

On classification of flow regimes in a channel with sudden expansion

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Supersonic flows of gas in the vicinity of the bottom region known as flows with sudden expansion have been considered. On the basis of extensive experimental studies, authors have proposed a complete classification of flow regimes: stationary, oscillating, and transient. Hysteresis of the regimes change at total gas pressure increasing and decreasing in front of the nozzle has been found. Typical shock-wave configurations emerging at the jet flowing in a channel at different modes have been determined. The type of shock-wave structure and the nature of interaction of the mixing layer of a jet with the wall or reverse flow flowing into the channel from ambient medium determine the appropriate mode. Combination of physical and numerical experiment with bottom pressure calculation according to the developed semi-empirical model have revealed new flow regimes that were not studied earlier.

Key words: bottom pressure, bottom region, flow regimes, non-stationary processes.

Introduction

Topicality of investigation of supersonic jet flow in a coaxial cylindrical channel with sudden expansion (SEF) is caused by the fact that the regimes found in this simple case recur in more complicated technical devices with separation flows. One of such applications is the Eiffel chamber in a wind tunnel, where rarefaction occurs due to ejection properties of the jet. It allows imitating the conditions of the rocket flight in the upper layers of the atmosphere at the lower pressure. There are other technical applications of SEF as well. For instance, these are nozzle blocks, ejectors, supersonic combustion chambers, afterburners of air-feed jet engines, etc. All of them have the same technical problem — bottom pressure.

The process of supersonic jet flow in a channel with sudden expansion is accompanied with a wide range of gas-dynamic phenomena. Among them the most interesting are non-stationary processes including self-oscillating flow regimes realized in strictly defined ranges determinant for these flows. The results of investigation of the peculiarities of such flows and possibility of their development or avoidance are used at designing mufflers or, vice versa, acoustic generators and devices of pulsating blowing in metallurgy. Thus combination of a wide range of various gas-dynamic processes and their vast practical applications makes the considered problem topical both for fundamental and exploratory development.

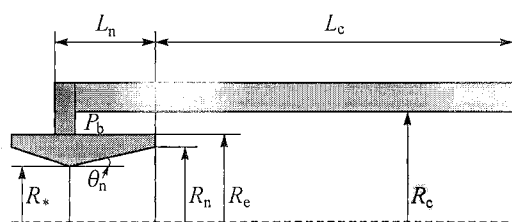


Fig. 1. The geometry of the channel with sudden expansion.

Radii of the nozzle section: R_* — the nozzle throat radius, R_n — the inner outlet section radius, R_e — the outer exit section radius; θ_n — the nozzle section half-angle; R_c — the channel radius, L_c — the channel length, L_n — the length of the nozzle extension into the channel; P_b — bottom pressure.

From the whole diversity, the authors of this work have selected the round supersonic jet flowing to axisymmetrical channel. It contains practically all elements of more complicated flows and may serve as their model. At preset configuration of the nozzle and channel (Fig. 1), the flow is fully determined by the multitudes of gas-dynamic variables: stagnation parameters of the working gas flowing from the nozzle F_0 ; parameters of gas filling a channel before the start of the jet flowing F_i . Multitudes F make up thermodynamic and thermophysical variables determinant for the state of the working and ambient gas: pressure p , temperature T , adiabatic exponent $\gamma = C_p/C_v$, etc., affecting the bottom pressure P_b in the vicinity of the outlet section of Laval nozzle.

The stated problem is to find all flow regimes and the order of their change depending on stagnation parameters F_0 under the given conditions F_i in ambient medium. It is necessary to consider that at some combinations of the channel and nozzle configuration, some regimes may be absent.

Physical phenomena that accompany the separation flows in the vicinity of the back ledge, the bottom section, and various models of the bottom areas are considered in detail in the monograph [1]. The proper notion of the bottom pressure and approaches to its determination are formulated in the fundamentals works [2, 3].

The first experimental works on the problem of the bottom pressure determination appeared in the middle of 50-ies. The works [4, 5] considered influence of the channel length on P_b and starting conditions in the channel, and works [6, 7] considered phases of flow formation in the channel and determined the typical graph of P_b dependence on P_0 ; here dependence of relative bottom pressure $\bar{P}_b = P_b/P_i$, on the nozzle and channel geometry and dimensionless parameters of the working gas stagnation in the receiver were studied as well. Since in the majority of investigations, the experiments were performed with the use of inertial pressure sensors, the nonstationary regimes were not found. Authors of the work [8] considered the flow in a rectangular channel with the bottom area taking into account the nonstationary processes. As a result, the ideas on specific graph $P_b(P_0)$ have been modernized (Fig. 2). In the work [9], there is data on the flow picture in the area of jet mixing layer flowing on the channel wall.

Recently attention to SEF has grown due to the development of jet engines for hypersonic aircraft as well as engine nozzles intended for a wide range of the flight modes [10, 11].

