Improvement of the Earth rotation theory: recent advances and prospects

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1. Introduction

- Improving the accuracy of the observation and theoretical modelling of the Earth rotation is an urgent and extremely important need.

- It is required for many practical purposes, as well as for improving our knowledge of the changing Earth.

- Some of the reasons are:

- The so-called Earth Orientation Parameters (EOP) provide the link between the “International Terrestrial Reference Frame” (ITRF) and the “International Celestial Reference Frame” (ICRF).

- Since years ago, the realization of the ITRF is strongly dependent on the EOP determination. Therefore, the accuracies of ITRF and EOP limit each other

- The monitoring of the geodetic signals of global change, e.g. the observed sea level trends, imposes very stringent requirements to the accuracy and stability of the reference frames and thus to the their mutual orientation – the EOP.

1. Introduction

- The frame accuracy goals pursued by GGOS, the Global Geodetic Observing System of the International Association of Geodesy (IAG) are:
    - *1 mm in position*
    - *0.1 mm/yr in stability*

- They correspond roughly to 30 µas (micro arc seconds) and 3 µas/yr, in terms of Earth rotation - geocentric angles and angular velocities.

- Those values are well above the actual accuracies (or precision / repeatability) of the EOP determinations and predictions according to the last IERS Annual Reports and of the existing Earth rotation theories.

- Therefore, the improvement of Earth rotation theories to achieve the required levels of accuracy and consistency is a very difficult challenge.

- In this context, the International Astronomical Union (IAU) and the IAG approved the creation of an IAU/IAG Joint Working Group on Theory of Earth Rotation (JWG_ThER), with the purpose of promoting the needed advances.
All the authors are members of such working group, chaired by the first.

2. On the IAU/IAG JWG_ThER

❑ The structure of this JWG is rather complex, since it follows the characteristics of the current set of 5 EOP as well as the fields of specialization of researchers.

❑ The people in charge are:

- **Chair**: Jose M. Ferrándiz (mainly IAU)
- **Vice-Chair**: Richard Gross (mainly IAG)

❑ The WG is structured in three Sub-Working Groups **(SWG):**

1. **Precession/Nutation** (Chair: Juan Getino)
2. **Polar Motion and UT1** (Chair: Aleksander Brzezinski)
3. **Numerical Solutions and Validation** (Chair: Robert Heinkelmann)

❑ These SWG should work in parallel for the sake of efficiency and they should be linked together as closely as the needs of consistency demand.

❑ The group is intended to operate during to terms.

❑ Information about its activity, including presentations and reports, is available at the working group web site:

http://web.ua.es/en/wgther

2. IAU/IAG JWG_ThER and our contributions

- The Desired Outcomes of the JWG ThER are:

1. Contribute to improving the accuracy of precession-nutation and ERP theoretical models by proposing both new models and corrections to existing models.
2. Clarify the issue of consistency among conventional ERP, their definitions in various theoretical approaches, and their practical determination.
3. Establish guidelines or requirements for future theoretical developments with improved accuracy.

- **In the rest of the paper we selected a few of the results contributed by the authors within this framework**, related to
  - corrections of nutation models reaching some tens of µas: ***effect of the rotation of the solid inner core*** (Escapa et al 2012), not included in conventional theories due to its simplification of the Earth model
  - ***deficiencies in the theoretical foundations of nutation theory IAU2000*** found recently (Ferrándiz et al 2015)
  - ***second order additional precession terms*** appearing in Earth models with a ***fluid core*** (Ferrándiz et al 2004, Baenas et al 2014))

3.1 Direct effects of the rotation of the solid inner core (SIC)

- The **usual solutions for the rotation of three-layer Earth** models are partially inconsistent, since they consider a kind of **“virtual” motion of SIC**:
  It only ***affects the frequencies of free oscillation but does not produces any variations of the Earth’s tensor of inertia*** and thus its gravitational potential. Therefore, the usually accounted effects can be considered somehow as “indirect”.

