating them.

We fix intensity $\bar{K}$ through the ration of parameters of streamline deceleration $k = f_{02}/f_{03}$or entropy difference $\Delta s = c_v \ln(\vartheta_1/\vartheta_2)$, where $\vartheta = p/\rho^{\gamma}$ is the Laplace-Poisson invariant. For at $\bar{K}p_2 = p_3, \Delta s = c_v \ln(\rho_3/\rho_2)^{\gamma}$. As the gas flow in the Riemann wave is isentropic, the deceleration parameters in $\overline{R_r}$- wave do not change and intensity $\bar{K}$ is expressed by the total parameters ratio$k = f_{02}/f_{03} = 1$. At the shock wave the deceleration parameters undergo a discontinuity which should characterize the entropy intensity (Arkhipova, 2013):

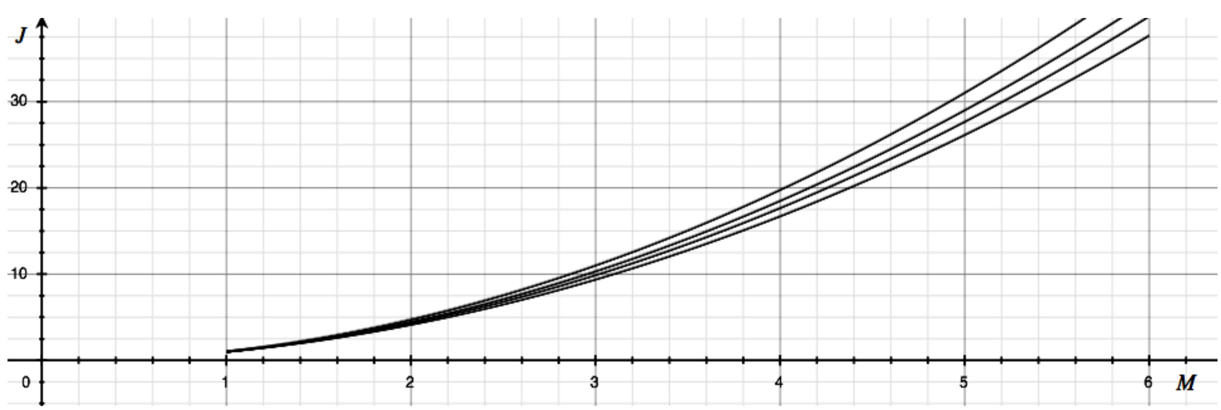

Fig. 4: Dependence of maximal intensity of shock J_m on Mach number at the fixes adiabatic index γ =1.1 (upper curve), 1.25, 1.4, 1.67 (lower curve)

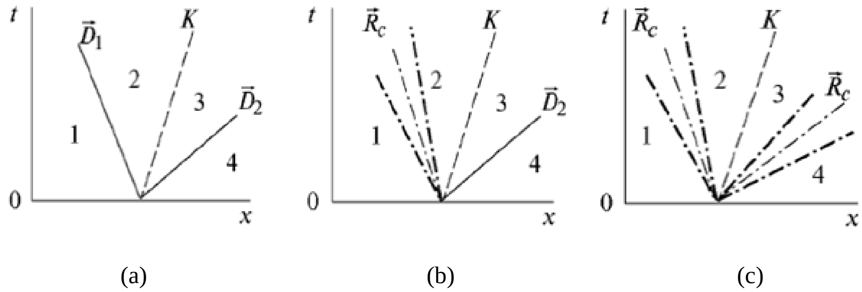

Fig. 5: Types of an arbitrary discontinuity breakup; D-shock waves; R-isentropic expansion wave; K-contact discontinuity

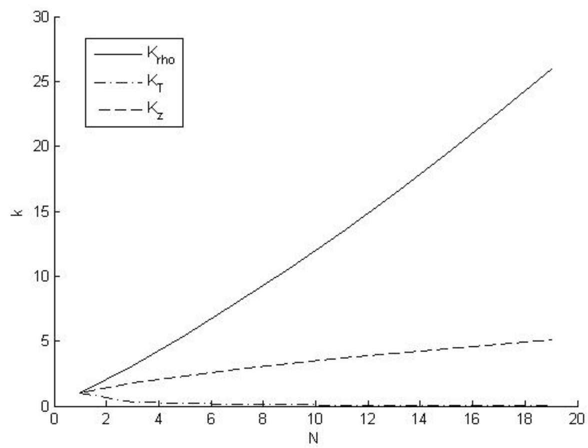

Fig. 6: Diagram of intensity k_ρ(N)

$$\Delta s_D = c_{v_2} \ln\left(J_D E_D^{\gamma}\right) \tag{11}$$

Figure 6 shows dependence of the contact discontinuity separating differently directed shock wave and centered Riemann centered depression wave propagating in the mediums initially having different static pressure p_{01} and p_{04}, $N = p_{01}/p_{04}$.

CONCLUSION

In the generic form, the gas-dynamic discontinuity conception was introduced. At the same time, we have used the V.N. Uskov`s theory of interference of stationary (Arkhipova, 2013) and non-stationary (Kochin *et al*., 1963) gas-dynamic discontinuities. As the normal wave (discontinuities) intensity we used the ratio of pressures before and after the discontinuity. In case of contact or tangential discontinuity, these pressures are equal, therefore the intensity is understood as the total pressures ratio. We showed the connection of non-stationary waves and stationary discontinuities. We introduces the conception of the compression shock (standing shock wave) as an individual case of a moving shock wave. We demonstrated that the basic difference between them is in the character of entropy reorganization. At the stationary discontinuity the total het content does not undergo a discontinuity, but at the shock wave it changes unevenly. The direct compression shock and contact discontinuity calculation results are given.

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