The authors of the present work carried out systematic experimental investigation of the separation flow in a channel with sudden expansion [12]. The relative area of the channel remained constant in the experiments: $F_c/F_* = (R_c/R_*)^2 = 64.3$. This condition allowed setting nozzles with Mach number on the section up to $M_n = 7$. In investigations

the nozzles with $M_n = 1-6$ and half-angles on the nozzles sections $\theta_n = 0, 8, 15, 30$, and 40° were used.

Physical picture of supersonic jet flowing in a channel with sudden expansion was visually investigated with application of a shadow column instrument IAB-451 and setups with plane transparent walls. Experiments were performed in the mode of controlled total pressure P_0 increase in the receiver with the rate of about 5–7 atm/s. At such rate of P_0 increase in the channel as a rule stationary fluctuations of bottom pressure developed. Typical duration of blowing was 25–40 s. Results were registered either on paper with oscillograph or with magnetograph on magnetic tape. Such experimental methodology based on the application of inertialess pressure sensors allowed determining frequency and amplitude of oscillating processes and in a number of cases the form of an oscillating cycle as well.

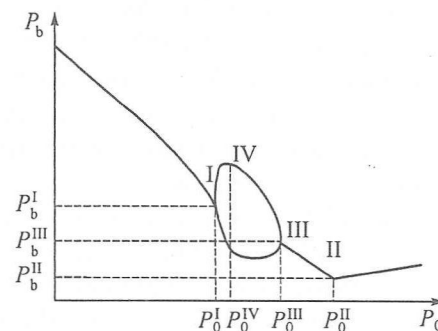
Purely experimental studies [12] are insufficient for the development of comprehensive classification of the flow modes in a channel with sudden expansion. High rate of total pressure increase in front of the nozzle has resulted in the neglect of some transient processes in the experiments [12]. Semi-empirical model of the flow in a channel developed by the authors [13] further allowed predicting a number of nonstationary regimes that were not found during the experiments. Their existence was proved by numerical calculations and new experiments. Below there are results obtained on the basis of the analysis of experimental data as well as the results of numerical calculations and calculations with the use of semi-empirical methodology.

Since semi-empirical method contained rather approximate model of main turbulent part of the jet under the conditions of positive longitudinal pressure gradient, then for its verification and specification of the qualitative picture of the flow, numerical calculations were performed in axisymmetrical statement with the use of Reynolds-averaged Navier — Stokes equations (RANS). The finite volume method of the second order in space and time was applied. In stationary modes, implicit difference scheme was used.

The dimensions of the calculation area were selected equal to physical dimensions of the full-scale experimental unit. Prior to calculations, the problem solution convergence was studied depending on the dimensions and thickness of the difference grid. Series of experiments performed for the nozzles with the numbers $M_n = 2-3$ and half-angles of the nozzle 8° and 15° , has shown that the order of the regime change is reproduced in the same way regardless of the type and thickness of the difference grid. Only P_0 values, at which one regime is switched to another, change. Since the order of the regimes change was the subject of studies for calculations the authors used standard structured difference grid with 0.25 mm cell in

Fig. 2. Specific pressures on typical plot of $P_b(P_0)$.

Points: I — beginning, III — end of oscillations, II — minimal bottom pressure, IV — maximal range of bottom pressure.



critical section of the nozzle. The boundary layers on the nozzle walls were simulated with the use of the method of standard wall functions using the logarithmic velocity profile near the wall.

Testing of different turbulence models has shown that the best coincidence with experimental results in the part of P_b value as well as of the moments of regimes switching, start and completion of low-frequency oscillations is given by the version of $k-\varepsilon$ turbulence model known as realizable $k-\varepsilon$ model [14]. The advantage of the realizable $k-\varepsilon$ model [15] is its more accurate prediction of the jet dissipation rate and provision of the best forecasts of separation and recirculation flows as well as the ones with the developed secondary flows exposed to strong pressure gradients.

The term "realizable" means that the model meets mathematical restrictions for normal stresses that agree with physics of turbulent flows, i. e., the negative values of eddy viscosity at the calculations of high-gradient flows are excluded. In all further calculations, according to recommendations of the monograph [15], the realizable $k-\varepsilon$ turbulence model was used. At transient process simulation with the use of numerical methods, it was important to accurately reproduce the degree of jet expansion and the pressure gradient along the channel wall. The main problem was to determine the moment of the beginning of the jet mixing layer interaction with the channel edge, and of the jet flowing on the channel wall with its starting and main parts, as well as calculation of average flow parameters in reverse flow on the annular gap between the channel wall and the jet.

Thus, the role of theoretical investigations was to determine the regimes that were not found in the earlier experiments.

1. Analysis of flow modes on specific dependence graph $P_b(P_0)$

With the increase of total pressure P_0 in the receiver in front of the nozzle, there is gradual reduction of the bottom pressure P_b in the channel from the value equal to the outer (atmospheric) pressure P_i , to the minimal value determined by the structural peculiarities of the device; after that P_b starts growing again (see Fig. 2). Thus, there are two branches of the graph: the descending one when with P_0 growth the bottom pressure drops and the ascending one when the bottom pressure grows. The specific loop in Fig. 2 coincides with the area of the oscillating regime which starts at the value of total pressure P_0^I and ends up at P_0^{III} . At point P_0^{IV} , the oscillations of the bottom pressure of maximal amplitude are realized.