- Even in a simplified non–elastic situation, the mass redistribution caused by the inner core rotation affects the nutations beyond the usual “indirect” effect gathered in the transfer (or generating) functions

- Using the Hamiltonian formalism, **Escapa et al (2012) derived the analytical direct effects on the motion of the figure axis due to the mass redistribution caused by the SIC differential rotation**.

- Some **amplitudes** of the resulting direct nutation terms are not negligible, since they **reach the magnitude of some tens μas**.

- They are thus of the same order than other second order effects on nutation and below the accuracy target of GGOS

3.2 Direct effects of the rotation of the inner core: Results

- In the following table the amplitudes larger than 10 μas are marked in red
- The **largest** contributions has a **semi-annual** period

Argument					Period	Figure axis (μas)	
l_M	l_S	F	D	$\overline{\Omega}$	Days	$\Delta\psi$	$\Delta\varepsilon$
+0	+0	+0	+0	+1	-6793.48	2.79	-0.31
+0	+0	+0	+0	+2	-3396.74	0.00	-0.01
+0	+1	+0	+0	+0	365.26	14.95	9.29
+0	-1	+2	-2	+2	365.25	-1.78	0.48
+0	+0	+2	-2	+2	182.63	44.61	-19.92
+0	+1	+2	-2	+2	121.75	1.64	-0.72
+1	+0	+0	+0	+0	27.55	-2.22	0.02
+0	+0	+2	+0	+2	13.66	7.17	-3.08
+0	+0	+2	+0	+1	13.63	1.22	-0.63
+1	+0	+2	+0	+2	9.13	0.96	-0.41

4.1 Reference Frames in Earth Rotation Theories

- The EOP determined from observations give the relative orientation of the conventional terrestrial and celestial frames, ITRF and ICRF

- However, nutation theories provide the orientation of a hypothetic terrestrial reference system (TRS) with respect to an assumed inertial frame

- The TRS in IAU2000 is the 'i-system' introduced by Mathews et al (1991), defined by the following 3 properties (Shapiro and Mathews 1992)

  - *it is a **"crust-fixed"** reference frame – required since observatories are fixed to the Earth crust*

  - *it is a **Tisserand mean axis** system – required since axes are not solely defined in non-rigid bodies*

  - *it is a "body-fixed**" principal axis** system - required to align their axes with the axes of the underlying rigid Earth theory (REN2000), since MHB2000 uses a transfer function approach*

- The three conditions are not strictly compatible, but the authors claimed they can be satisfied up to the *O(me) accuracy* order – *m being the amplitude of polar axis motion, e the Earth's ellipticity*

4.2 TRF in IAU2000 theory versus ITRF

- IAU200A is used to model the observed nutation offsets dX, dY, referred to ITRF and ICRF. Therefore, it must be clarified whether their associated terrestrial frames can be identified or not, within the present accuracy limits.

- Ferrándiz et al (2015, EGU) studied the motion of the principal axes of inertia with respect to the ITRF. The analysis was based on:

  - the CSR/UT monthly values of the normalized geopotential coefficients of degree 2, C_{2m} and S_{2m} derived mainly by SLR analysis in the period 2011-2015 (RL05 solution, documented in Cheng et al 2011).

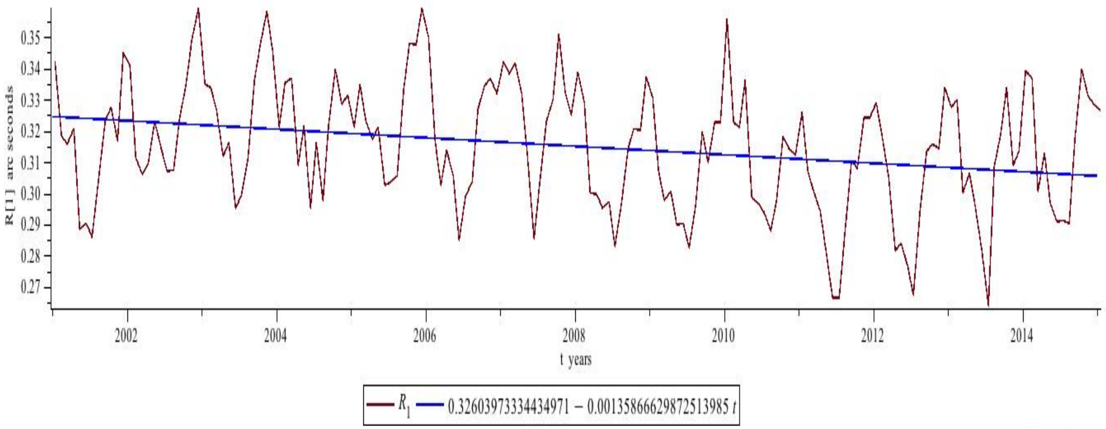
  - The analytical expressions of the Cartesian coordinates of the poles of the principal axes $O\xi$, $O\eta$, $O\zeta$ resulting from small variations of the inertia tensor derived by Barkin and Ferrándiz (2000, AAT)