Figure 3 shows the photographs of typical shock-wave structures of the jets flowing in the channel with sudden expansion as well as the respective flow schemes. Here q_p is the dimensionless gas flow rate ejected from the bottom area related to the gas flow rate through the nozzle, q_v is the gas flow rate supplied to the bottom area from the ambient medium or from the area of mixing layer flowing on the channel wall. Since shadow photographs were taken on the units with plane transparent walls distortion appeared in the flow picture. For its adjustment in the stationary modes other methods of flow visualization were used as well, for instance, with oil coating of internal channel walls. Among the stationary modes we reckon: the mode of open bottom area (OBA) (the respective SWS is given in Fig. 3a), non-self-similar mode (NSM) with closed bottom area (CBA) (see Fig. 3b), and self-similar mode (SM) (Figs. 3d and 3e).

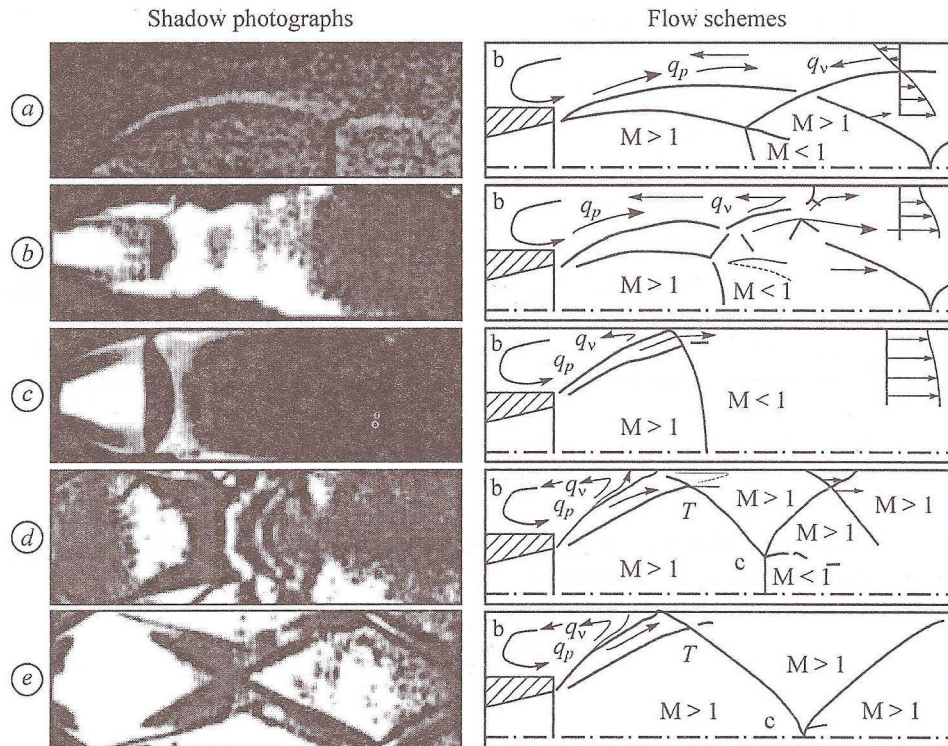


Fig. 3. Typical shock-wave structures in a channel with a SEF. $L_c > L_{co}$. $M_a = 2$. Modes: *a* — Mode OBA, $P_0 = 30$ atm, $P_0 < P_0^I$; *b* — Mode NSM CBA, $P_0 = 40$ atm, $P_0^{III} > P_0 > P_0^I$; *c* — minimal P_b , $P_0 = 60$ atm, $P_0 = P_0^II$; *d* — SM, $P_0 = 70$ atm, $P_0 > P_0^II$; *e* — SM, $P_0 = 90$ atm, $P_0 \gg P_0^II$.

In Fig. 4, dependences $P_b(P_0)$ are schematically presented at different channel lengths. On the graphs there are all stationary flow modes, oscillating and transient process (TPi) experimentally found by now. It is seen that Fig. 4 is noticeably different from the classical graph $P_b(P_0)$ in Fig. 2.

The flow modes were determined with the use of the developed semi-empirical methodology of bottom pressure calculation (see [13]); then results were compared with available experimental data. For all modes and transient processes determined in calculations, fair experimental proof has been found.

At small values of P_0 the flow of working gas with gas-dynamic discontinuities flowing from the nozzle section and reverse flow from the ambient medium to the bottom area through the annular gap between the wall and the jet boundary are formed (Fig. 3*a*). Since P_b in this case depends on F_i , the OBA mode is non-self-similar. The graphs separate depending on the channel length. The shorter the channel is the higher the part of the graph complying with OBA lies. It is especially noticeable in short (1 in Fig. 4*a*) and medium channels (2 in Fig. 4*a*). Short, medium, and long channels differ in the presence or absence of one or other modes. All flow modes are realized in series only during the experiments with long channels. In the medium and short channels, part of the modes may be absent.

The notion of optimal channel length L_{co} , i. e., the one at which there is an absolute minimum of P_b may be introduced. This channel length complies with the lowest total pressure P_0 , at which the mode of supersonic gas flow is formed over the entire channel section.

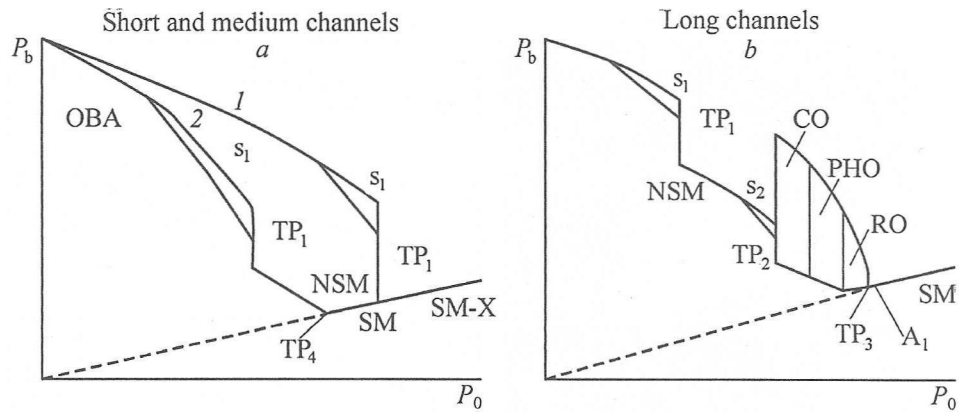


Fig. 4. Modes of channel flow with sudden expansion on a typical plot of $P_b(P_0)$.
a — short channels (1), medium channels (2); b — long channels.

On the results of experimental data processing the empirical formula determining the optimal channel length was obtained:

$$L_{co} = \frac{3.15}{2(0.7 + \text{tg } \theta_n)} M_n D_c,$$

where D_c is the diameter of channel.

With P_0 increase the flow picture does not change qualitatively until the reverse flow over the annular gap becomes transonic in the section complying with maximal diameter of the first jet barrel. Oscillations s_1 accompanied by chaotic change of the parameter amplitude in the bottom area with large frequency arise. This type of oscillations was observed in absolutely all performed experiments.

Analysis of oscillations s_1 (Fig. 5) has shown [16, 17, 18] that they can not be considered the random ones as it was supposed earlier; therefore, this mode is called the mode of stochastic oscillations.

The OBA modes always end up with nonstationary transient process TP_1 . In long and medium channels, TP_1 is followed by [19] NSM CBA at which the jet flows on the channel walls with the turbulent part (Fig. 3b). Since the graph $P_b(P_0)$ consists of the descending and ascending branches there is always a point complying with minimal

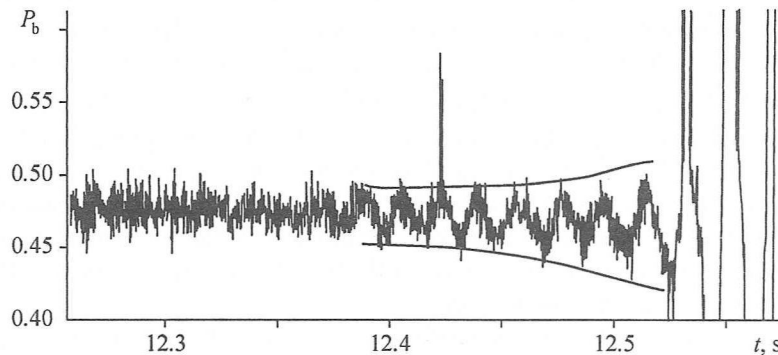


Fig. 5. Stochastic fluctuations in s_1 and transient process before the regime of composite fluctuations.

$M_n = 3$, $P_0 = 40$ atm, $P_0 \approx P_0^I$, $L_c = 6.3$, $L_c > L_{co}$, long channel.

bottom pressure (point II in Fig. 2 and Fig. 3c). As numerous experiments show [12], at minimal bottom pressure the entire transverse section of the channel is blocked by the powerful central shock wave after which the flow is subsonic. At further P_0 increase, the growth of $P_b(P_0)$ occurs linearly, the straight line passes through the coordinate origin, and jet off-design $n = P_n/P_b$ (here P_n is pressure at the nozzle exit) remains constant. The flow picture has no dependence on the outer pressure (Fig. 3d). It is said that the self-similar flow mode was set. If SM follows right after the flow mode with open bottom area such channels are called short. In the channels with medium length, the flow switches from non-self-similar mode with closed bottom area to the similar one during the transient process TP₄. This is how short channels differ from the ones with medium length. Further increase in P_0 results in gradual edging of central shock wave "s" (Figs. 3d and 3e) downstream to the channel section from the intersection point T of the suspended shock wave "c" with the reflected one.