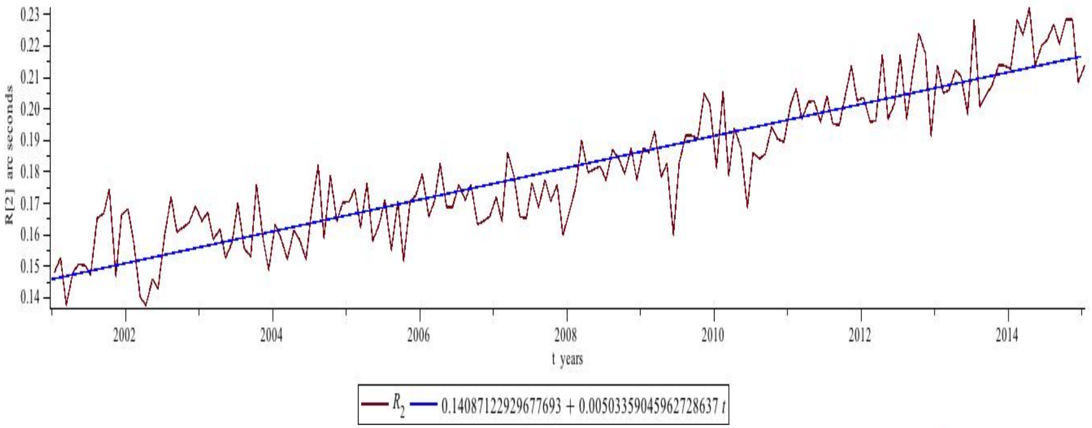
- The transformation was expressed by means of 3 infinitesimal rotations, from the ITRF to the determined principal axes, given up the first order by:

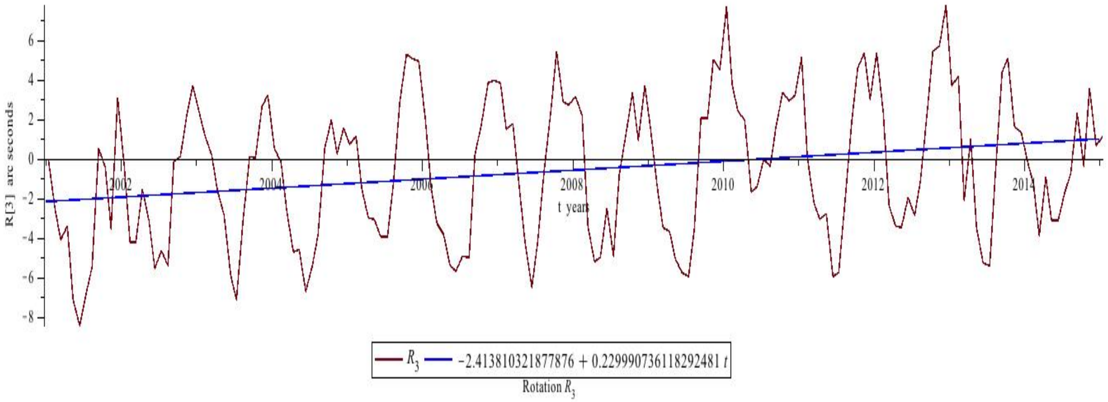
$$R_1 = -\frac{\Delta S_{21}}{2C_{22}^0 + C_{20}^0}, R_2 = -\frac{\Delta C_{21}}{2C_{22}^0 - C_{20}^0}, R_3 = \frac{1}{2}\frac{\Delta S_{22}}{C_{22}^0}$$

- ***The next slide shows that the ideal Tisserand, principal mean axes assumed in IAU2000 drift away the ITRF***.