2. Nonstationary and transient modes

On the descending branch of the graph $P_b(P_0)$ at jet flowing to long and medium channels (see Fig. 4), there are areas of oscillating modes. All types of oscillations and transient processes are non-self-similar. Their occurrence is accompanied with hysteresis of parameters at the increase and release of P_0 .

The transient processes TP₁₋₄ differ from other nonstationary modes since at constant total pressure P_0 , parameters in the bottom area change during the finite time interval tending to stationary state or limit cycle. Thus, transient processes are responsible for switching from one mode to another, and these modes may be both stationary and oscillating.

Reasons and mechanism of transient processes have been finally determined only with the use of semi-empirical method [13].

During calculations, it was found that TP₁ mode complies with closing of the reverse flow from ambient medium to bottom areas and formation of critical section with Mach number $M = 1$ (see [20, 21]), rather than outcome of the current line with constant mass to the channel edge as it was considered earlier [12]. Figure 6 shows the calculation of the jet in the channel at P_0 agreeing with the beginning of the transient process. Besides, distribution of Mach number is shown.

As initial conditions in the channel the velocity distribution meeting continuity equation as TP₁, well as pressure and temperature complying with the conditions of standard atmosphere were set. As boundary conditions total pressure in front of the nozzle P_0 , as well as atmospheric pressure on the boundary of the calculation area located in the distance equal to $0.25 L_c$ from the channel section were set. Calculations were performed at the step increase of P_0 . Calculations with the steadying method have resulted in stationary solutions.

It is apparent that in reverse flow there is an area with the velocity equal to local sound velocity (the light area adjacent to the channel wall). It should be noted that the flow picture depends on whether P_0 increases (Fig. 6a) or decreases (Fig. 6b), in other words, whether we approach the point on the graph $P_b(P_0)$ (Fig. 4) from the side of OBA mode or from the side of non-self-similar mode with CBA. Thus, there is an area of ambiguity of $P_b(P_0)$ dependence, i. e., there is hysteresis.

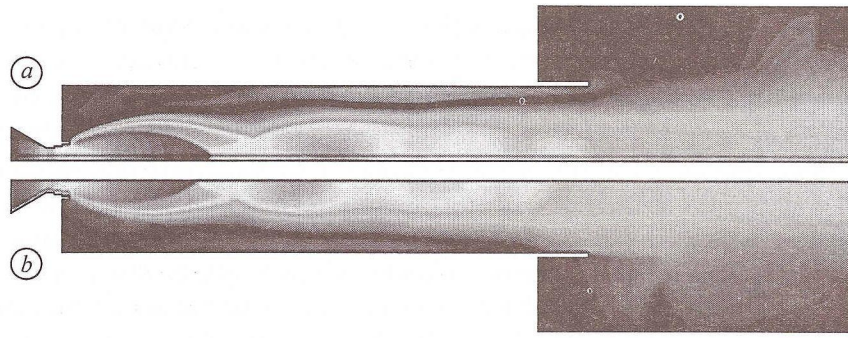


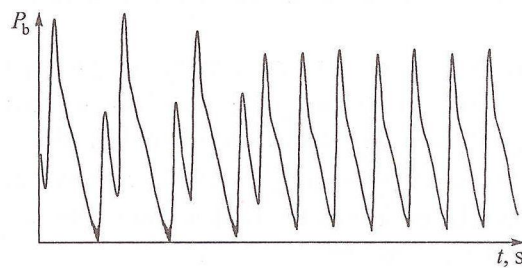
Fig. 6. Transient process between TP_1 OBA \leftrightarrow CBA.
 $M_n = 2$, $L_c = 4.02$, $L_c > L_{co}$. Short channel.

Low-frequency oscillations with large amplitude (oscillation frequency of about 100–300 Hz, oscillation frequency up to 0.6 atm) are always preceded with [21, 22] the oscillating mode s_2 (Fig. 4) of stochastic high-frequency oscillations (specific frequency of approximately 1–10 kHz and amplitude of 0.05–0.1 atm). The mode s_2 relates to non-self-similar modes with CBA.

Thorough experimental study proved by numerical calculation [18, 20, 22] has shown that the low-frequency oscillations start with Hopf subcritical bifurcation that complies with the transient process TP_2 , and end up [23] with subharmonic cascade (TP_3 process).

Three types of low-frequency oscillations [19], following each other along with P_0 change, have been found. These oscillations differ in the form of oscillating cycle and mechanism of sustaining. At P_0 increase, first composite oscillations arise (CO), when the oscillating cycle consists of two semi-cycles: part of an oscillating cycle occurs at the bottom area opening and another one at the closed one. CO may be absent in the long channels. With P_0 increase CO are always followed by pseudoharmonic oscillations (PHO); on the form of their oscillating cycle they are similar to van der Pol oscillator. Figure 7 shows a moment of transition from composite oscillations to pseudoharmonic ones during the experiment. The image was obtained from the oscillograph screen.

The form of the oscillating cycle of low-frequency oscillations is determined solely by power supply and discharge mechanism of the bottom area. For each value of P_0 and P_b it is possible to calculate configuration of the jet boundary and interaction of the mixing layer with the channel wall, and determine gas rate in the bottom area (whether filling or discharge of the bottom area occurs). Neither the amplitude, nor frequency, nor the form of the oscillating cycle of CO and PHO depend on the turbulence parameters. This experimentally determined fact [21] serves to apply quasi stationary problem statement for low frequency oscillations calculation.



Pseudoharmonic oscillations take place between two utmost positions complying with jet flowing on

Fig. 7. Transition from composite oscillations CO to pseudoharmonic oscillations PHO.

$M_n = 2$, $P_0 = 50$ atm, $P_0^{III} > P_0 > P_0^I$, $L_c = 4.02$,
 $L_c > L_{co}$, short channel.

the wall with the turbulent part and the first barrel. The bottom area is closed. Comparison of the records (kinograms) obtained during numerical calculations with experimental oscillograms allowed concluding that the form of PHO oscillating cycle along with the increase of Mach number at the nozzle exit changes from almost harmonic to saw-tooth. In the work [24] PHO were called shock-pattern oscillation. At last at further growth of P_0 relaxation oscillations with constraints (RO) are set. The oscillating cycle of RO has specific saw-tooth form.

In the channels with the nozzles with $M_n \leq 1.5$, low-frequency oscillations arise in a very narrow range P_0 . Sometimes it is only several atmospheres. At large rate of P_0 increase, the mode of low-frequency oscillations is easy to miss, which occurred in earlier experiments [12]. As a result, the authors came to erroneous conclusion that at $M_n = 1-1.5$, low-frequency oscillations do not arise at all [21].

As a rule, the low-frequency oscillations end up in the vicinity of the graph point complying with the minimal bottom pressure. Will there be a decrease of oscillations amplitude to zero in the course of time if at this moment P_0 is fixed or it will remain constant? In other words, is there a transient process depending solely on time t ? Series of experiments where P_0 changed slowly have shown that such transient process TP_3 does exist (Fig. 8).

Its duration as a rule is five periods T of low-frequency oscillations. In this section, usually there are three pairs of P_b splashes with smaller amplitude of set oscillations. The distance between respective peaks equals $2T$. Then amplitude of oscillations drops for the time T to the value of chaotic pulsations P_b , and the value of each further pulsation is approximately 2.5 times lesser than the previous one.

The transient process TP_4 (Fig. 4a) between non-self-similar mode with the closed bottom area and self-similar mode in the channels with medium length consists in displacement of the point of the jet boundary flowing on the wall from the area of turbulent section to the area of the first barrel [22, 23] at P_0 complying with minimal bottom pressure. The process was first predicted by the authors in 1997 at the development of semi-empirical model [13, 25, 26, 27, 28] of supersonic jet in the channel, later found in experimental oscillograms and reproduced during numerical calculations. In long channels at the numbers $M_n > 2$, when oscillations lengthen to the point of minimal bottom pressure, TP_4 may be absent (Fig. 4b). At $M_n < 1.5$, the range of oscillations ($P_0^I - P_0^{III}$) narrows, and after the completion of oscillations there is the graph section complying with NSM CBA, and mechanism of the transient process TP_4 is the same as in the channels of medium length.

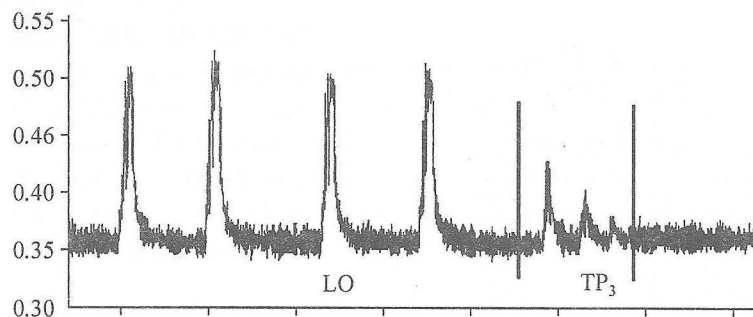


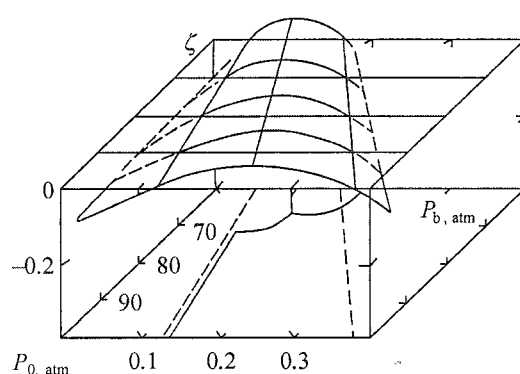
Fig. 8. Transition process TP_3 ,
 $M_n = 3$, $P_0 = 58$ atm, $P_0 \approx P_0^{III}$, $L_c = 6.3$, $L_c > L_{co}$, long channel.