4.3 Results







Units: arc seconds & years.

Vertical scales are different.

- **R1** is clockwise, moving the positive Oz axis towards Oy with a velocity of **-1.36 ± 0.37 mas/yr** (roughly -4 cm/yr on the Earth's surface)
- **R2** is counter clockwise, bringing the Oz axis towards Ox with velocity **5.03 ± 0.18 mas/yr** (about 15 cm/yr on the surface)
- **R3** drifts the Ox axis Eastwards, with a higher velocity of **230 ± 36 mas/yr** (7 m/yr on Earth's equator) - *since the main term in the denominator is C_{22} instead of C_{20}*

5.1 Second order effects on precession in the rigid case

- Kinoshita (1977) derived the first accurate rigid Earth rotation solution using the Hamiltonian approach and the Hori's perturbation method. The solution was improved together with Souchay (1990) and then other co-authors, the final release being REN2000.

- The original Hamiltonian H is transformed into a new one K, free of periodic terms. Both Hamiltonians and the generating function W are expressed as sum of terms of different orders of perturbation

$$H = H_0 + H_1 + H_2 + \cdots$$
$$\Downarrow \qquad W = W_1 + W_2 + W_3 + \ldots$$
$$K = K_0 + K_1 + K_2 + \cdots$$

- The second order contribution to precession comes from the secular term $K_2 = \frac{1}{2}\{H_1, W_1\}_{sec}$

- The value of the second order effect on the precession in longitude is - 46.8 mas/cy (K&S 1990, reported by Williams 1994)

- The Hamiltonian method was used for other authors to treat the non-rigid Earth (Kubo and mostly Getino and Ferrándiz, who derived the solution GF2000 for a three layers Earth model)

5.2 Second order effects on precession due to the Fluid Core

- Ferrándiz et al (AJ 2004) first proved that the fluid core gives rise to not negligible second orders effects on the precession in longitude, cast as:

$$\Delta\left(\frac{d\psi}{dt}\right) = \frac{1}{M\sin I}\frac{\partial K_2}{\partial I} = dp_0 + dp_{10} + dp_{11}.$$

- The three dp_x terms above are originated by zonal, sectorial and tesseral Hamiltonian terms (long-period, diurnal and semidiurnal bands)

- Alike observational errors, non accounted corrections to the precession induces changes in the value of the dynamical ellipticity ***H*.**

- The next table displays the total second order effects and the equivalent relative variation of H – after the already accounted rigid contribution is subtracted

Contribution to precession (mas/cy units)					
Model	dp_0	dp_{10}	dp_{11}	Total	ΔH ppm
Rigid	-45.3641	-.3979	-.2976	-46.0596	–
Poincaré	-45.3641	-22.5908	-1.2625	-69.2174	4.8
Poincaré + Elas. Man.	-45.3641	-12.5195	-.7433	-58.6269	2.5
Net effect (new)				-12.5673	

5.3 Indirect effects on nutations

❑ They come from **the change** in the value of the **dynamical ellipticity** ***H*. Indirect effects are larger** than the differences with respect to the rigid case of the direct effects

Argument					Period	Figure axis (μas)	
l_M	l_S	F	D	$\overline{\Omega}$	Days	$\Delta\psi$	$\Delta\varepsilon$
+0	+0	+0	+0	+1	-6793.48	-44.64	23.93
+0	+0	2	-2	2	-182.62	-3.42	1.49

6. Conclusions

❑ We have shown that there are many evidences supporting the need of carrying out a thoughtful revision of the Earth rotation theories along the next years, in order to fulfil the stringent accuracy requirements of IAG/GGOS (at the millimetre level) adopted by the IAU/IAG JWG ThER

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