3. Substantiating of modes classification by analyzing jet in a channel as a dynamic system

To characterize the jet in a channel as a dynamic system we introduced the notion of the misbalance of mass consumptions $\zeta = (q_p - q_v)/Q_n$, supplied in the bottom area (q_v) from ambient medium or from the area of jet attachment to the channel wall and ejected from the bottom area with the jet (q_p), related to the rate of the working gas through the nozzle Q_n (Fig. 3). Misbalance ζ is a criterial value characterizing the overall state of gas-dynamic system in the channel. If at the set P_0 we can determine the jet depending on the value of bottom pressure P_b , for instance, with the use of semi-empirical method [25, 26], then with integral methods [27, 28] it is possible to find characteristics of the mixing layer on its boundary. Having calculated interaction of the mixing layer with the channel wall [25], or characteristics of reverse flow [26] in the modes with open bottom area we may calculate q_v , q_p and find misbalance of ζ consumptions. With the abovementioned methods, it was ascertained that at the jet flowing on the channel wall with gas-dynamic or turbulent part the function of misbalance $\zeta(P_b)$ at $P_0 = \text{const}$ in the limits of the permissible values P_b may be non-monotone and have two roots. One of these roots is stable and another one is not (dotted lines on the lower plane $P_0 - P_b$ in Fig. 9).

Stable root agrees with experimental value of the bottom pressure (in Fig. 9 — full line on the bottom surface $P_0 - P_b$). If at the set P_0 misbalance ζ equals zero then the system is in a stationary position, otherwise the bottom pressure changes. If function $\zeta(P_b)$ in the range of permissible values P_b has no roots then at the given P_0 the jet in the channel can not be in the stationary position. Investigation of gas-dynamic system behavior in space $\zeta - P_b$ is analogous to investigation of its dynamic properties on the phase plane with single parameter P_0 . Change of this parameter is accompanied by transformation of shock-wave structure, birth and destruction of limit cycles, and various transient processes TP_i .

The behavior of $P_b(\zeta, t)$ function significantly depends on the parameter P_0 . For its analysis it is possible to use the well developed apparatus of time series [29, 30]. At some critical values of P_0 , the qualitative change of gas-dynamic system state (catastrophe) occurs. These moments in the developed semi-empirical model agree with the transient processes TP_i . Beyond the catastrophe points the system performs chaotic fluctuations in the vicinity of attractor (stationary position or limit cycle).



Analysis of misbalance function behavior $\zeta(P_b)$ along with P_0 change allows introducing full formal classification of the flow modes containing all modes fixed experimentally (Fig. 4). Thus, the analysis performed in section 1 is correct but may be incomplete.

Fig. 9. Dependence of misbalance ζ on P_b and P_0 .

The classification is based on several classification features:

- the type of the bottom area (open or closed);
- the flow self-similarity;
- the character of $P_b(t)$ dependence on time at $P_0 = \text{const}$.

On the type of the bottom area all modes are classified to the ones with open bottom area when the jet flowing from the nozzle does not interact with the channel walls and air from the ambient medium flows in the bottom area, and the modes with the closed bottom area when in this area there is only gas passed through the nozzle, and the reverse flow from ambient medium is absent.

The authors differentiate self-similar and non-self-similar modes. Sometimes self-similar modes with cellular structure of shockwaves (SM-X), where the areas of subsonic flow may be neglected not only near the channel walls but over the entire area of transverse section as well, are distinguished. However, if we consider the system “jet in a channel” from the point of view of its dynamic properties on the phase plane $\zeta - P_b$, then SM-X does not differ from SM. At non-self-similar modes, the bottom pressure depends on P_0 , L_c , and F_i .

On the character of P_b dependence on time at constant P_0 stationary modes, when at the set P_0 the bottom pressure remains unchanged, and nonstationary ones are distinguished. Non-stationary modes include transient processes (TP₁₋₄) and oscillating ones (s₁₋₂, CO, PHO, RO, A₁). The oscillating modes on specific frequency may be divided into the low-frequency (CO, PHO, RO) and high-frequency ones (s₁₋₂, A₁). The names of oscillations are determined by the form of the oscillating cycle and physical mechanism of their sustaining. Theoretical analysis has shown that self-similar mode probably shall be preceded by the mode of high-frequency oscillations A₁, excited by interaction of acoustic waves propagating from the area of jet flowing on the channel wall with the jet boundary and bottom volume. Presence of such mode has not been proved experimentally yet.

Formal classification on the basis of transformation of $\zeta - P_b$ function allowed predicting presence of the mode s₂ and relating it to the class of chaotic attractors. Chaotic oscillations in the channel named the random ones were known in earlier works as well. Along with P_0 increase, their amplitude grows that was earlier explained by the increase of perturbations with average velocity approaching in reverse flow to local sound velocity. However, for the mode s₂ such explanation is not suitable since the calculation shows that the local reverse flow in this case is significantly subsonic. So the question arises as to whether oscillations s₁ and s₂ have random character and are similar to “white noise”. For verification of this hypothesis, autocorrelation function was calculated on time series $P_b(t)$ obtained during the targeted experiment where P_0 remained constant. If oscillations s₁₋₂ are random then values of this function shall be equal to zero. They changed in wide limits [0, 1]. Hence, the mode of low-frequency oscillations is preceded by the determinate chaos from which the transient oscillating process TP₂ with increasing amplitude is formed.

All modes found by now are present in Fig. 10. Each mode in this figure is designated by the combination of letters, for instance, SM means “self-similar mode”.

Classification of flow regimes with sudden expansion

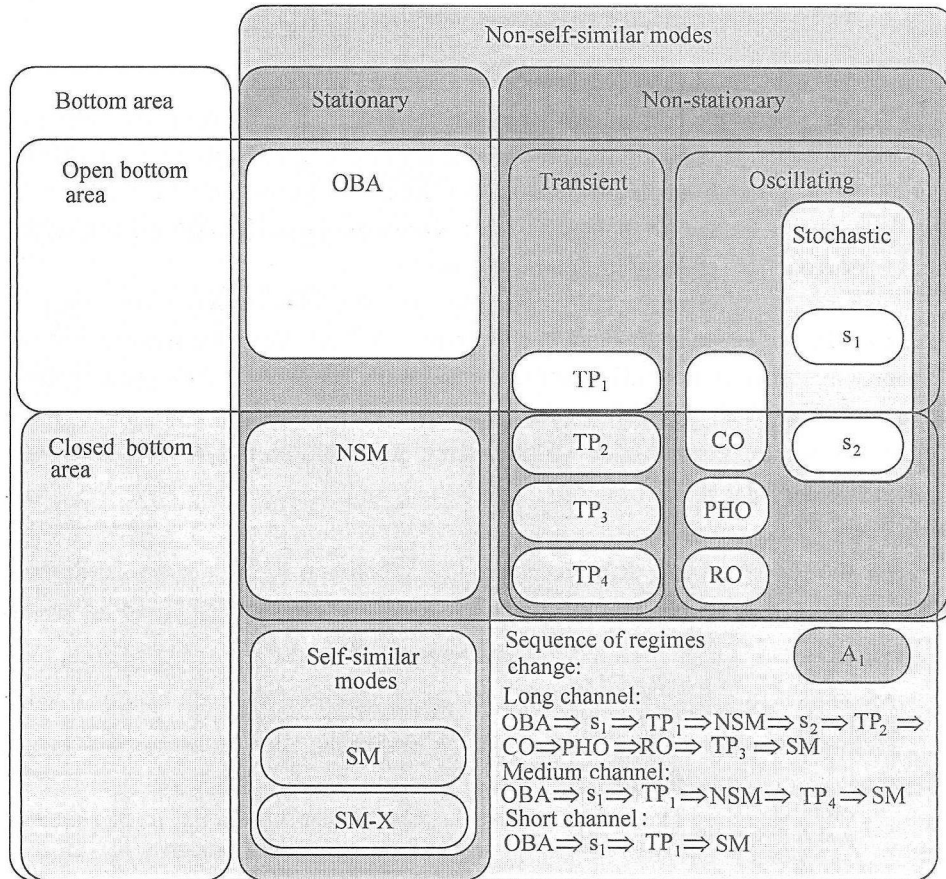


Fig. 10. Classification of flow modes with sudden expansion.

The class of specific mode may be determined in Fig. 10 on the designated area inside which it lies. For instance, TP_1 is a non-self-similar nonstationary transient mode with open bottom area (in the intersection of OBA, "Nonstationary", "Open bottom area"). NSM is non-self-similar stationary mode with closed bottom area (lies in the intersection of NSM areas, "Stationary", "Closed bottom area"). RO is non-self-similar, nonstationary, oscillating mode with the closed bottom area, etc.

Conclusions

On the basis of the performed systematic experiments, theoretical investigations and calculations main modes of separation flow including transient and oscillating ones in the channel with sudden expansion have been found and classified; the physical factors determinant for such flows have been explained. This classification of the modes explains regularities of the bottom pressure change, the order of the changing modes, and realized gas dynamic structures of such flows at gas parameters' change in front of the nozzle and serves a basis for further analysis of amplitude-frequency characteristics of arising oscillating processes and hysteresis phenomena in the channel.

The developed semi-empirical method of bottom pressure calculation, the performed analysis of the jet in the phase space $\zeta - P_b$, and series of computing experiments in combination allowed significantly widening understanding on the modes of flows with sudden expansion compared with the earlier purely experimental works [12].

At the same time, a number of mechanisms of nonstationary mode change require further investigation. In particular, it is not completely clear whether pseudoharmonic and relaxation oscillations are manifestations of the same physical process or if these are independent phenomena. The developed formal classification predicts a number of new nonstationary processes such as transformation of a fine structure of oscillating processes, superposition of background chaotic oscillations on main low-frequency cycle, intermittency of the first and second type, and transient processes in the form of direct and inverse subharmonic cascades. They all require additional investigation and experimental verification.

The obtained data for the channel with the set single Laval nozzle may be used as a basis for the explanation of flow properties in more complex devices such as nozzle blocks, gas dynamic lasers, flow-through chambers with supersonic combustion, etc.

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